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An experimental study of costly coordination

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Abstract

This paper reports data for coordination game experiments with random matching. The experimental design is based on changes in an effort-cost parameter, which do not alter the set of Nash equilibria nor do they alter the predictions of adjustment theories based on imitation or best response dynamics. As expected, however, increasing the effort cost lowers effort levels. Maximization of a stochastic potential function, a concept that generalizes risk dominance to continuous games, predicts this reduction in efforts. An error parameter estimated from initial two-person, minimum-effort games is used to predict behavior in other three-person coordination games.

JEL classification: C72; C92

Keywords: Coordination games; Stochastic potential; Logit equilibrium; Out-of-sample prediction

1. Introduction

After the prisoner's dilemma, the coordination game is perhaps the most widely discussed paradigm in game theory. Interest in coordination games stems from the presence of multiple Nash equilibria that can be Pareto ranked, which raises the possibility of "getting stuck" in an outcome that is undesirable for all players. For this reason, this

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class of games is of interest to macroeconomists (Bryant, 1983; Cooper and John, 1988; Romer, 1996). Since (generically) all equilibria are strict, standard refinements leave the set of Nash equilibria unchanged, which has prompted game theorists to search for new selection criteria. An array of alternative theories of behavior in coordination games have been put forward, both static and dynamic.[1]

Some theorists argue that coordination game experiments are useless for game theory because the Nash equilibrium and its refinements have no predictive power in this case and, as a consequence, "anything goes." We feel that the opposite is true: the unexpected empirical regularities observed in coordination experiments (such as the ones reported in this paper) can guide further theoretical work. For instance, previous experiments have shown that coordination problems cannot be ruled out by an assumption that agents somehow find the Pareto-dominant equilibrium. Indeed, some of the most widely cited results from laboratory experiments provide cases where subjects end up at the Nash equilibrium that is *worst* for all concerned (Van Huyck et al., 1990; Cooper et al., 1992; and the survey of Ochs, 1995). Since much of the theoretical work was motivated by the need to explain coordination failures in the laboratory, it is now time to return to the laboratory and carry out experiments designed explicitly to evaluate some of these theories.

This paper reports the results of several new coordination experiments. The first game to be considered is one in which pairs of subjects choose an effort level, and the resulting payoff is the *minimum* of the efforts minus the cost of one's own effort. This payoff structure can arise from a joint production process in which the group output is proportional to the minimum of the individual inputs, as is the case with perfect complementarity. The different treatments are based on a change in the common cost per unit of effort. As long as this cost is less than one, the best response to any set of others' efforts is just the minimum of those efforts, so (non-critical) changes in the cost of effort will not alter the set of Nash equilibria in pure strategies, nor will they change the predictions of any dynamic theory that is based on adjustment toward the best response to efforts observed in the previous period. Changes in the cost of effort do affect the relative costs of "errors" in overshooting or undershooting the minimum of other's efforts, so theories like risk dominance and maximum stochastic potential (discussed below) that take into account the costs of errors will be sensitive to the effort cost parameter.

The qualitative predictions that follow from maximizing the stochastic potential are supported by this first experiment. The data are used to estimate the "noise" parameter of the model, which is then used for out-of-sample prediction in six new sessions with three-person games. These sessions include both minimum-effort and median-effort coordination games.

The paper is organized as follows: the theoretical motivation for the experimental design is discussed in more detail in Section 2, and laboratory results for two- and three-person

[1] Static approaches include Pareto dominance (Harsanyi and Selten, 1988), risk dominance (Harsanyi, 1995; Carlsson and van Damme, 1993), and "noisy" equilibrium models (Anderson et al., 2001; Carlsson and Ganslandt, 1998). Dynamic models of coordination behavior can be roughly divided into evolutionary models (Kandori et al., 1993; Young, 1993; Crawford, 1991; Anderson et al., 2004), adaptive learning models (Crawford, 1995; Van Huyck et al., 1997), and "noisy" learning models (Battalio et al., 2001; Camerer and Ho, 1999).

games are presented in Sections 3 and 4, respectively. Section 5 describes the estimation of the equilibrium model, and the final section concludes.

2. Pareto dominance, risk dominance, and maximum stochastic potential

The experiment involves a series of single-period coordination games with groups of randomly matched subjects who make independent "effort" choices. Efforts were restricted to a continuous interval $[\underline{e}, \bar{e}]$, i.e. fractional efforts were allowed. Let player i's effort be denoted by $e_i \in [\underline{e}, \bar{e}]$, $i = 1, \ldots, n$. The payoffs for a symmetric, n-person minimum-effort game are:

$$\pi_i(e_1, \ldots, e_n) = \min\{e_1, \ldots, e_n\} - ce_i, \quad i = 1, \ldots, n, \tag{1}$$

where c is the effort cost. As long as c is less than 1, payoffs are maximized when all players choose the highest possible effort. Note, however, that *any* common effort level constitutes a Nash equilibrium, since a costly unilateral increase in effort will not raise the minimum, and a unilateral decrease will reduce the minimum by more than the cost when $c < 1$. This argument does not depend on the number of players, so non-critical changes in c and n will not alter the set of Nash equilibria in pure strategies, despite the reasonable expectation that efforts should be high for sufficiently low effort costs and low numbers of participants.[2]

