

OPTIMAL PRIVATISATION USING QUALIFYING AUCTIONS*

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This article explores use of auctions for privatising public assets. In our model, a single ‘insider’ bidder possesses information about the asset’s common value. Bidders are privately informed about their costs of exploiting the asset. Due to the insider’s presence, uninformed bidders face a strong winner’s curse in standard auctions. We show that the optimal mechanism discriminates against the informationally advantaged bidder. It can be implemented via a two-stage ‘qualifying auction’. In the first stage, non-binding bids are submitted to determine who enters the second stage, which consists of a standard second-price auction augmented with a reserve price.

The World Bank’s ‘toolkit’ or ‘practitioners’ guide’ to privatisation describes the following two-stage auction procedure for privatising public assets (Welch and Frémont, 1998, p. 32): in the first stage non-binding expressions of interest are received from interested buyers. Based on these expressions of interest and a review of the financial capacity of potential bidders a short list of potential buyers is selected. These bidders then move to the second stage of the process, which consists of a more traditional auction with binding bids.

Welch and Frémont (1998) mention several practical advantages of this procedure.¹ In this article, we abstract away from these practical issues and investigate how the mechanism performs in terms of more traditional economic criteria such as efficiency and revenue.² An increase in efficiency is often cited as one of the main goals of privatisation but in many cases generating high revenues is at least as important. For instance, European countries that face the straightjacket of the *Stability and Growth Pact* have massively turned to privatisation of public assets to reduce deficits and government debt.³ Similar trends are seen outside the EU, where revenue is an equally important objective of privatisation.⁴ Moreover, the International Monetary Fund and the World Bank explicitly mention using the revenue from privatisation to reduce government debt and enhance fiscal stability in their *Guidelines for Public Debt Management*.⁵

In our model, bidders’ valuations for the asset consist of both a private and a common value component. The former corresponds to the bidder’s cost of exploiting the asset, which is privately known to the bidder. In addition, one ‘insider’ bidder (e.g. the state-owned firm’s incumbent management) is better informed about the asset’s common value. Besides privatisation, there are many other situations of interest where exactly one

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¹ See also Gibbon (1996) and other ‘How-to-Guides’ published by Privatization.org.

² For instance, we do not model potential problems with bidders’ financial capacities.

³ See, for instance, the *Economist* (2002b), the *Economist* (2004), and the OECD (2003).

⁴ See, e.g., the *Economist* (2002a) on Central Europe, Prasad (2004) on China, and the International Monetary Fund (2003) on a number of other countries.

⁵ See the International Monetary Fund and World Bank (2002).

insider bidder is better informed than the seller and other bidders. For example, when a company decides to outsource its catering division it may solicit bids from outside catering companies as well as from its current in-house caterer. Similarly, in management and employee buyouts (MEBOs) the firm's current management has a clear informational advantage relative to other investors. As a final example, consider licences for spectrum usage, which typically have a lease period of eight to fifteen years. When such a licence is re-auctioned the current user is better informed about its profitability.⁶

We are interested in situations where privatisation involves substantial risk, i.e. when uncertainty about the asset's common value is large relative to private-value cost differences. For example, when bidding for a contract to collect garbage, differences in fleet operating costs are likely to be small while the contract's common value may vary depending on people's willingness to sort their garbage voluntarily into several categories (paper, plastic, glass etc.). Likewise, when market licences are auctioned among firms with access to a common technology, private-value differences are negligible compared to uncertainty about common market characteristics. Finally, when a high-visibility art work is sold, bidders' valuations depend crucially on its authenticity. This common value aspect introduces a strong winner's curse element especially if one of the bidders has inside information.⁷

We show that in such environments with relatively small private-value differences, standard auctions perform poorly in terms of expected revenue due to the possibility of a winner's curse. Both the second price auction and the English auction are dominated by the 'qualifying auction' we study, which is modelled after the two-stage procedure employed by the World Bank. In the first stage of the qualifying auction, bidders place non-binding bids and all but the lowest bidder are allowed to participate in the second stage, which is a standard second price auction. We prove that the qualifying auction augmented with a reserve price implements the optimal (revenue-maximising) mechanism.

The reason why the qualifying auction outperforms standard auctions is that it eliminates the adverse effects of the winner's curse. Indeed, there exists an equilibrium of the qualifying auction where in the first stage every bidder bids the unconditional expected value for the asset. The intuition is that since first-stage bids are non-binding, the expected value does not have to be conditional on winning. If the insider bidder places a very low bid in the first stage, uninformed bidders observe the negative news about the asset's common value and account for this via their second-stage bids. Since the bidder with the lowest first-stage bid is not allowed to participate in the second stage, there is no incentive for the informed bidder to signal bad news if, in fact, he possesses good news about the asset's common value.

⁶ A related example is the auctioning of petrol stations along the Dutch highways. In this auction, the current owner competes with other oil companies for the rights to exploit the petrol station in the next lease period.

⁷ Consider the following excerpt taken from <http://www.maineantiquedigest.com/articles/vangogh.htm>. 'Want to take a chance on a Van Gogh? The bidding starts at \$150,000 and there are no guarantees. You have to act fast. The painting is referred to as 'Mystery Vincent Van Gogh painting' and as 'Sunflowers and Oleanders'. The attorney for the trustee in the bankruptcy case already has a bid for \$125,000 so the bidding will have to go to \$150,000 to top that and then can proceed in \$10,000 increments. What's the catch? Opinion is definitely divided on whether the painting is a Van Gogh, whether it's signed, and lots else. If you get a \$15 million painting for \$150,000, good for you. If you get a nice decoration for \$150,000, good for the owners.'

Qualifying auctions were first studied by Ye (2004) who refers to it as a process of 'indicative bidding'.⁸ Ye's important contribution focuses on a different aspect of the two-stage qualifying auction, i.e. costly entry. In his model, bidders have some preliminary but inconclusive information about the asset for sale. In the first stage, they submit non-binding bids, which are used to select a few bidders for the second stage. Those that qualify for the second stage incur a large cost in determining their true, or final, value for the asset.⁹ Ye shows efficient entry cannot be guaranteed with indicative bidding and he proposes alternative two-stage formats to avoid this problem. The main difference from our article is that in Ye's (2004) model, bidders are symmetric and entry, or acquiring information, is costly. In contrast, in our model a single insider bidder possesses information that affects all, while entry is cost less. We show that qualifying auctions perform quite well in these situations.

Our article is related to the work of Bulow *et al.* (1999) who consider pure common-value takeover auctions where a single bidder has a (small) private-value toehold in the company being acquired. They find that in the ascending auction the existence of a single toehold bidder can have disastrous effects for revenue because other bidders face a strong winner's curse. This is akin to the situation that uninformed bidders face in our model. We show that the qualifying model solves the winner's curse problem by discriminating against the informationally advantaged bidder.¹⁰

The qualifying auction helps to maximise revenue from privatisation. This differs from Cornelli and Li (1997) who consider privatisation where different bidders have different objectives. This makes it possible that the highest bidder will not maximise the value of the company. To illustrate, a western car maker may buy a privatised car plant in Eastern Europe and close it down to increase its market power. Even if this bidder makes the highest offer, the government may prefer to sell to another bidder who plans to operate the plant. Cornelli and Li (1997) show how the government can use the fraction of shares sold to screen potential buyers with different plans.

Finally, there are a few papers that show technical similarities. Larson (2005) provides a thorough investigation of existence and uniqueness issues in pure common value auctions. By introducing small private-value disturbances that vanish in the limit, Larson is able to pin down a unique equilibrium for the pure common value case. Hernando-Veciana and Trøge (2004) determine circumstances under which the insider bidder in an English auction is better off disclosing his information. Their analysis of the English auction is parallel to that of Section 1. Hernando-Veciana (2004) shows that in pure common value second-price auctions, uninformed bidders may have higher expected payoffs than the single insider bidder. None of these papers determine the optimal mechanism for this context nor do they discuss the qualifying auction, which is the main focus of this article.

The article is organised as follows. The next Section introduces the privatisation model and shows that standard auctions perform poorly in terms of revenue. Section 2 solves for the optimal mechanism that maximises revenue from privatisation. Section 3

⁸ See Kagel *et al.* (2004) for an experimental test of Ye's (2004) model.

⁹ More precisely, bidder i 's first-stage(second-stage) information is X_i (Y_i) and his value for the asset is $X_i + Y_i$. To enter the second stage and learn Y_i , qualifying bidders have to pay a large entry cost. See also Calzolari and Pavan (2006) and Eso and Szentes (2003) for related approaches.

