Zero Intelligence in an Edgeworth Box

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Abstract

This paper expands on the continuous double auction (CDA) trader behavior literature, providing a general-equilibrium (GE) suited version of Gode and Sunder's (1993) "zero intelligence" (ZI) model. While the original authors of ZI have proposed a venture into trader behavior in the Edgeworth box (Gode, Spear, and Sunder, 2004), the model proposed here generalizes to a much further extent and provides a lower level of "zero" intelligence. Agents participate as two-way traders with utility functions and allocations giving them a natural market-side of preference. Orders are chosen randomly upon entry, with a lattice of bundles of the market's two goods providing the potential landing spots (upon potential future full acceptance of the order). I test the major rule variants of the model as well as many of the rules implicit in the CDA environment via simulation, providing a GE investigation analogous to the partial equilibrium (PE) literature's analysis of the original ZI model.

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"[T]he main stream (of models) for the last 50 years has entered the wilderness of bounded rationality through the rationality gate... there have been a few who have entered through the other (zero intelligence) gate... both of these groups are needed... though we need 'tunnelers' from both directions and we don't have enough coming from the minimal intelligence direction." -J. Doyne Farmer (2020)[1]

1 Introduction

Agent-based models have been a prominent process by which researchers have entered the zero/minimal intelligence gate. The study of trader behavior and price formation in markets, including the continuous double auction (CDA) institution, can especially benefit from such simple, parsimonious modelling given how easily a market's underpinnings can become overcomplicated.

Gode and Sunder (1993) postulated a model for double auction behavior which enters through the gate of no intelligence in a quite literal sense, introducing "zero intelligence" traders to the literature.[2] Zero intelligence traders provide a counter to the traditional rational traders who hold perfect utility maximizing capabilities when placing orders. Instead, prices are randomly chosen within the set of feasible prices $[0, M]$, or some no-loss constrained subset of this. Both the partial equilibrium and general equilibrium (Gode, Spear and Sunder(2004)) versions of the model provide glimpses into the natural equilibrating tendencies of the CDA. However, the models rely on a few restrictive assumptions, either institutional or behavioral in nature.

This paper proposes a more generalized lower-bound model of zero intelligence in a continuous double auction, set in the general equilibrium environment of an Edgeworth box. In continuity with Gode et al. (2004) (GSS henceforth), agents act as two-way traders in a

[1]This quote is from The First Conference on ZI/MI Intelligence Agents in 2020, in a discussion on agent-based modelling in economics.

[2]Hurwicz et al. (1975) was likely the first proposal of minimal or zero intelligence in a pure exchange setting. Many thanks to John Ledyard for the reference.

simple two good economy, participating in a series of market periods where their endowments are reset at the beginning of each period. The main departure of this paper's model lies in the intelligence given to traders when "randomly" placing orders. GSS provides traders with three major choice tendencies: (1) a constraint restricting traders from selecting orders which would allow them to lose utility, (2) an order choice method which has prices chosen via a uniform draw over a set of order vector slopes (in radians[3]), and (3) a set step-size on the length of the order vector. The model presented in this paper relaxes all of the constraints, thus imparting traders with a much lower, truer level of 'zero' intelligence. Traders instead select bundles over the set of feasible 'next' allocations on the side of entry, via uniform draw from a fine lattice.

A stream of other agent-based models emerged in the late 20th century alongside Gode and Sunder (1993). Wilson's (1987) game theoretic venture, while informative, showed the limitations complexity places on a strategic approach to modelling CDA trader behavior. Friedman (1991) and Easley and Ledyard (1993) both posited non-strategic models with variations on a reservation price mechanism, yet differed in the dynamics of interest. Friedman studied within-period pricing dynamics, while Easley and Ledyard focused on across-period dynamics. Gjerstad and Dickhaut (1998) proposed a belief-based model shortly after, though moving away from zero intelligence towards higher complexities of trader intelligence. Each model in this group is oriented in a partial equilibrium setting; very few models (or even non-theoretical works) have focused on general equilibrium dynamics in these simple CDA markets (Gode and Sunder (2004) and Crockett et al. (2008) are a couple such papers). This paper adds to the general equilibrium sub-literature, with the hope of growing the discussion moving forward.

Much research has addressed the assumptions made in Gode and Sunder (1993). Cliff and Bruten (1997) provide insight into the funneling power of the markets containing strictly decreasing demand schedules and increasing supply schedules, while Gjerstad and Shachat

[3] A uniform draw over this set places substantially more weight on lower prices. For example if $(\pi/2, \pi)$ is the feasible set of angle choices, half of the support yields prices between 0 and 1, while the other half covers $(1, \infty)$.