The theoretical construct most commonly used to "select" an equilibrium in 2×2 coordination games is Harsanyi and Selten's (1988) notion of risk dominance. One appealing feature of risk dominance is its sensitivity to cost that determines the losses associated with deviations from best responses to others' decisions. To illustrate the concept of risk dominance, consider the two-person minimum-effort game shown in Table 1 in which efforts are constrained to be the integers 1 or 2. When both players are choosing efforts of 1, the cost of a unilateral deviation to 2 is just the cost of the extra effort, c, which will be referred to as the "deviation loss." Similarly, the deviation loss at the $(2, 2)$ equilibrium is $1 - c$, since a unilateral reduction in effort reduces the minimum by 1 but saves the marginal effort cost c. The deviation loss from the low-effort equilibrium is greater than that from the high-effort equilibrium if $c > 1 - c$, or equivalently, if $c > 1/2$, in which case we say that the low-effort equilibrium is risk dominant.[3] Risk dominance, therefore, has the desirable property that it selects the low-effort outcome if the cost of effort is sufficiently high.[4]

There is, however, no consensus on how to generalize risk dominance for games with more players, a continuum of decisions, etc. A related concept that does generalize is the

[2] Anderson et al. (2001) show that there are a continuum of (two-point) mixed Nash equilibria, but that each of these has the perverse comparative statics property that an increase in the effort cost will *raise* the probability associated with the higher of the two effort levels over which randomization occurs.

[3] The application of risk dominance for asymmetric two-person games is equivalent to comparing the product of the two players' deviations losses at each equilibrium.

[4] Laboratory experiments based on 2×2 coordination games show that the risk dominant outcome may have a lot of drawing power even though play usually starts out near the Pareto-dominant equilibrium (see e.g. Straub, 1995).

Table 1
A 2×2 coordination game

		Player 2's effort	
		1	2
Player 1's effort	1	$1-c,\ \ 1-c$	$1-c,\ \ 1-2c$
	2	$1-2c, 1-c$	$2-2c, 2-2c$

notion of maximization of a "potential" of a game.[5] Loosely speaking, the idea behind potential is to find a function that is maximized by a Nash equilibrium for the game. More precisely, a potential function for a game is a function of all players' decisions with partial derivatives that match those of individual players' payoffs with respect to their own decisions. The class of games that admit a potential function includes some well-known and interesting examples.[6] For instance, it is straightforward to show that the potential function for the 2×2 coordination game in Table 1 is given by: $V = p_1 p_2 - (1-c)(p_1 + p_2)$, where p_i denotes the probability with which player i chooses the low effort 1.[7] Hence, the potential is maximized in the low-effort outcome ($p_1 = p_2 = 1$) when $c > 1/2$ and it is maximized in the high-effort outcome ($p_1 = p_2 = 0$) when $c < 1/2$. For the 2×2 coordination game shown in Table 1, risk-dominance and maximum potential thus coincide, and this equivalence holds more generally for any symmetric 2×2 game.

In contrast to risk-dominance, the notion of maximum potential can be generalized to more general settings. For instance, for the n-player minimum effort game given in (1), the potential function is simply the common production function that determines a *single* player's payoff, minus the sum of *all* players' effort costs:

$$V(e_1, \ldots, e_n) = \min\{e_1, \ldots, e_n\} - c \sum_{i=1}^{n} e_i. \tag{2}$$

The inclusion of all effort costs is needed to ensure that $\partial V_i / \partial e_i = \partial \pi_i / \partial e_i$, $i = 1, \ldots, n$, for all feasible vectors of decisions, when these derivatives exist. The maximization of potential will obviously require equal effort levels. At any common effort, e, the potential in (2) becomes: $V = e - nce$, which is maximized at the lowest effort when $nc > 1$, and is maximized at the highest effort when $nc < 1$. In two-person games, this condition reduces to the risk dominance comparison of c with $1/2$. Hence, the Nash equilibrium that maximizes potential in this game is sensitive to parameters that may affect actual behavior.

The notion of maximum potential can be used to evaluate results from previous coordination-game experiments.[8] However, laboratory data often show some amount of

[5] Rosenthal (1973) first used a potential function to study properties of a Nash equilibrium. Monderer and Shapley (1996) provide a general treatment.

[6] For instance, all 2×2 games admit a potential function, as do some versions of public goods games, Cournot oligopoly games, etc. See Monderer and Shapley (1996) and Anderson et al. (2004) for further examples.

[7] Player i's payoff of choosing the low effort with probability p_i is: $\pi_i(p_i, p_j) = p_i p_j - p_i(1-c) - p_j + (2-2c)$, and it is straightforward to check that $\partial V_i / \partial p_i = \partial \pi_i / \partial p_i$ for $i = 1, 2$.

[8] The most widely cited coordination experiment is that of Van Huyck et al. (1990), who conducted games with 14 to 16 players and an effort cost of either 0 or $1/2$, so nc was either zero or about seven. Compared to the

"randomness," which cannot be explained by any deterministic theory of this type. We will use the idea of maximum "stochastic potential" and derive its implications for the parameters of the experiments reported below. We also explain its relationship with the logit quantal response equilibrium (McKelvey and Palfrey, 1995).