¹⁰ See also the work by Angeles de Frutos and Pechlivanos (2006).

shows that the optimal mechanism can be implemented using a qualifying auction augmented with a reserve price. Section 4 provide further developments, Section 5 concludes and the Appendix contains most of the proofs.

1. Standard Auctions for Risky Privatisation

We assume that bidder i 's value for the asset consists of a common value component $V + \theta$, with V known and θ unknown, minus a private cost c_i . In the 'good' state of the world, G , the common value is $V + 1/2\theta$ while in the 'bad' state of the world, B , the common value is $V - 1/2\theta$ (where $\theta > 0$). There are n bidders competing for the asset, $n - 1$ of which are uninformed about the common value. In particular, these uninformed bidders possess only the prior information that both states are equally likely, $P(G) = P(B) = 1/2$. In contrast, the insider bidder receives a noisy signal $\vartheta \in \{g, b\}$ about the state of the world, where $P(g|G) = P(b|B) = q > 1/2$ and $P(b|G) = P(g|B) = 1 - q$.

To simplify notation we define an uninformed bidder's private value as $s_i \equiv V - c_i$, for $i = 2, \dots, n$, where the s_i have distribution function $F(\cdot)$ with associated density $f(\cdot)$ defined on the support $[\underline{s}, \bar{s}]$. This way the uninformed bidders' values for the asset can be written as $s_i + P$.¹¹ The private value for the insider bidder depends on whether privatisation occurs.¹²

For instance, empirical evidence shows that privatisation of a government-owned firm enhances efficiency even when the current management remains in place (e.g., Hilke, 1993; Chapters 8, 9 and 10 in Shleifer and Vishny, 1998). Explanations include the introduction of a hard budget constraint for managers after privatisation and the reduced influence of politicians on how the firm is run. Hence incumbent management (if it keeps running the firm) faces changed circumstances. This is likely to affect incumbent's valuation s_1 . We assume the insider's private value for the asset is given by $s_1 = \underline{s}$ without privatisation¹³ and s_1 is a draw from $F(\cdot)$ defined on $[\underline{s}, \bar{s}]$ otherwise.¹⁴

ASSUMPTION 1. *The density $f(\cdot)$ is symmetric and log-concave on $[\underline{s}, \bar{s}]$.*

Log-concavity implies among other things that the density is single-peaked and that the hazard rate $f(s)/[1 - F(s)]$ is non-decreasing.¹⁵ Symmetry implies that the mean and median of the distribution are given by $(\underline{s} + \bar{s})/2$.

¹¹ We assume that $\underline{s} > \theta$ so that bidders' values are strictly positive.

¹² If the insider's private value would not change due to privatisation, it can be deduced by the seller from past performance. The informational advantage for the insider would then only involve the asset's common value. This would simplify the optimal mechanism considerably but does not appear to be realistic.

¹³ Setting s_1 equal to \underline{s} simply means that outside firms with lower private values cannot profitably win the auction.

¹⁴ This assumption is made mainly for notational simplicity. Our results can easily be extended to the case where the insider's private-value distribution $F_1(\cdot)$ is different from $F(\cdot)$.

¹⁵ Note that

$$\frac{f(s)}{1 - F(s)} \geq -\frac{\int_s^{\bar{s}} f'(u) du}{\int_s^{\bar{s}} f(u) du} = -\frac{\int_s^{\bar{s}} [f'(u)/f(u)] f(u) du}{\int_s^{\bar{s}} f(u) du} \geq -\frac{f'(s)}{f(s)},$$

where the final inequality holds because $f(s)$ is log-concave so $f'(s)/f(s)$ is non-increasing in s . Hence, $f(s)^2 + f'(s)[1 - F(s)] \geq 0$ or $\{f(s)/[1 - F(s)]\}' \geq 0$.

Our main interest concerns the case where the asset to be privatised carries substantial risk, which occurs when the variation in the common-value is large relative to the variation in private costs. In other words, we consider ‘almost common-value’ auctions as in Bulow *et al.* (1999).¹⁶ Our model is also related to that of Larson (2005) who focuses on the case where the private-value differences are vanishingly small.¹⁷

ASSUMPTION 2. *Private-value differences are small compared to common-value differences, i.e. $\bar{s} - \underline{s} < (q - \frac{1}{2})\theta$.*

This assumption captures the idea that it is mainly the common-value component that determines the asset’s value. Note that the expected value of the asset to the least efficient informed bidder when receiving good news, $\vartheta = g$, is given by $\underline{s} + q(\frac{1}{2}\theta) + (1 - q)(-\frac{1}{2}\theta)$. Likewise, the expected value to the most efficient bidder after receiving bad news, $\vartheta = b$, is $\bar{s} + q(-\frac{1}{2}\theta) + (1 - q)(\frac{1}{2}\theta)$. Hence, in expected terms, the risky asset is worth more (less) to the least (most) efficient bidder who received good (bad) news than to the most (least) efficient uninformed bidder. This separation of expected values by the insider’s common-value signal is one (technical) reason to invoke Assumption 2. (In Section 4 we discuss how our results extend to the case when this assumption is relaxed.) More importantly, the applications we have in mind (see the Introduction) are all characterised by a large degree of common-value uncertainty while private-value differences are small.

We first determine the bidding behaviour in an English auction, which is modelled using proxy bids. First, the informed bidder submits a single maximum bid and uninformed bidders submit their maximum bids for the case when the informed bidder is active. Second, if one or more uninformed bidders outbid the insider, they get the opportunity to revise their maximum bids for the case when the informed bidder is no longer active. If there is a tie among the revised bids then the original proxy bids are used to allocate the object to the uninformed bidder with the highest private value (efficient tie breaking) and to determine his payment in terms of the second-highest private value among the uninformed bidders.

The optimal bid for the insider is easy to characterise: drop out at the net expected value $b_I(s_I | \vartheta = g) = s_I + (q - \frac{1}{2})\theta$ in the case of good news and at $b_I(s_I | \vartheta = b) = s_I - (q - \frac{1}{2})\theta$ in the case of bad news. The following Lemma characterises the optimal bids for uninformed bidders, see Figure 1, where we assume that the informed bidder can be identified.¹⁸ An advantage of the qualifying auction, discussed below, is that we do not need to make this assumption.

LEMMA 1. *The following constitutes an equilibrium of the English auction with proxy bids. The insider bidder drops out at $b_I(s_I | \vartheta)$. An uninformed bidder drops out at*

¹⁶ The main difference is that the latter paper considers the effects of small value differences (due to ‘toeholds’) that are commonly known by the bidders. In contrast, we consider the effects of large informational differences.

¹⁷ See also Hernando-Veciana (2004) and Hernando-Veciana and Tröge (2004).

¹⁸ See also Hernando-Veciana and Tröge (2004) for the two-bidder case with 1 insider and 1 uninformed bidder. Note that the equilibrium of Lemma 1 does not depend on whether Assumption 2 holds or not.

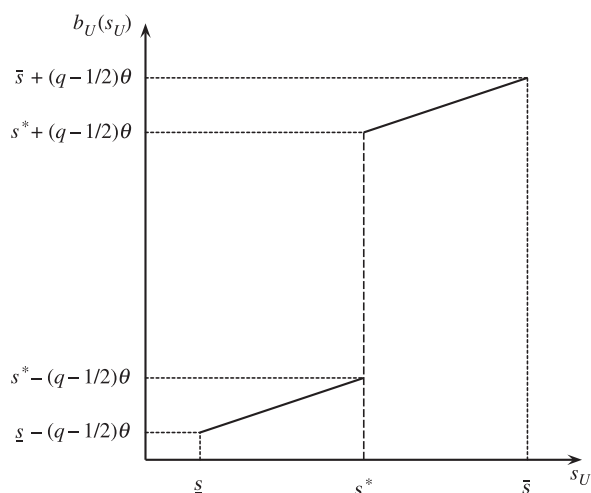


Fig. 1. Uninformed Bidder's Optimal Bidding Function in the English Auction

$$b_U(s_U) = \begin{cases} s_U - (q - \frac{1}{2})\theta & \text{if } s_U < s^* = \frac{1}{2}(\underline{s} + \bar{s}) \\ s_U + (q - \frac{1}{2})\theta & \text{if } s_U \geq s^* = \frac{1}{2}(\underline{s} + \bar{s}) \end{cases}$$

when the insider is still active. If the insider dropped out at some price level, p_I , and one or more uninformed bidders are still active, they will revise their drop-out levels to $b_U(s_U | p_I) = p_I + \max(0, s_U - s_I)$ where s_I solves $p_I = b_I(s_I | \vartheta)$ for $\vartheta = b$ or $\vartheta = g$. Ties among revised proxy bids are resolved efficiently (based on $b_U(s_U)$), in which case the winner's payment is based on the second-highest private value among uninformed bidders.