If a game has a potential function V, the stochastic potential is based on a consideration of probability distributions of decisions that determine the expected value of potential (Anderson et al., 2001). In particular, the stochastic potential for given distributions of players' decisions is the expected value of the ordinary potential, denoted $E\{V\}$, plus terms that will make the maximand sensitive to noise in the choice distributions. These terms that determine the value of dispersion correspond to the physical concept of "entropy." In the case of a continuous density function, $f_i(e_i)$, entropy is defined as $-\int f_i \ln(f_i)\,\mathrm{d}e_i$. The entropy for the system is the sum of the entropy terms for individual players' distributions, weighted by an error parameter μ. Thus the stochastic potential is: $E\{V\} - \mu\sum_i \int f_i \ln(f_i)\,\mathrm{d}e_i$, where the sum is over all player indices and the integral is over the range of feasible effort choices. Since entropy is maximized by complete randomness (a uniform distribution of decisions), the distribution that maximizes expected potential plus μ times entropy will be more dispersed as the error parameter increases. In the other limit as $\mu \to 0$, the entropy term becomes irrelevant and the maximization of stochastic potential becomes equivalent to the maximization of ordinary potential, which leads to a Nash equilibrium in this context. Thus the maximization of stochastic potential provides a generalization of Nash that is parameterized by an error parameter μ.

For the case of a two-player minimum effort game, the expected value of the potential function in (2) contains a term that is the expected value of the minimum of two decisions. If player i uses a continuous choice density $f_i(e_i)$, with corresponding distribution function $F_i(e_i)$, then the distribution function for the minimum of the two efforts is: $1-(1-F_1(e_1))(1-F_2(e_2))$. The stochastic potential, V_S, is calculated by adding weighted entropy terms to the expected value of the minimum and subtracting the expected effort costs:[9]

$$V_S = \int_{\underline{e}}^{\bar{e}} \prod_{i=1}^{2}\bigl(1-F_i(e)\bigr)\,\mathrm{d}e - c\sum_{i=1}^{2}\int_{\underline{e}}^{\bar{e}}\bigl(1-F_i(e)\bigr)\,\mathrm{d}e - \mu\sum_{i=1}^{2}\int_{\underline{e}}^{\bar{e}} f_i(e)\log\bigl(f_i(e)\bigr)\,\mathrm{d}e. \tag{3}$$

critical nc value of 1, these parameter choices appear rather extreme, which may explain why their data exhibit a huge shift in effort decisions. By the last round in the experiments in which $nc = 0$, almost all (96%) participants chose the highest possible effort, while over three-quarters chose the lowest possible effort when nc was around seven. One purpose of Van Huyck et al.'s experiment was to show that a Pareto-inferior outcome may arise in coordination games. Other experiments were conducted with 2 players, but the payoff parameters were such that nc exactly equaled the critical value 1, and, with a random matching protocol, the data showed a lot of variability.

[9] Recall that the expected value of a random variable with distribution function F can be written as the integral of $1-F$ (ignoring possible boundary terms that are independent of F).

Anderson et al. (2001) show that maximization of the stochastic potential requires symmetry across players, i.e. $F_1(e) = F_2(e) = F(e)$, as is the case without noise. Maximization of the stochastic potential with respect to the common distribution $F(e)$ is a straightforward calculus-of-variations problem, and the necessary condition can be expressed:[10]

$$\mu f'(e) = f(e)\bigl(1 - F(e) - c\bigr), \tag{4}$$

which is a differential equation in the common distribution function. Consider the intuition behind (4). If the other player is using an effort distribution, $F(e)$, then an increase in effort at e will raise the minimum with probability $1 - F(e)$ and increase the cost at a rate c, so the $1 - F(e) - c$ term is the derivative of the expected payoff with respect to one's own effort. It follows that (4) can also be written as a differential equation in the equilibrium density function: $\mu f'(e) = \pi^{e\prime}(e) f(e)$, which defines the continuous version of the "logit equilibrium" (McKelvey and Palfrey, 1995).[11] Anderson et al. (2001) show that a solution to (4) exists, is unique, and that an increase in the effort cost lowers efforts in the sense of first-degree stochastic dominance. Thus the prediction of this stochastic-potential approach is consistent with the intuitive notion that reductions in the effort cost will increase efforts, although not necessarily all the way to the maximum possible effort.

One final issue is how (boundedly rational) subjects are supposed to find the maximum of stochastic potential. In Anderson et al. (2004), we specify a continuous gradient-based adjustment process with Brownian motion, i.e. the time rate of change in a player's decision is equal to the slope of the expected payoff function (locally) plus a continuous-time normal random error. We show that any stable steady state of this process produces a distribution of decisions that maximizes stochastic potential (i.e. produces a logit equilibrium).[12]

3. Laboratory results for the two-person minimum effort game

Contrary to the continuous nature of most real-world effort decisions, most coordination experiments conducted to date involved only a few possible effort choices. To gain realism, and to reduce the possibility of "extreme" behavior (i.e. boundary decisions), subjects in the experiments reported here choose from a *continuous* interval: [110, 170]. We chose this

[10] Recall that the Euler condition for maximizing $\int I(F, f, x)\,\mathrm{d}x$ is: $\delta I/\delta F = \partial I/\partial F - \mathrm{d}/\mathrm{d}x\{\partial I/\partial f\} = 0$, or in the present context: $-2(1-F) + 2c + 2\,\mathrm{d}/\mathrm{d}x\{\mu + \mu \ln f\} = -2(1-F-c) + 2\mu f'/f = 0$, where f' denotes the derivative of the density function. This result can be rearranged to obtain the expression in (4).

[11] The correspondence between this differential equation and the logit equilibrium can be seen by integration to express the density $f(e)$ as proportional to $\exp(\pi^e(e)/\mu)$.

[12] The concepts of logit equilibrium and maximization of stochastic potential differ in a subtle way. In particular, the variational condition in (4) is a first-order condition, and therefore, a logit equilibrium may be a local minimum of the stochastic potential. Anderson et al. (2004) show that local minima are unstable for a dynamic gradient-based adjustment process with Brownian motion, whereas local maxima are stable. Since we have proved that the logit equilibrium is unique for this game (Anderson et al., 2001), it is globally stable for the evolutionary adjustment process. Incidentally, the noisy evolutionary adjustment process explains the symmetric adjustment patterns in Fig. 2 (see Goeree and Holt, 1999). It is worth noting that there is a one-to-one correspondence between logit equilibria and extreme points of stochastic potential, whereas this equivalence does not hold for the twin concepts of the Nash equilibrium and (deterministic) potential. For example, there is a continuum of Nash equilibria for the coordination game, but only one maximizes potential when $cn \neq 1/2$.

particular range with the objective of avoiding a highly focal number like 50 or 100, and we did not want 150 to be at the midpoint of the range. (Although focalness plays no role in our theory, we believe that it can be important, especially in coordination games.)[13] Given the knife-edge properties of $c = 1/2$ for two-person coordination games, we conducted one treatment with $c = 1/4$ and another with $c = 3/4$. As noted above, this change does not alter the predictions of theories based on best responses to others' decisions, e.g. pure-strategy Nash equilibria.

The experimental design involved six sessions, each with 10 student subjects recruited from undergraduate economics classes at the University of Virginia. No subject had previously participated in a coordination game. Upon arrival, participants were seated in visually isolated booths. We began by reading the instructions.[14] The payoffs were explained in words and with symbols, e.g.: "you will receive a penny amount that equals the minimum of the two efforts chosen, minus the cost of your own effort, which is 0.25 times your own effort choice." There were no numerical examples in the instructions, in order to avoid focal suggestions. Questions were asked and answered privately to avoid suggestive statements.

Subjects were told that there would be 10 periods of random pairings. At the start of each period, subjects were prompted to choose effort levels. Effort choices were restricted to the interval [110, 170], with fractional efforts allowed, which they could select by using decimal points. Subjects were then randomly matched and each person was informed privately about own earnings and the "other person's decision." The process took about one hour.[15]

Three sessions were conducted under the high-cost treatment ($c = 3/4$) and three under the low-cost treatment ($c = 1/4$).[16] The period-by-period averages for each session are shown as thin lines in Fig. 1, and the averages for all sessions in each treatment are shown as thick lines. The data exhibit a couple of interesting features. First, the averages of all sessions begin near the midpoint of the range of feasible effort choices on the vertical axis.

Figure 2 shows the histograms of the effort decisions in the first and in the last three periods for the high-cost treatment (light) and low-cost treatment (dark). The null hypothesis that the initial distributions are equal cannot be rejected at the 5 percent level using a standard Kolmogorov–Smirnov test.[17] Second, even though all sessions start out similarly, a clear separation is apparent by the fifth period. For later periods, the null hypothesis

[13] Furthermore, this choice facilitates the comparison of our results with those of Van Huyck et al. (1990) who let subjects choose integer effort levels that ranged from 1 to 7.

[14] See http://www.people.virginia.edu/~cah2k/datapage.html for the instructions and data.

[15] The instructions stated that the 10 periods of random matching would be followed by "a different experiment." In fact, these two-person coordination games were followed by a series of 6–9 one-period games of chicken, matching pennies, etc.

[16] This created somewhat of a dilemma, since earnings are much lower under the high-cost treatment. We dealt with this issue by increasing the fixed payment from the customary level of \$6 to a level, \$12, that would ensure reasonable earnings for the first hour, even for the high-cost treatment. (The \$6 initial payment was used in session 1, with the low-effort-cost treatment, but the higher initial payment was used in all subsequent sessions.) Including the fixed payment, most subjects' earnings were in the \$7 to \$9 range in 0.25 treatment, and in the \$16 to \$18 range in the 0.75 treatment. These earnings were augmented in the one-period games that followed.

[17] Furthermore, comparing the empirical distribution functions with a uniform distribution results in a Kolmogorov–Smirnov statistic of 0.2 for both treatments, while the critical value is 0.22 for a sample size of 30 and a confidence level of 10 percent. Hence, the null hypothesis that the first-period empirical distributions are equal to a uniform distribution for the two treatments cannot be rejected at the 10% level.

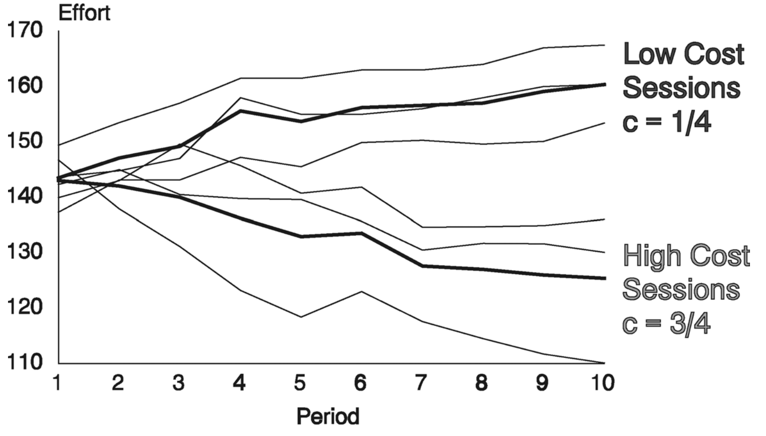

Fig. 1. A coordination game: average effort decisions by period. *Key*: Fine lines are session averages. Bold lines are averages across all sessions in a treatment.