Note that the English auction does not necessarily result in an efficient allocation. For example, when $s_I < s_U < s^*$, inefficiencies occur when the insider receives good news about the asset's value. Likewise, when $s^* < s_U < s_I$, inefficiencies occur when the insider receives bad news about the asset's value. These inefficiencies are small in magnitude as they involve differences of bidders' private values, which are negligible for large θ . We next show that the impact of a single insider on the auction's revenue is more profound compared to the case with n uninformed bidders.

PROPOSITION 1. *When an asset is privatised using the English auction, the loss in revenue due to the presence of a single insider grows (at least) linearly with θ as θ grows large.*

What about the second price auction? With one informed and one uninformed bidder the uninformed bidder's strategy is the same as in Lemma 1 as there is no opportunity for the uninformed bidder to learn about the asset's common value. With additional uninformed bidders, however, the winner's curse problem is exacerbated compared to the English auction. In the latter, active uninformed bidders can quit immediately after the insider drops out at a low price. By contrast, in the second price auction, an uninformed bidder may have to pay another uninformed bidder's high bid when the insider's information is bad. To avoid such a costly scenario, uninformed bidders will have to bid more cautiously in a sealed-bid second-price auction compared to the English auction.

LEMMA 2. *The following constitutes an equilibrium of the second-price auction. The insider bids $b_I(s_I | \vartheta)$. An uninformed bidder bids*

$$b_U(s_U) = \begin{cases} s_U - (q - \frac{1}{2})\theta & \text{if } s_U < s^{**} \\ B(s_U) & \text{if } s_U \geq s^{**} \end{cases}$$

with $B(s_U)$ strictly increasing in s_U and $\underline{s} + (q - \frac{1}{2})\theta < B(s_U) < s_U + (q - \frac{1}{2})\theta$ for all $s^{**} < s_U < \bar{s}$ and $s^{**} > s^* = \frac{1}{2}(\bar{s} + \underline{s})$.

Note that uninformed bidders' optimal bids in the second price auction are more conservative than those in the English auction in two ways. First, high bids in the English auction exceed those in the second price auction.¹⁹ Second, the probability that an uninformed bidder bids high in the second price auction is lower than in the English auction ($1 - F(s^{**}) < 1/2$). The latter effect becomes dominant when uncertainty about the common value grows.

LEMMA 3. *With two or more uninformed bidders $\lim_{\theta \rightarrow \infty} s^{**} = \bar{s}$.*

In other words, as the uncertainty about the common value grows the loss in revenue for the seller is proportional to θ irrespective of the state of the world. Recall that in the English auction, a loss proportional to θ occurs only in the bad state of the world or when all uninformed bidders have below average private values. We thus have:

PROPOSITION 2. *When an asset is privatised using the second price auction, the loss in revenue due to the presence of a single insider is even worse than in the English auction when the uncertainty about the common value grows large.*

While our focus is on revenue it is interesting to note that also efficiency is lower in the second price auction.²⁰

2. Optimal Mechanism

Here we derive the optimal (revenue-maximising) mechanism for privatisation where private-value differences are small compared to common-value differences. We assume the designer can observe the insider's identity, e.g. in an auction it is known which bid is placed by the state-owned firm's incumbent management.²¹ We consider direct revelation mechanisms where each uninformed bidder truthfully reveals her one-dimensional type, s_i , and the informed player 1 truthfully reveals her two-dimensional type (ϑ, s_1) . Let $x_i(s_1, \dots, s_n | \vartheta)$ denote the probability that player i is awarded the asset as a function of bidders' reports, $t_i(s_1, \dots, s_n | \vartheta)$ her payment, and $u_i(s_1, \dots, s_n | \vartheta)$ her

¹⁹ We say an uninformed bidder's bid is 'high' when it exceeds the insider's optimal bid in case of good news with positive probability.

²⁰ Hernando-Veciana and Michelucci (2008) introduce the notion of second-best efficiency, i.e. the highest efficiency attainable taking into account bidders' incentive constraints. It can be shown that the English auction with proxy bids is second-best efficient in the environment studied here. In contrast, with more than two bidders, the second-price auction is not second-best efficient nor is the standard English auction (without proxy bids) because ties are not resolved efficiently.

²¹ The next Section discusses a practical implementation of the optimal mechanism, which does not require this assumption.

utility. For $i \geq 2$ we define bidder i 's expected probability of winning $x_i(s_i) = E_{S_{-i}, \vartheta} \{x_i(s_1, \dots, s_n | \vartheta)\} = 1/2 \int_{S_{-i}} [x_i(s_1, \dots, s_n | b) + x_i(s_1, \dots, s_n | g)] dF(S_{-i})$, where $S_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$. Bidder i 's expected payment $t_i(s_i)$ and expected utility $u_i(s_i)$ are defined analogously. Let $x_1(s_1 | \vartheta) = E_{S_{-1}} \{x_1(s_1, \dots, s_n | \vartheta)\}$ denote the expected probability of winning for the informed bidder 1, with similar definitions for $t_1(s_1 | \vartheta)$ and $u_1(s_1 | \vartheta)$.

An uninformed bidder $i \geq 2$ announces type \hat{s} that maximises

$$u_i(s_i) = \max_{\hat{s}} E_{\vartheta} \left(x_i(\hat{s} | \vartheta) \left\{ s_i + \frac{1}{2} [P(G | \vartheta) - P(B | \vartheta)] \theta \right\} - t_i(\hat{s} | \vartheta) \right).$$

Using the envelope theorem we have

$$\frac{\partial u_i(s_i)}{\partial s_i} = E_{\vartheta} [x_i(s_i | \vartheta)] \equiv x_i(s_i),$$

which can be integrated to give

$$u_i(s_i) = u_i(\underline{s}) + \int_{\underline{s}}^{s_i} x_i(s) ds. \tag{1}$$

This necessary condition for truth telling is also sufficient if $x_i(s)$ is non-decreasing in s .

Next we turn to the informed bidder 1. The same steps leading up to (1) show that for the insider to truthfully reveal s_1 we must have, for $\vartheta \in \{b, g\}$,

$$u_1(s_1 | \vartheta) = u_1(\underline{s} | \vartheta) + \int_{\underline{s}}^{s_1} x_1(s | \vartheta) ds. \tag{2}$$

In addition, for bidder 1 to reveal her common-value signal ϑ truthfully we must have:

$$u_1(s_1 | \vartheta) = \max_{\hat{s}, \hat{\vartheta}} \left(x_1(\hat{s} | \hat{\vartheta}) \left\{ s_1 + \frac{1}{2} [P(G | \hat{\vartheta}) - P(B | \hat{\vartheta})] \theta \right\} - t_1(\hat{s} | \hat{\vartheta}) \right).$$

Lemma 4 provides sufficient conditions for bidders to participate and truthfully reveal their private information. The proof uses the following insight: if the insider lies about her common value information after receiving good news she also reports a private value $\hat{s} = \bar{s}$ so as to maximise her probability of winning. Likewise, if the insider lies about her common value information after receiving bad news she reports a private value $\hat{s} = \underline{s}$. The conditions of Lemma 4 ensure that neither deviation is profitable.

LEMMA 4. *Bidders participate and truthfully reveal their types if (1) and (2) hold and*

(i) $u_i(\underline{s}) \geq 0$ for $i \geq 2$, $u_1(\underline{s} | b) \geq 0$ and

$$u_1(\underline{s} | g) = u_1(\bar{s} | b) + x_1(\bar{s} | b) [(2q - 1)\theta - (\bar{s} - \underline{s})]. \tag{3}$$

(ii) $x_i(s_i)$ is non-decreasing in s_i for $i \geq 2$, $x_1(s_1 | \vartheta)$ is non-decreasing in s_1 for $\vartheta \in \{b, g\}$ and

$$x_1(\underline{s} | g) = x_1(\bar{s} | b). \tag{4}$$

Equations (3) and (4) patch together the insider's expected utility and probability of winning across the two information signals. They are sufficient but not necessary

conditions, i.e. the equality signs in (3) and (4) can be relaxed to some degree.²² Strict equalities follow, however, when we restrict attention to revenue-maximising mechanisms, which we discuss next.