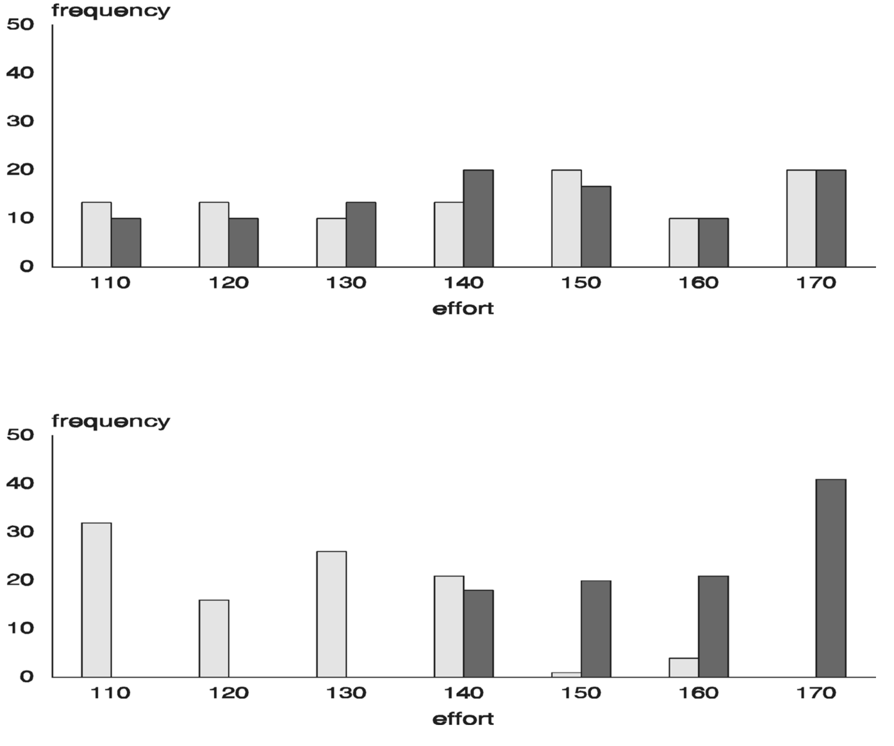

Fig. 2. Effort choice frequencies in period 1 (top) and periods 8–10 (bottom). *Key*: Light bars correspond to high effort cost, dark bars correspond to low effort cost.

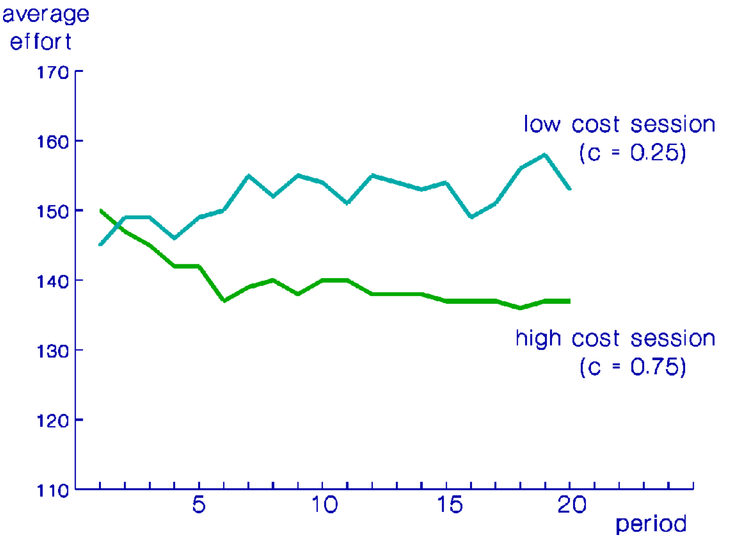

Fig. 3. Average efforts for two sessions with twenty periods.

of no treatment effect can be rejected at the 5 percent level of significance using a non-parametric test.[18] In the last three periods, all decisions in the low-cost sessions are above the midpoint (140), while almost all decisions are below the midpoint with a high cost, as shown in Fig. 2. Finally, the average effort trajectories seem to spread symmetrically around the midpoint: the upward trend for the three low-effort-cost sessions is reflected by an essentially symmetric downward trend for the three high-effort-cost sessions.

A casual impression that one might get from Fig. 1 is that the data have not fully converged by period 10. In particular, there may be some tendency for data averages to move closer to the boundaries with more periods. To test this conjecture, we ran two additional sessions that lasted for twenty periods. The procedures used for these new sessions were somewhat different because we used computerized interactions to save time. The average effort levels by period are shown in Fig. 3. Note that there is no tendency to move towards the boundaries after periods 6 to 7. If anything, these two sessions most resemble the least extreme sessions in Fig. 1 (i.e. the two sessions with data averages closest to the midpoint 140).

4. Minimum and median-effort experiments with three players

Past research on coordination games seems to suggest that two is the critical number of players for efficient coordination. For instance, in the minimum-effort coordination exper-

[18] The intuition behind the test is clear. There are "six-take-three" = 20 possible ways that the effort averages could have been ranked, and of these the most extreme ranking was observed, with all three low-c sessions having the highest ranks. The probability of this outcome under the null is, therefore, $1/20 = 0.05$.

iments of Knez and Camerer (1994), coordination gets steadily worse with larger groups and the biggest decrease in efficiency occurs when going from two to three players. Their explanation is that with more than two players, beliefs about others' behavior become ambiguous: while two players only have to worry about each others' beliefs about one another, the introduction of additional players forces everyone to think about beliefs opponents have about others.

Knez and Camerer's observation pertains to $c = 1/2$, in which case moving from two to three players is an important step. The concept of maximizing potential generalizes their intuition to group sizes larger than two: depending on the value of nc, average effort levels may be either low or high with two players (see Fig. 1), and the same is true for three (or more) players. To test this prediction, we ran two new minimum-effort coordination sessions, now with cohorts of twelve subjects being randomly matched in groups of three. The effort-cost was $1/2$ in the high-cost treatment and $1/10$ and in the low-cost treatment, so that nc is greater than 1 in the high-cost treatment and less than one in the low-cost treatment.[19] The period-by-period average effort levels for both treatments are shown in Fig. 4. Both sessions start out at the same level, which falls in the same range (between 140 and 150) as in the sessions with random pairings. As predicted, however, average effort levels in the high-cost session fall while average effort levels rise in the low-cost session.

Another characteristic of coordination experiments done to date is that when payoffs are determined by the *median* effort, the dynamics exhibit strong history-dependence: i.e. final outcomes are largely determined by initial play (Van Huyck et al., 1991). In all of the twelve sessions that they report, the median choice remained the same in each period and the final outcome was completely determined by first-period play. In addition, sub-

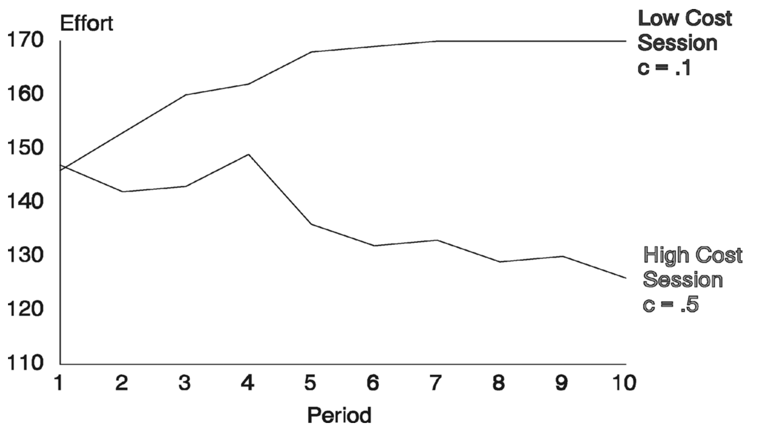

Fig. 4. A three-person minimum-effort coordination game: average effort decisions. *Key*: Averages by period for $c = 1/10$ (top) and $c = 1/2$ (bottom).

19 We would liked to have set $c = 1/6$ (instead of $c = 1/10$) in the low-cost treatment to preserve symmetry around $nc = 1$, but we felt that, since the experiments were done by hand, this would complicate payoff calculations too much and slow down the experiments (especially since effort choices could be any fractional amount).

jects' behavior showed little variation over time, in contrast with the adjustment patterns in minimum-effort games (see also Crawford, 1995).

The payoff structure of Van Huyck et al. (1991) differs from (1) in two ways. The minimum of all efforts is replaced by the median, and, more importantly, a cost is added that is quadratic in the distance between a player's effort and the median of all effort choices. The latter change may have an effect on behavior and could be part of the reason why the data show such strong history-dependence. We will consider a three-person median-effort coordination game with a payoff structure that is more closely related to (1) and that admits a potential. In particular, all three players receive the median, or middle, effort choice minus the cost of their own effort: $\pi_i(e_1, e_2, e_3) = \text{median}\{e_1, e_2, e_3\} - ce_i$, with c the effort-cost parameter.

As before, the potential function is simply the common production function that determines a *single* player's payoff, minus the sum of *all* players' effort costs:

$$V(e_1, \ldots, e_n) = \text{median}\{e_1, \ldots, e_n\} - c\sum_{i=1}^{n} e_i. \tag{5}$$

In contrast to the minimum-effort game, there is no symmetric pure-strategy equilibrium that maximizes (5): if all three players choose a common effort level, $e > \underline{e}$, one player could lower the total costs by deviating to the lowest possible effort $\underline{e}$ without affecting the median. Instead the (expected) potential is maximized by the Nash mixed-strategy equilibrium in which players pick the *lowest* effort, $\underline{e}$, with probability p^* and the highest effort, $\bar{e}$, with probability $1 - p^*$. A straightforward calculation shows that $p^* = (1 - (1 - 2c)^{1/2})/2$ for $c \leqslant 3/8$ and $p^* = 1$ otherwise.[20,21] Of course, introducing noise via the stochastic potential function yields less extreme and more continuous comparative statics. In the next section, we compare point predictions that follow from maximizing stochastic potential using a noise parameter estimated out-of-sample.

We conducted four sessions with median-effort-based payoffs, using effort-cost parameters of $c = 0.1$, $c = 0.4$ (2 sessions), and $c = 0.6$, respectively. Figure 5 shows the period-by-period averages for each treatment. Average efforts start at roughly the same level, but rise in the session with the lowest effort cost and fall in the one with the highest cost. The sessions with the intermediate effort cost have relatively flat trajectories, which is consistent with history-dependence, but the final effort levels for all sessions are inversely related