The seller’s expected revenue, R , equals the sum of the expected transfers from the bidders plus the status quo revenues that occur when no privatisation takes place. Using standard manipulations (Myerson, 1981), the expected revenue can be written as

$$\begin{aligned}
 R = & \underline{s} + \frac{1}{2} \sum_{i=1}^n \int_{\underline{s}}^{\bar{s}} \cdots \int_{\underline{s}}^{\bar{s}} \left[s_i - \underline{s} - \frac{1 - F(s_i)}{f(s_i)} \right] x_i(s_1, \dots, s_n | b) dF(s_1) \cdots dF(s_n) \\
 & + \frac{1}{2} \sum_{i=1}^n \int_{\underline{s}}^{\bar{s}} \cdots \int_{\underline{s}}^{\bar{s}} \left[s_i - \underline{s} - \frac{1 - F(s_i)}{f(s_i)} \right] x_i(s_1, \dots, s_n | g) dF(s_1) \cdots dF(s_n) \quad (5) \\
 & - \frac{1}{2} [u_1(\underline{s} | b) + u_1(\underline{s} | g)] - \sum_{i=2}^n u_i(\underline{s}).
 \end{aligned}$$

The seller chooses the assignment functions, $x(\cdot | b)$ and $x(\cdot | g)$, and the utilities of the lowest types, $u_i(\underline{s})$ for $i \geq 2$ and $u_1(\underline{s} | b)$ and $u_1(\underline{s} | g)$, so as to maximise expected revenue subject to the incentive compatibility and participation constraints of Lemma 4.

Let $s = s_{\min}$ denote the solution to

$$s - \frac{1 - F(s)}{f(s)} = \underline{s}. \quad (6)$$

Assumption 1 ensures the left side is strictly increasing in s so the solution s_{\min} is unique. We say the insider reports good (bad) news if the reported common-value signal is $\hat{v} = g$ ($\hat{v} = b$).

PROPOSITION 3. *If the informed bidder reports bad news, the optimal mechanism assigns the risky asset to the uninformed bidder with the highest reported private value if this value exceeds s_{\min} ; otherwise no privatisation takes place. If the informed bidder reports good news, the optimal mechanism assigns the risky asset to the bidder with the highest reported private value if this value exceeds s_{\min} ; otherwise no privatisation takes place.*

Proof. First, note from (5) that it is optimal to set $u_i(\underline{s}) = 0$ for $i \geq 2$ and $u_1(\underline{s} | b) = 0$. Equation (3) becomes

$$u_1(\underline{s} | g) = \int_{\underline{s}}^{\bar{s}} x_1(s_1 | b) ds_1 + x_1(\bar{s} | b)[(2q - 1)\theta - (\bar{s} - \underline{s})].$$

Combining this with expression (5) for the expected revenue, we have

²² Equations (3) and (4) can be relaxed to $x_1(\underline{s} | g) \geq x_1(\bar{s} | b)$ and $x_1(\bar{s} | b)[(2q - 1)\theta - (\bar{s} - \underline{s})] \leq u_1(\underline{s} | g) - u_1(\bar{s} | b) \leq x_1(\underline{s} | g)[(2q - 1)\theta - (\bar{s} - \underline{s})]$.

$$\begin{aligned}
 R = & \underline{s} + \frac{1}{2} \sum_{i=1}^n \int_{\underline{s}}^{\bar{s}} \cdots \int_{\underline{s}}^{\bar{s}} [\widetilde{MR}_i(s_i) - \underline{s}] x_i(s_1, \dots, s_n | b) dF(s_1) \cdots dF(s_n) \\
 & + \frac{1}{2} \sum_{i=1}^n \int_{\underline{s}}^{\bar{s}} \cdots \int_{\underline{s}}^{\bar{s}} [MR_i(s_i) - \underline{s}] x_i(s_1, \dots, s_n | g) dF(s_1) \cdots dF(s_n) \tag{7} \\
 & - \frac{1}{2} x_1(\bar{s} | b) [(2q - 1)\theta - (\bar{s} - \underline{s})],
 \end{aligned}$$

where we defined

$$MR_i(s_i) = s_i - \frac{1 - F(s_i)}{f(s_i)}$$

for $i = 1, \dots, n$, and

$$\widetilde{MR}_1(s_1) = s_1 - \frac{2 - F(s_1)}{f(s_1)},$$

and $\widetilde{MR}_i(s_i) = MR_i(s_i)$ for $i = 2, \dots, n$.

Figure 2 shows $MR_i(s) - \underline{s}$ in case of good news (right panel) and $\widetilde{MR}_1(s) - \underline{s}$ in case of bad news (left panel). If $\widetilde{MR}_1(s_1) - \underline{s}$ were everywhere negative then the seller should obviously never allocate the asset to the insider in case of bad news. Suppose $\widetilde{MR}_1(s_1) - \underline{s}$ is positive for some values of s_1 as shown in the Figure. At the upper end $s_1 = \bar{s}$ we have

$$\widetilde{MR}_1(\bar{s}) - \underline{s} = (\bar{s} - \underline{s}) - \frac{1}{f(\bar{s})} \leq 0,$$

since Assumption 1 implies that $f(\cdot)$ is minimised at the boundaries so $1 = \int_{\underline{s}}^{\bar{s}} f(s) ds \geq f(\bar{s})(\bar{s} - \underline{s})$. Hence, when the marginal revenue $\widetilde{MR}_1(s_1)$ is positive for some values of s_1 it is non-monotonic. Since the assignment function $x_1(s_1 | \vartheta)$ has to be non-decreasing in s_1 we need to ‘iron out’ the marginal revenue curve (see, for instance, Bulow and Roberts, 1989). This yields the $\mu\mu'\mu''\mu'''$ curve in Figure 2. The main point is that the part of the curve that pertains to bad news lies everywhere below the zero line that connects \underline{s} and \bar{s} . To understand this result, note that

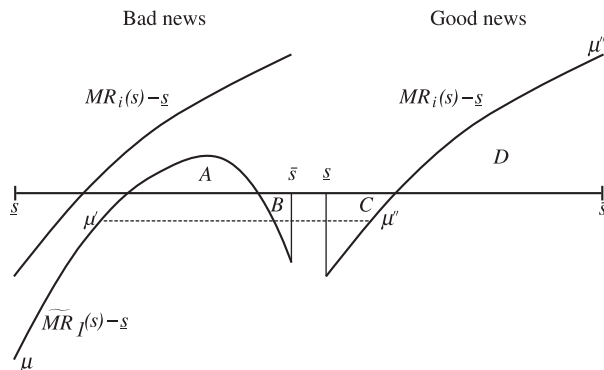


Fig. 2. Ironing Out the Insider's Marginal Revenue Curve

$\int_{\underline{s}}^{\bar{s}} MR_i(s)f(s)ds = \underline{s}$. (To sell to all types, revenue has to equal \underline{s} since that it is the maximum price at which all types are willing to buy.) The weighted D , C areas (weighted by the density function $f(\cdot)$) are therefore equal: $D - C = 0$. Obviously, $D > A - B$ so $A - B - C < 0$. The line connecting μ' to μ'' thus has to lie below the zero to replicate this negative area. As a result, the seller should never award the asset to the insider when the insider reports bad news. In other words, $x_1(s_1 | b) = 0$ for all $s_1 \in [\underline{s}, \bar{s}]$.

This implies that the final term in (7) is zero. Hence, revenue is maximised by allocating the project to the uninformed bidder with the highest $\widetilde{MR}_i(s_i) - \underline{s}$ in case of bad news and to the bidder with the highest $MR_i(s_i) - \underline{s}$ in case of good news conditional on these values being positive. The log-concavity Assumption 1 ensures $MR(s_i)$ is an increasing function and it is positive if and only if $s > s_{\min}$. So in case of bad news the project is assigned to the uninformed bidder with the highest private value if this private value exceeds s_{\min} otherwise no privatisation takes place. In case of good news the project is assigned to the bidder with the highest private value (whether or not the bidder is informed) if this value exceeds s_{\min} , otherwise no privatisation takes place. *Q.E.D.*

Note that the optimal mechanism discriminates against the informationally advantaged bidder to ensure truthful information revelation and protect uninformed bidders from the winner’s curse. This is akin to Myerson’s (1981) solution for the case of asymmetric value distributions where bidding credits are assigned to ‘weaker’ bidders to enhance competition and force the advantaged bidder to bid closer to her true value.

3. Qualifying Auctions

In this Section we describe a practical implementation of the optimal mechanism, consisting of a two-stage qualifying auction. In the first stage all n bidders place a bid. The $n - 1$ highest bids qualify for the second stage, which consists of a standard second price auction augmented with a reserve price. All qualifying bidders learn only the lowest first-stage bid, as does the seller who sets an optimal reserve price based on this information. The following Lemma describes an equilibrium in this context (another equilibrium is discussed below).

LEMMA 5. *In the first stage of the qualifying auction, optimal bids are*

$$b_I(s_I | \vartheta) = s_I + \frac{1}{2}[P(G | \vartheta) - P(B | \vartheta)]\theta,$$

$$b_U(s_U) = s_U.$$

All but the lowest bidder qualify for the second stage. The seller and qualifying bidders get to know the lowest first-stage bid from which they can perfectly infer the insider’s information ϑ . The reserve price is set at $r = b_I(s_{\min} | \vartheta)$ and qualifying bidders’ optimal second-stage bids are

$$b(s) = \begin{cases} b_I(s | \vartheta) & \text{if } s \geq s_{\min}, \\ \text{‘no bid’} & \text{if } s < s_{\min}, \end{cases}$$

with s_{\min} the unique solution to (6).

Proof. Since all bidders bid their unconditional expected values in the first stage, the lowest bid is either in the range $[\underline{s} - (q - \frac{1}{2})\theta, \bar{s} - (q - \frac{1}{2})\theta]$ or in the range $[\underline{s}, \bar{s}]$, which are disjoint intervals by assumption 2: $(q - \frac{1}{2})\theta > \bar{s} - \underline{s}$. In the former case, the seller and qualifying bidders know $\vartheta = b$ while in the latter case they know $\vartheta = g$.

To show that the above strategies constitute an equilibrium note that, given first-stage behaviour, qualifying bidders know ϑ and hence it is optimal for them to bid their expected values in the second stage. Now consider the qualifying stage. Suppose the insider's information is $(\vartheta = g, s_I)$: can the insider gain by bidding something different than $b_I(s_I | \vartheta)$? There are two possibilities: (i) the deviating bid is the lowest first-stage bid and (ii) it is not. In the former case, the insider will forgo the opportunity to bid in the second stage, which cannot be profitable. In the latter case, the lowest first-stage bid is determined by an uninformed bidder (as it would be in equilibrium), in which case the seller and qualifying bidders infer that $\vartheta = g$. Again such a deviation by the insider is not profitable. What if the insider's information is $(\vartheta = b, s_I)$? In this case she might deviate such that her bid is no longer the lowest first-stage bid. But then the seller and other qualifying bidders infer $\theta = g$, in which case the insider will not be able to profitably win the second stage.

Finally, given the second price nature of the second stage auction, it is easy to show that uninformed bidders have no incentive to deviate from truthful bidding in the first stage. *Q.E.D*

Note that the qualifying auction implements the optimal mechanism of the previous section and its revenue is independent of the degree of common value uncertainty unlike that of the standard auctions discussed above. Importantly, the qualifying auction is revenue maximising when the insider's identity is unknown or cannot be (legally) used. Moreover, the qualifying auction is robust in the sense that it is optimal also when there is no insider bidder. In this case, all bidders are symmetric and the first stage simply gets rid of the bidder with the lowest private value with no consequences for seller revenues.

PROPOSITION 4. *The qualifying auction implements the optimal mechanism in situations with and without informational asymmetries whether or not the insider's identity can be observed.*

It is important to point out that the qualifying auction also admits a 'babbling' equilibrium in which no information is revealed after the first stage. In particular, if the maximum bid is denoted $\bar{b} < \infty$, then all bidders submit \bar{b} in the first stage, one randomly chosen bidder is removed from the auction and no information is revealed.²³ Note that this equilibrium makes all uninformed bidders worse off as the winner's curse reappears in the final stage.

4. Discussion

Recall that Assumption 2 implies the insider bidder's value lies outside the set of unconditional expected (private) values of the uninformed bidders. Here we drop this restriction. Suppose the insider bidder gets one of three signals: $-\theta, 0, \theta$ with probabilities

²³ If there is no upper-bound on bids, such an equilibrium, strictly speaking, does not exist.

$\frac{1}{2}p$, $1 - p$, and $\frac{1}{2}p$ respectively (to simplify notation we set $q = 1$). Then with probability $1 - p$ the insider is no more informed than the rest and the set of insider's expected values is identical to that of the uninformed bidders. In other words, the insider's values are no longer separated from those of the uninformed bidders. We keep assumption 2 written as: $\bar{s} - \underline{s} < \frac{1}{2}\theta$. In addition, we require that the insider is more likely to be informed about either state than to be uninformed, so $1/2p \geq 1 - p$ or $p \geq 2/3$.²⁴

To derive the optimal mechanism, we need the expressions for marginal revenue. As before they depend on the insider's common value signal:

$$MR_1^{\hat{\theta}}(s) - \underline{s} = \begin{cases} s - \underline{s} - \frac{(2/p) - F(s)}{f(s)} & \text{if } \hat{\theta} = -\theta \\ s - \underline{s} - \frac{[(1 - \frac{1}{2}p)/(1 - p)] - F(s)}{f(s)} & \text{if } \hat{\theta} = 0 \\ s - \underline{s} - \frac{1 - F(s)}{f(s)} & \text{if } \hat{\theta} = \theta. \end{cases} \tag{8}$$

It is readily verified that $p \geq \frac{2}{3}$ and $f(\bar{s}) \leq 1/(\bar{s} - \underline{s})$ (from Assumption 1) imply that the marginal revenues are non-positive at the top in case of bad or no news, i.e. $MR_1^{\hat{\theta}}(\bar{s}) - \underline{s} < 0$ for $\hat{\theta} = -\theta$ and $\hat{\theta} = 0$. Following the steps of the proof of Proposition 3, we can now show that the ironed out $MR(s) - \underline{s}$ curve lies below zero in case of bad news ($\hat{\theta} = -\theta$) or no news ($\hat{\theta} = 0$). In other words, the insider bidder is never assigned the asset when she did not receive good news.

The following three stage qualifying auction implements the optimal mechanism. In the first two stages all (remaining) bidders place non-binding bids and in each round the lowest bidder drops out. The other bidders learn the lowest bid. The final stage consists of a standard second price auction among the remaining $n - 2$ bidders. As above, the seller adjusts the reserve price in the final round based on the losing bids in the first two rounds.

Using similar arguments as in the previous Section, we can show that in case of bad news the insider bidder bids $s_1 - \theta < \underline{s}$ and everyone learns from this lowest bid that the common value equals $-\theta$. If the lowest bid is above \underline{s} , uninformed bidders update their expectation of the common value to

$$\frac{1 - p}{1 - \frac{1}{2}p} 0 + \frac{\frac{1}{2}p}{1 - \frac{1}{2}p} \theta = \frac{p}{2 - p} \theta \geq \frac{1}{2} \theta$$

for $p \geq 2/3$. Hence, in the second round, the lowest uninformed bid exceeds $\underline{s} + \frac{1}{2}\theta$ which exceeds the highest informed bid in case of no news, \bar{s} . So after learning the lowest bid in the second round, uninformed bidders and the seller can distinguish between no news and good news. The qualifying auction therefore implements the optimal mechanism even in the case when the informed bidder's valuation overlaps with the uninformed bidders' unconditional expected valuations.

Finally, suppose $\frac{1}{2}\theta \leq \bar{s} - \underline{s} < \theta$ so that the informed bidder's valuations in case of good (bad) news and no news are overlapping. In this case, the qualifying auction helps to

²⁴ As p gets smaller the informed bidder becomes less informed and in the limit when $p = 0$ all bidders are equally (un)informed. For our discussion here we maintain $p \geq 2/3$ to ensure that the informational advantage of the informed bidder is substantial.

reduce the winner's curse but it no longer implements the optimal mechanism. In the first stage, uninformed bidders learn whether the informed bidder has bad news about the asset since $\bar{s} - \theta < \underline{s}$. However, in the second stage the uninformed bidders may not be able to infer whether the informed bidder has no news or good news (as their updated bids in the second round overlap with $[\underline{s}, \bar{s}]$). Uninformed bidders therefore face a reduced winner's curse in the final stage (roughly speaking the winner's curse has been halved), which results in more aggressive bidding and higher revenues. In this sense, the qualifying auction still improves on other formats although it is no longer optimal.²⁵

5. Conclusion

High-stakes auctions for privatising public assets are often plagued by substantial bidder asymmetries, which may have devastating effects for the auction's revenue. Previous literature has typically assumed that these asymmetries take the form of (known) private-value differences (Myerson, 1981; Bulow *et al.*, 1999). In contrast, in this article we analyse the effects of *informational asymmetries* that arise when one insider bidder has superior information about the asset's common value while private-value differences are small. To illustrate, consider the sale of the Los Angeles licence in the 1995 FCC spectrum auction for mobile-phone broadband licences. The following discussion is taken from Klemperer (2002).

While the license value was hard to estimate, it was probably worth similar amounts to several bidders. But Pacific Telephone, which already operated the local fixed-line telephone business in California, had distinct advantages from its database on potential local customers and its familiarity with doing business in California.