[20] When players choose $\underline{e}$ with probability p and $\bar{e}$ with probability $1 - p$, the potential is given by: $V(p) = p^3\underline{e}(1 - 3c) + 3p^2(1 - p)(\underline{e}(1 - 2c) + c\bar{e}) + 3p(1 - p)^2(\bar{e}(1 - 2c) + c\underline{e}) + (1 - p)^3\bar{e}(1 - 3c)$. The first-order condition for maximization yields a quadratic equation, $2p(1-p) = c$, that is solved by $p^* = (1-(1-2c)^{1/2})/2$. Note that when the others choose $\underline{e}$ with probability p^* and $\bar{e}$ with probability $1 - p^*$, a player's expected payoff of choosing an effort level, e, becomes: $\pi^e(e) = (p^*)^2\underline{e} + 2p^*(1 - p^*)e + (1 - p^*)^2\bar{e} - ce$, which is independent of e since p^* solves $2p^*(1 - p^*) = c$. So choosing $\underline{e}$ with probability p and $\bar{e}$ with probability $1 - p$, constitutes a symmetric mixed-strategy Nash equilibrium. The potential evaluated at p^* is $V(p^*) = (1 - 3c)(\bar{e} + \underline{e})/2 + (1 - 2c)^{3/2}(\bar{e} - \underline{e})/2$, which is only greater than or equal to $\underline{e}(1 - 3c)$ when $c \leqslant 3/8$. So the potential is maximized at $p^* = (1 - (1 - 2c)^{1/2})/2$ when $c \leqslant 3/8$ and at $p^* = 1$ when $c > 3/8$.

[21] This median-effort game has a continuum of asymmetric Pareto-ranked Nash equilibria in which two players choose a common effort level, e, and the third player chooses the lowest possible effort $\underline{e}$. Such asymmetric outcomes are unlikely to be observed when players are randomly matched and drawn from the same pool.

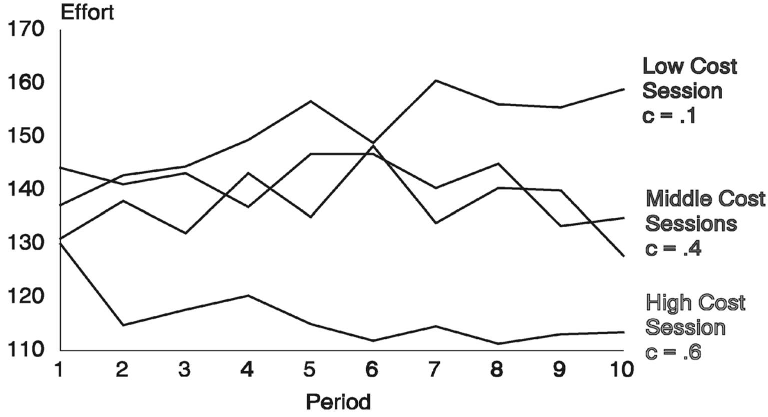

Fig. 5. A median-effort coordination game: average effort decisions. *Key*: Averages by period for $c = 0.1$ (top), $c = 0.4$ (middle), and $c = 0.6$ (bottom).

to c. For later periods, the null hypothesis that the product c has no treatment effect can be rejected at the 10 percent level of significance using a non-parametric test.[22]

5. Estimation of the logit equilibrium model

We used data from the two-person minimum-effort coordination-game experiment to estimate the equilibrium model in (4) directly, by dividing the interval [110, 170] into one-cent intervals and replacing the density function in (4) with probabilities.[23] Thus (4) becomes a set of simultaneous equations that determine the equilibrium probabilities for each effort level, and for a given value of μ the equations in (4) can be solved using numerical methods. The likelihood is the product of the calculated probabilities of the decisions actually observed and is maximized by iterating over μ. This yields an estimated value of $\mu = 7.4(0.5)$, with the standard error in parentheses.[24]

Table 2 summarizes the data and logit predictions for the two-person minimum effort experiment. It shows the average effort levels (standard deviations) in the final three periods by session and pooled over all three sessions in each treatment. For both values of the effort cost, two of the three session averages are within one standard deviation of the average predicted by the logit equilibrium that maximizes stochastic potential for the estimated $\mu = 7.4$. There are, however, unexplained differences between different sessions in the same treatment (cohort effects). Consider, for instance, session 2 of the high-cost treat-

[22] There are 24 possible ways that the effort averages could have been ranked, and of these only two rankings are as extreme as the one observed. The probability of this outcome under the null is, therefore, $2/24 = 0.09$.

[23] Clearly, the data show some systematic time patterns in the early periods, which is why we only used the last three periods to estimate the equilibrium value of μ.

[24] We did the estimation for sessions 1–6. We did not include session 7 and 8 (with 20 periods) in this estimation because of procedural differences and in particular computerization, which makes these sessions less comparable to the three-person game experiments.

Table 2
Average effort levels in periods 8–10 (standard deviations)

	First session	Second session	Third session	Pooled	Logit equilibrium
Low cost ($c = 1/4$)	151 (10)	166 (5)	159 (12)	159 (11)	154 (12)
High cost ($c = 3/4$)	131 (11)	112 (5)	135 (11)	126 (14)	126 (12)

ment, in which initial behavior in the first three periods is more extreme than in the other high-cost sessions. Subsequent effort choices are lower and gravitate towards the lower boundary 110, presumably because beliefs are more pessimistic. This history-dependence is not picked up by equilibrium models such as the one implied by maximizing stochastic potential. Nevertheless, the predictions that follow from maximizing stochastic potential are remarkably accurate when we aggregate the sessions in the same treatment. The averages for the two treatments end up at about 126 (14) for the high-cost treatment and at 159 (11) for the low-cost treatment, which is only slightly more extreme than the stochastic potential predictions of 126 (12) and 154 (12) based on the estimated error parameter.