Before bidding for the California phone license, Pacific Telephone announced in the Wall Street Journal that 'if somebody takes California away from us, they'll never make any money' (Cauley and Carnevale, 1994, p. A4). Pacific Telephone also hired one of the world's most prominent auction theorists to give seminars to the rest of the industry to explain the winner's curse argument that justifies this statement, and reinforced the point in full page advertisements that ran in newspapers of cities where their major competitors were headquartered (Koselka, 1995, p. 63).

The auction was a standard ascending (English) auction. And the result was that the bidding stopped at a very low price. In the end, the Los Angeles license yielded only \$26 per capita. In Chicago, by contrast, the main local fixed-line provider was ineligible to compete and the auction yielded \$31 per capita even though Chicago was thought less valuable than Los Angeles because of its lower household incomes, lower expected population growth, and more dispersed population.

²⁵ If the seller can (legally) identify the informed bidder, separate thresholds can be set for the informed bidder to qualify for the next round. In this way, more information on the common value can be generated in this case.

We interpret the California example as follows: Pacific Telephone had inside information about the market they were already operating in and, hence, they knew much better than other bidders how much the licence would be worth. With differences in private valuations being small, Pacific Telephone could thus credibly claim that if its bid would be topped the winning bidder would have to fall prey to the winner's curse. As a result the licence sales price remained low (see Proposition 1) especially after a renowned auction theorist explained the logic behind the winner's curse to *other* bidders, making sure uninformed bidders would opt for a cautious strategy (see Lemma 1).

The Chicago example shows that one way to resolve this problem is to exclude the insider bidder. This solution, however, is not always legally possible nor is it optimal. Practical literature concerning the divestment of government-owned assets, such as the World Bank's 'How-to-Guide' for privatisation (Welch and Frémont, 1998), suggests a completely different approach based on a 'qualifying auction'. This auction consist of two stages. In the first stage, bidders place non-binding bids and all but the lowest bidder are allowed to participate in the second stage, which is a standard second price auction augmented with a reserve price.

In this article, we demonstrate that this simple format implements the revenue-maximising mechanism in situations where a single insider bidder has superior information about the asset's common value. The reason why the qualifying auction outperforms other formats is that it eliminates the adverse effects of the winner's curse. Indeed, there exists an equilibrium of the qualifying auction where in the first stage every bidder bids the unconditional expected value for the asset (see Lemma 5). The intuition is that since first-stage bids are non-binding, the expected value does not have to be conditional on winning. If the insider places a very low bid in the first stage, uninformed bidders observe the negative news about the asset's common value and account for this via their second-stage bids. And since the bidder with the lowest first-stage bid is not allowed to bid in the second stage, there is no incentive for the insider to signal bad news if, in fact, she possesses good news.

Through the addition of a qualifying stage the auction discriminates against the informationally advantaged bidder to ensure truthful information revelation (see Proposition 3), which is reminiscent of Myerson's (1981) recipe for how to deal with value-advantaged bidders. An important difference, however, is that the qualifying auction treats bidders in a symmetric manner and neither the seller nor the bidders need to know the identity of the insider. Moreover, qualifying auctions remain optimal in situations where informational asymmetries are small or non-existent (Proposition 4). Their long-time use in the sales of complex and risky assets lends further credence to their effectiveness in combatting the adverse effects of large informational asymmetries.

Appendix: Proofs

Proof of Lemma 1. Without loss of generality, label the uninformed bidders $1, \dots, n-1$ and the insider n . Take the point of view of bidder 1 with private value s_U and let $Z = \max_{2, \dots, n-1}(s^i)$ denote the highest private value of the other uninformed bidders, and let $Y = \max_{2, \dots, n}(s^i) = \max(Z, s_n)$ denote the highest private value of all other bidders. We distinguish two cases:

(I) bidder 1 bids low, $s'_U - (q - \frac{1}{2})\theta$, for some $0 < s'_U < 1$, or (II) bidder 1 bids high, $s'_U + (q - \frac{1}{2})\theta$, for some $0 < s'_U < 1$.

Case I: Bidding low

First, suppose bidder 1 bids $s'_U - (q - \frac{1}{2})\theta$ where $s'_U < s^*$. In this case she wins only in case of bad news and all other bidders have private values less than s'_U . Her expected payoffs are:

$$\frac{1}{2} \int_0^{s'_U} \int_0^{s'_U} [s_U - \max(s_I, z)] dF(z)^{n-2} dF(s_I) = \frac{1}{2} \int_0^{s'_U} (s_U - y) dF(y)^{n-1}. \quad (\text{A.1})$$

Next, suppose bidder 1 bids $s'_U - (q - \frac{1}{2})\theta$ where $s'_U > s^*$. Now she wins only in the case of bad news, all other uninformed bidders have private values less than s^* , and the insider has a private value less than s'_U . Bidder 1's expected payoffs are:

$$\begin{aligned} \frac{1}{2} \int_0^{s'_U} \int_0^{s^*} [s_U - \max(s_I, z)] dF(z)^{n-2} dF(s_I) &= \frac{1}{2} \int_0^{s^*} (s_U - y) dF(y)^{n-1} \\ &+ \frac{1}{2} F(s^*)^{n-2} \int_{s^*}^{s'_U} (s_U - s_I) dF(s_I). \end{aligned} \quad (\text{A.2})$$

Note that (A.1) exceeds (A.2) for $s_U < s^*$ while the reverse is true for $s_U > s^*$. In other words, if bidder 1 chooses to bid low, she will set $s'_U < s^*$ ($s'_U > s^*$) iff $s_U < s^*$ ($s_U > s^*$). Furthermore, in either case, bidder 1's expected payoffs are maximised by setting $s'_U = s_U$. Bidder 1's expected payoffs therefore are:

$$\pi_{low} = \frac{1}{2} \int_0^{s_U} (s_U - y) dF(y)^{n-1} \quad (\text{A.3})$$

for $s < s^*$ and

$$\pi_{low} = \frac{1}{2} \int_0^{s^*} (s_U - y) dF(y)^{n-1} + \frac{1}{2} F(s^*)^{n-2} \int_{s^*}^{s_U} (s_U - s_I) dF(s_I) \quad (\text{A.4})$$

for $s > s^*$.

Case II: Bidding high

If bidder 1 bids $s'_U + (q - \frac{1}{2})\theta$ where $s'_U < s^*$, she wins only if all other uninformed bidders have private values less than s^* . Expected payoffs are:

$$\begin{aligned} \frac{1}{2} \int_0^{s^*} \int_0^{s^*} [s_U - \max(s_I, z)] dF(z)^{n-2} dF(s_I) &+ \frac{1}{2} \int_{s^*}^1 \int_0^{s^*} (s_U - s_I) dF(z)^{n-2} dF(s_I) \\ &+ \frac{1}{2} \int_0^{s'_U} \int_0^{s^*} (s_U - s_I) dF(z)^{n-2} dF(s_I). \end{aligned} \quad (\text{A.5})$$

Here the two terms on the first line correspond the case of bad news and the term on the second line to the case of good news.

If bidder 1 bids $s'_U + (q - \frac{1}{2})\theta$ where $s'_U > s^*$, it is useful to distinguish several scenarios. In the case of bad news, bidder 1 wins if all other uninformed bidders have private values less than s'_U . Her payment, however, depends on whether the other uninformed bidders' private values are below or above the threshold level, s^* . In the former case, bidder 1's payment is determined by the highest of all others values. In the latter case, bidder 1's payment is determined by the highest of the uninformed bidders' private values. In case of good news, bidder 1 wins only if all others' values are less than s'_U . Again, her payment depends on whether the other uninformed bidders' private values are below or above the threshold level, s^* . Expected payoffs now are:

$$\begin{aligned}
 & \frac{1}{2} \int_0^{s^*} \int_0^{s^*} [(s_U - \max(s_I, z))dF(z)^{n-2}dF(s_I) + \frac{1}{2} \int_{s^*}^1 \int_0^{s^*} (s_U - s_I)dF(z)^{n-2}dF(s_I) \\
 & + \frac{1}{2} \int_0^1 \int_{s^*}^{s'_U} (s_U - z)dF(z)^{n-2}dF(s_I) + \frac{1}{2} \int_0^{s'_U} \int_0^{s^*} (s_U - s_I)dF(z)^{n-2}dF(s_I) \\
 & + \frac{1}{2} \int_0^{s^*} \int_{s^*}^{s'_U} (s_U - z)dF(z)^{n-2}dF(s_I) + \frac{1}{2} \int_{s^*}^{s'_U} \int_{s^*}^{s'_U} [s_U - \max(s_I, z)]dF(z)^{n-2}dF(s_I).
 \end{aligned} \tag{A.6}$$

Here the first three terms correspond the case of bad news and the final three terms to the case of good news. Note that when one or more other uninformed bidders bid high, bidder 1 always pays the difference between her private value and the next highest private value of the other uninformed bidders irrespective of the private value of the insider.

Note that (A.5) exceeds (A.6) for $s_U < s^*$ while the reverse is true for $s_U > s^*$. In other words, if bidder 1 chooses to bid high, she will set $s'_U < s^*$ ($s'_U > s^*$) iff $s_U < s^*$ ($s_U > s^*$). Furthermore, in either case, bidder 1's expected payoffs are maximised by setting $s'_U = s_U$. Bidder 1's expected payoffs therefore are:²⁶

$$\begin{aligned}
 \pi_{high} &= \pi_{low} + \frac{1}{2} \int_{s_U}^{s^*} (s_U - y)dF(y)^{n-1} \\
 &+ \frac{1}{2} F(s^*)^{n-2} \left[\int_0^{s_U} (s_U - s_I)dF(s_I) + \int_{s^*}^1 (s_U - s_I)dF(s_I) \right]
 \end{aligned} \tag{A.7}$$

for $s < s^*$ and²⁷

$$\begin{aligned}
 \pi_{high} &= \pi_{low} + \frac{1}{2} \int_{s^*}^{s_U} (s_U - z)dF(z)^{n-2} + \frac{1}{2} \int_{s^*}^{s_U} (s_U - y)dF(y)^{n-1} \\
 &+ \frac{1}{2} F(s^*)^{n-2} \left[\int_0^{s^*} (s_U - s_I)dF(s_I) + \int_{s_U}^1 (s_U - s_I)dF(s_I) \right]
 \end{aligned} \tag{A.8}$$

for $s_U > s^*$.

Symmetry of $F(\cdot)$ implies that the term in brackets in (A.7) is negative for $s_U < s^*$, and so is the other term. In other words, for $s_U < s^*$ we have $\pi_{high} < \pi_{low}$ and it is optimal to bid low $B(s_U) = s_U - (q - \frac{1}{2})\theta$. Moreover, symmetry of $F(\cdot)$ implies that the term in brackets in (A.8) is positive for $s_U > s^*$, and so are the other two terms. In other words, for $s_U > s^*$ we have $\pi_{high} > \pi_{low}$ and it is optimal to bid high $B(s_U) = s_U + (q - \frac{1}{2})\theta$.

Proof of Proposition 1. The inefficiencies mentioned in the main text may cause the bidder with the highest private value to determine the price (instead of the bidder with the second highest private value) with positive effects on revenue. A *lower* bound for revenue loss results by computing revenue as if it is *always* the bidder with the highest private value that determines the price. If Y_n^k denotes the k^{th} highest from n private value draws the sales price is then based on Y_n^1 (instead of Y_n^2). The sales price is raised by $(q - \frac{1}{2})\theta$ when the insider receives good news and the

²⁶ The additional terms are explained as follows: in case of bad news, bidder 1 now also wins when both Z and s_I are between s_U and s^* (this yields the $\frac{1}{2} \int_{s_U}^{s^*} (s_U - y)dF(y)^{n-1}$ term), and when Z is less than s^* and s_I is between s^* and 1 (this yields the $\frac{1}{2} F(s^*)^{n-2} \int_{s^*}^1 (s_U - s_I)dF(s_I)$ term). In case of good news, bidder 1 now also wins when Z is less than s^* and s_I is less than s_U (the $\frac{1}{2} F(s^*)^{n-2} \int_0^{s_U} (s_U - s_I)dF(s_I)$ term).

²⁷ The additional terms are explained as follows: in case of bad news, bidder 1 now also wins when Z is between s^* and s_U for any value of s_I (this yields the $\frac{1}{2} \int_{s^*}^{s_U} (s_U - z)dF(z)^{n-2}$ term), and when Z is less than s^* and s_I is between s_U and 1 (this yields the $\frac{1}{2} F(s^*)^{n-2} \int_{s_U}^1 (s_U - s_I)dF(s_I)$ term). In case of good news, bidder 1 now also wins when both Z and s_I are less than s_U (this yields $\frac{1}{2} \int_0^{s_U} \int_0^{s_U} [s_U - \max(s_I, z)]dF(z)^{n-2}dF(s_I) - \frac{1}{2} \int_0^{s^*} \int_0^{s^*} [(s_U - \max(s_I, z)) - (s_U - s_I)]dF(z)^{n-2}dF(s_I) = \int_{s^*}^{s_U} (s_U - y)dF(y)^{n-1} + \frac{1}{2} F(s^*)^{n-2} \int_0^{s^*} (s_U - s_I)dF(s_I)$).

highest of the uninformed bidders' private values exceeds s^* , and it is lowered by $(q - \frac{1}{2})\theta$ otherwise. The former event occurs with probability $\frac{1}{2}[1 - (\frac{1}{2})^{n-1}]$. Hence, a lower bound for the loss in revenue is $(q - \frac{1}{2})\theta - \frac{1}{2}[1 - (\frac{1}{2})^{n-1}]2(q - \frac{1}{2})\theta - [E(Y_n^1) - E(Y_n^2)] = (\frac{1}{2})^{n-1}(q - \frac{1}{2})\theta - [E(Y_n^1) - E(Y_n^2)]$, which is linearly increasing in θ .

Proof of Lemma 2. We say the uninformed bid is high (low) when it has a positive (zero) chance of winning against the equilibrium bid of an insider who received good news. The expected payoff of an uninformed bidder with private value s_U when bidding low, $s'_U - (q - \frac{1}{2})\theta$, equals

$$\frac{1}{2} \int_{\underline{s}}^{s'_U} \left\{ \left[s_U - \left(q - \frac{1}{2} \right) \theta \right] - \left[s - \left(q - \frac{1}{2} \right) \theta \right] \right\} dF(s)^{n-1}.$$

Clearly, it is optimal to set $s'_U = s_U$ in this case.

Optimal high bids can be derived from the following marginal argument: in equilibrium, $B(s_U)$ follows from the condition that the costs and benefits of raising the bid to $B(s_U + \epsilon)$ cancel. Note that such an increase has an effect only when it turns the uninformed bidder into a winner while she was previously losing. This occurs when

- (i) the insider received bad news and by raising her bid the uninformed bidder just beats the highest of the other uninformed bidders,
- (ii) the insider received good news and by raising her bid the uninformed bidder just beats the highest of the other uninformed bidders, and
- (iii) the insider received good news and by raising her bid the uninformed bidder just beats the insider.

Case (i) occurs when the highest of the other uninformed bidders' private values lies between s_U and $s_U + \epsilon$ while the insider's private value can be anything. This event occurs with probability $\epsilon(n - 2)f(s)F(s)^{n-3}$ and the net gain in this case is $s_U - (q - \frac{1}{2})\theta - B(s_U)$. Similarly, case (ii) occurs with probability $\epsilon(n - 2)f(s)F(s)^{n-3}F[B(s_U) - (q - \frac{1}{2})\theta]$ where the extra term is included to capture the probability that the insider's bid in case of good news, $s_U + (q - \frac{1}{2})\theta$, is less than $B(s_U)$. The net gain in this case is $s_U + (q - \frac{1}{2})\theta - B(s_U)$. Finally, case (iii) arises when all other uninformed bidders have private values less than s_U and the insider's private value lies between $B(s_U) - (q - \frac{1}{2})\theta$ and $B(s_U + \epsilon) - (q - \frac{1}{2})\theta$. The probability of this event is $\epsilon F(s)^{n-2} B'(s_U) f[B(s_U) - (q - \frac{1}{2})\theta]$ and the net gain in this case is $s_U + (q - \frac{1}{2})\theta - B(s_U)$.

Adding the different scenarios yields the following differential equation for $B(s_U)$

$$\begin{aligned} 0 = & (n - 2)f(s_U)F(s_U)^{n-3} \left[s_U - \left(q - \frac{1}{2} \right) \theta - B(s_U) \right] \\ & + (n - 2)F \left[B(s_U) - \left(q - \frac{1}{2} \right) \theta \right] f(s_U)F(s_U)^{n-3} \left[s_U + \left(q - \frac{1}{2} \right) \theta - B(s_U) \right] \\ & + B'(s_U)f \left[B(s_U) - \left(q - \frac{1}{2} \right) \theta \right] F(s_U)^{n-2} \left[s_U + \left(q - \frac{1}{2} \right) \theta - B(s_U) \right] \end{aligned} \tag{A.9}$$

with boundary condition $B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta$.