Next, consider the three-person minimum effort game. In order to get an ex ante prediction for the average effort levels in the final periods, we shall use $\mu = 7.4$, which was estimated from the two-person experiment. The population density that maximizes the stochastic potential is characterized by the three-person generalization of Eq. (4):

$$\mu f'(e) = f(e)\big(\big(1 - F(e)\big)^2 - c\big). \tag{6}$$

Equation (6) can be derived as follows. Recall that, in general, the condition for stochastic potential maximization is given by the logit-equilibrium condition: $\mu f'(e) = \pi^{e\prime}(e) f(e)$. An increase in effort raises costs at a rate c and results in a higher minimum effort only if the others' efforts are higher, which occurs with probability $(1 - F)^2$. Hence marginal payoffs are: $\pi^{e\prime} = (1 - F)^2 - c$, which together with the logit condition yields (6). Using the estimated value of 7.4 for the error parameter, (6) can be solved numerically and the resulting predictions for the average effort levels are: 154 for $c = 1/10$ and 129 for $c = 1/2$, with a standard deviation of 8 in each case. The average efforts for the high-cost session end up quite close to the logit predictions. The low-cost session, however, provides an example of "lock-in dynamics": there is no more residual noise and behavior gets stuck at the upper boundary after period 7.

Finally, consider the three-person median game. Since subjects have no method of coordinating on asymmetric effort distributions when they are randomly matched and drawn from the same pool, it seems sensible to characterize the entire population of players by a common distribution function F, with corresponding density f. The marginal payoff function can be derived in the same manner as above. An increase in effort raises costs at a rate c and affects the median only if one of the other players is choosing a higher effort level and the other a lower effort level, which happens with probability $2F(1 - F)$. Hence, the condition for maximum stochastic potential becomes:

$$\mu f'(e) = f(e)\big(2F(e)\big(1 - F(e)\big) - c\big). \tag{7}$$

The predictions for the final-period average effort levels that follow from (7) (again with $\mu = 7.4$) are: 150 for $c = 0.1$, 140 for $c = 0.4$, and 130 for $c = 0.6$ with a standard deviation of 8 in each case. The observed average efforts in the last three periods for these sessions were 157 ($c = 0.1$), 136 and 138 ($c = 0.4$), and 113 ($c = 0.6$), respectively. There are some deviations from the theoretical predictions, and in the low-cost sessions behavior might have "locked in" at the upper boundary with more repetition (as it did in the three-person minimum-effort game, see Fig. 4). However, the overall pattern is tracked fairly well and the comparative statics predictions that follow from maximizing stochastic potential are borne out by the data.[25]

6. Conclusion

Coordination games are of interest to both macroeconomists and microeconomists because the presence of multiple, Pareto-ranked Nash equilibria raises the possibility of failure to coordinate on a "good" outcome. One direction of research has been to devise and study mechanisms that facilitate profitable coordination. In addition, theorists have studied coordination games extensively because the presence of multiple equilibria provides a useful platform for the analysis of strategic behavior. Not surprisingly, data from past coordination experiments have provided a rich testing ground for theoretical advances, and the original Van Huyck et al.'s (1990) experiments are some of the most widely cited in the experimental economics literature. This paper reports a new set of experimental data generated by changes in the economic variables, e.g. effort cost, which should affect the likelihood of successful coordination. These new experiments were designed in light of some recent theoretical advances in the analysis of equilibrium and dynamics with noisy behavior, and one objective of this paper is to add a new set of stylized facts to guide current theoretical work that is proceeding in several different and potentially promising directions.

A second purpose of this paper is to shed light on how the well-known notion of "risk dominance" in 2×2 games might be generalized. In the continuous minimum-effort game, a unilateral increase in effort above some common level will reduce one's payoff by c per unit effort, whereas a unilateral one-unit decrease in effort will reduce payoff by $1 - c$, since the minimum effort is reduced by 1. Thus any common effort level is a Nash equilibrium, but intuition suggests that the average effort levels should depend on the relative losses from over-shooting or under-shooting the other's effort, i.e. on whether c is greater than or less than $1/2$. Risk dominance uses these "deviation losses" to predict which outcome will occur in a two-decision game. One way to generalize risk dominance to economic situations with a continuum of decisions is to consider the equilibrium that maximizes a "potential function." In the two-person coordination game, this procedure selects the equilibrium with the highest possible effort when $c < 1/2$ and with the lowest possible effort when $c > 1/2$. This paper presents the results of a laboratory experiment using effort

[25] As noted above, the mixed-strategy Nash prediction for the median game involves randomizing between the lowest and highest possible effort level when $c = 0.4$. The effect of adding noise is to produce a bi-modal density function with considerable mass near the boundaries. This bi-modal pattern is corroborated by the data for the two sessions with $c = 0.4$. In the final three periods of these sessions, about two thirds (35 out of 54) of the effort choices were within 10 of the upper and lower boundary.

cost parameters of 1/4 and 3/4. The effort-cost treatment separates the data nicely, with symmetric increases for low effort costs and decreases for high effort costs, as shown by the dark lines in Fig. 1 that track the average efforts by treatment for each period.

The data clearly show some degree of randomness, reflecting noisy response to asymmetries in deviation losses. To capture this randomness, we use a “stochastic potential” function which includes an entropy term that is weighted by an estimated error parameter. The intuition for stochastic potential is that it will be maximized by gradient-based adjustments subject to normal random noise, just as the deterministic potential is maximized by deterministic adjustments in the direction of higher payoffs. The final-period averages are close to the levels that maximize stochastic potential. Follow-up experiments show that this approach is also useful in organizing the data from different contexts, e.g. three-person minimum and median effort-coordination games. Overall, this combination of theory and experiment provides a coherent picture of behavioral responses to key economic incentives and can be useful in designing mechanisms that facilitate coordination.

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