Note that for $n = 2$, (A.9) implies $B(s) = s + (q - \frac{1}{2})\theta$, which is the result of Lemma 1. For $n \geq 3$, we do not have an explicit solution to (A.9) but some insight can be gleaned as follows. First, the requirement that the bid is high implies $B(s_U) > \underline{s} + (q - \frac{1}{2})\theta$. Hence, by the risky asset Assumption 2, $B(s_U) > s_U$ and (A.9) thus implies that $B'(s_U) > 0$. Furthermore, (A.9) implies $B(s_U) < s_U + (q - \frac{1}{2})\theta$ for $s_U < \bar{s}$ since otherwise all terms on the right side are negative. Finally, the boundary condition $B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta$ is derived as follows. Suppose, in contradiction, $B(\bar{s}) < \bar{s} + (q - \frac{1}{2})\theta$. Since $B'(s_U) > 0$, $B(\bar{s}) > B(s)$ for all $s < \bar{s}$ so raising $B(\bar{s})$

leaves unchanged the probability of winning against an uninformed bidder but raises the probability of winning against an informed bidder with private value $s < \bar{s}$ who received good news, which is profitable. Hence $B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta$.

The differential equation in (A.9) thus characterises a well-defined increasing bidding function. Our final task is to determine the set of private values for which an uninformed bidder will bid low or high. Let s^{**} denote the private value for which the uninformed bidder is indifferent between a low and high bid. This cut-off value can be determined as follows: define the low bid $L \equiv s^{**} - (q - \frac{1}{2})\theta$ and the high bid $H \equiv B(s^{**})$ where $B(s_U)$ is determined by (A.9). The equilibrium expected payoffs of an uninformed bidder with private value s^{**} from bidding low is

$$\pi(L) = \frac{1}{2} \int_{\underline{s}}^{s^{**}} (s^{**} - s) dF(s)^{n-1},$$

and the equilibrium expected payoff from bidding high is

$$\pi(H) = \pi(L) + \frac{1}{2} F(s^{**})^{n-2} \left[\int_{s^{**}}^{\bar{s}} (s^{**} - s_I) dF(s_I) + \int_{\underline{s}}^{H - (q - \frac{1}{2})\theta} (s^{**} - s_I) dF(s_I) \right].$$

The additional terms on the right side arise as follows. In equilibrium, an uninformed bidder with cut-off private value, s^{**} , never wins against other uninformed bidders who bid high, which explains the $F(s^{**})^{n-2}$ term. Furthermore, the first (second) term in brackets pertains to the case where the insider received bad (good) news. An uninformed bidder with private value s^{**} is indifferent if and only if the terms in the brackets cancel, which implies

$$s^{**} = s^* + \int_{B(s^{**}) - (q - \frac{1}{2})\theta}^{s^{**}} (s^{**} - s) dF(s), \tag{A.10}$$

where $s^* \equiv E(s) = \frac{1}{2}(\underline{s} + \bar{s})$. This defines a negatively sloped curve in $(s^{**}, B(s^{**}))$ space that runs from $(s^*, s^* + (q - \frac{1}{2})\theta)$ to $(\bar{s}, \underline{s} + (q - \frac{1}{2})\theta)$. The differential equation (A.9) defines a positively sloped curve in $(s^{**}, B(s^{**}))$ space (since $B'(\cdot) > 0$) that starts below $s^* + (q - \frac{1}{2})\theta$ at $s^{**} = s^*$ (since $B(s_U) < s_U + (q - \frac{1}{2})\theta$ for all $s_U < \bar{s}$) and ends at $(\bar{s}, \bar{s} + (q - \frac{1}{2})\theta)$. The unique intersection of these two curves defines the cut-off value s^{**} . Since $B(s_U) < s_U + (q - \frac{1}{2})\theta$ for all $s_U < \bar{s}$, (A.10) implies $s^{**} > s^*$.

Proof of Lemma 3. Suppose, in contradiction, $\lim_{\theta \rightarrow \infty} s^{**} = \tilde{s} < \bar{s}$. For large θ , the probability that an uninformed bidder with private value $s > \tilde{s}$ pays another uninformed bidder's high bid in case of bad news is strictly positive ($\frac{1}{2}[F(s)^{n-2} - F(\tilde{s})^{n-2}] > 0$), resulting in a large loss. The uninformed bidder is better off bidding low, contradicting Lemma 2.

Proof of Lemma 4. Note that bidders' expected payoffs are non-negative so participation is guaranteed. To ensure bidder 1 does not report $\vartheta = b$ when in fact $\vartheta = g$ we require

$$u_1(s_1 | g) \geq \max_{\hat{s}} \left\{ x_1(\hat{s} | b) \left[s_1 + (q - \frac{1}{2})\theta \right] - t_1(\hat{s} | b) \right\}. \tag{A.11}$$

The incentive compatibility constraint for bidder 1 of type $(\vartheta, s_1) = (b, \bar{s})$ implies, for all \hat{s} ,

$$x_1(\bar{s} | b) \left[\bar{s} - (q - \frac{1}{2})\theta \right] - t_1(\bar{s} | b) \geq x_1(\hat{s} | b) \left[\bar{s} - (q - \frac{1}{2})\theta \right] - t_1(\hat{s} | b).$$

The risky asset Assumption 2 implies $(2q - 1)\theta > (\bar{s} - s_1)$ for all s_1 , and since $x_1(\cdot | b)$ is non-decreasing we have, for all \hat{s} ,

$$x_1(\bar{s} | b)[(2q - 1)\theta - (\bar{s} - s_1)] \geq x_1(\hat{s} | b)[(2q - 1)\theta - (\bar{s} - s_1)].$$

Adding the previous two inequalities yields, for all \hat{s} ,

$$x_1(\bar{s} | b) \left[s_1 + \left(q - \frac{1}{2} \right) \theta \right] - t_1(\bar{s} | b) \geq x_1(\hat{s} | b) \left[s_1 + \left(q - \frac{1}{2} \right) \theta \right] - t_1(\hat{s} | b).$$

Hence, the maximisation problem in (A.11) is solved by $\hat{s} = \bar{s}$ for all types s_1 . In other words, if the insider lies about her common-value information by reporting $\vartheta = b$ while, in fact, $\vartheta = g$, she also reports \bar{s} as her private value to maximise her probability of winning. Condition (A.11) reduces to

$$\begin{aligned} u_1(s_1 | g) &\geq x_1(\bar{s} | b) \left[s_1 + \left(q - \frac{1}{2} \right) \theta \right] - t_1(\bar{s} | b) \\ &= u_1(\bar{s} | b) + x_1(\bar{s} | b)[(2q - 1)\theta - (\bar{s} - s_1)]. \end{aligned} \tag{A.12}$$

To prove this inequality, note that for $\vartheta = g$ the incentive compatibility equation (2) yields

$$\begin{aligned} u_1(s_1 | g) &= u_1(\underline{s} | g) + \int_{\underline{s}}^{s_1} x_1(s | g) ds \\ &\geq u_1(\underline{s} | g) + x_1(\underline{s} | g)(s_1 - \underline{s}) = u_1(\underline{s} | g) + x_1(\bar{s} | b)(s_1 - \underline{s}), \end{aligned} \tag{A.13}$$

where we used that $x_1(\cdot | g)$ is increasing and (4). Inequality (A.12) now follows from (A.13) and (3).

Finally, to guarantee that an informed bidder with common-value signal $\vartheta = b$ never reports $\hat{\vartheta} = g$, we require

$$u_1(s_1 | b) \geq \max_{\hat{s}} \{ x_1(\hat{s} | g) \left[s_1 - \left(q - \frac{1}{2} \right) \theta \right] - t_1(\hat{s} | g) \}.$$

Proceeding in an analogous manner as above shows that the solution to the maximisation problem is given by $\hat{s} = \underline{s}$ for all s_1 , so the condition becomes

$$\begin{aligned} u_1(s_1 | b) &\geq x_1(\underline{s} | g) \left[s_1 - \left(q - \frac{1}{2} \right) \theta \right] - t_1(\underline{s} | g) \\ &= u_1(\underline{s} | g) - x_1(\underline{s} | g)[(2q - 1)\theta - (s_1 - \underline{s})]. \end{aligned}$$

Using (3) and (4) the expression in the second line can be rewritten as $u_1(\underline{s} | b) + \int_{\underline{s}}^{\bar{s}} x_1(s | b) ds - x_1(\bar{s} | b)(\bar{s} - s_1) \leq u_1(\underline{s} | b) + \int_{\underline{s}}^{s_1} x_1(s | b) ds = u_1(s_1 | b)$ where the inequality follows since $x_1(s | b)$ is non-decreasing in s .

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