(2021) contends the convergence of prices in ZI simulations and calls into question how 'harmless' the no-loss constraint really is. The second main contribution of this paper lies in an expansive investigation into how impactful the model assumptions, market rules and setting attributes are on market efficiency when supposedly unimpeded by trader behavior. At the model level, I test variation in assumptions over the set of feasible orders and the choice process placed over them. Each of the assumptions made, both alone and interacted with one another, crucially impact the behavior of the traders and the performance of the market. Evidence of convergence to equilibrium predictions is increased across the board when enforcing any of the assumptions.

Section 2 defines the institution and environment in focus, while Section 3 recounts the zero intelligence models of Gode and Sunder (1993) and Gode, Spear and Sunder (2004). An alternate general equilibrium model of zero intelligence is laid out in Section 4. Section 5 maps out and analyzes a vast simulation investigation into the underlying determinants of the model and environment. Section 6 concludes.

2 Double Auction

The double auction is one of the most ubiquitous institutions used in markets around the world. The most common variety is the continuous double auction. In this section, I'll give a brief formal overview of the institution.

A double auction implies at least one agent attempts to buy some amount of a good, while another agent wishes to sell said good. Each agent provides the price he is willing to pay or receive, respectively. If the payment (or bid) made by the buying agent exceeds the price requested (ask) by the seller, the agents transact the good. There are many conventions for determining the price to be paid; the one selected in this paper, and many other market focused papers, is the crossed price convention. In this case, whoever posted the price first between the two transacting agents has his price granted. Depending on restrictions on order quantity, a crossing can either mean the full clearing of both participating orders, or just one. Unless a set unit size is required, partial clearance is not unlikely, in which case the order with any remaining desire to trade (i.e. remaining quantity) would continue to exist in the orderbook. In a similar fashion, double auctions that are unrestricted in such a way can accommodate one order crossing with multiple others on the opposite side of the book.

The continuous part of this institution is the ability for any agent in the market to place an order at any given time, regardless of whether an order exists on the other side of the market or not. Orders can also be cancelled or replaced at any moment in time. Two main restrictions are generally imposed, however. First, the agent must be able to completely fill their part of a transaction at the time of crossing, meaning he cannot post an ask for more units than he currently owns or a bid for a price at which he is not liquid. The second restriction is dependent on the timing convention of orders. The CDA studied in this paper does not allow for expiration times on orders, meaning an order only leaves the market if cancelled, replaced or filled. As such, the market restricts the trader to at most one order on each side of the market at any given time.[4]

3 Zero Intelligence

Partial Equilibrium

Gode and Sunder (1993) took the pattern of reducing intelligence in non-strategic trader behavior models for the double auction to the limit, creating the "zero intelligence" model. While most of the related models assume some kind of history- or time-dependent driving force behind pricing decisions, ZI agents pay no mind to the state or history of the market. The model can be briefly summarized as follows.

Traders are given roles as either buyers or sellers. Buyers all hold redemption value schedules $\{r_1, .., r_{n_b}\}$ where n_b denotes the number of units the buyers hold at the beginning of the market. Sellers place orders to recover unit costs for their n_s units of the single good,

[4]In markets with limit orders that contain expiration times, order-shredding (or submitting multiple limit orders with staggered expirations) is feasible.

following the cost schedule $\{c_1, .., c_{n_s}\}$. In the base form of the model, all traders choose their order price from $U[0, M]$ via i.i.d. draws. A preferred version, ZI-Constrained (ZI-C), suggests that agents participate in a market which enforces order placement conducive with unit resale at no-loss to the agents. This means buyers draw from $U[0, r_i]$ and sellers draw from $U[c_j, M]$.

Orders are restricted to single unit quantities and must reduce the best bid-ask spread to be placed. Units are transacted in order (i.e. lowest redemption value and highest cost first). After a bid crosses an ask, or vice versa, the orderbook is reset. Traders are held from re-entry until all traders have traded their k^{th} unit. Simulated markets with these traders are run until all intramarginal units have been cleared, eliminating one of the two main drivers of inefficiency.[5]

General Equilibrium

Gode, Spear and Sunder (2004) returned to the ZI paradigm about a decade later to bring zero intelligence to general equilibrium settings (I refer to this model as GSS henceforth). Their environment of choice was the simple two-good Edgeworth box. Adjustments to the original partial equilibrium model are described below.

Traders may participate on either side of the market, and in fact, participate on both sides simultaneously at each entry. Instead of redemption value and cost schedules, traders are induced with Cobb-Douglas preferences. Utility function parameters and initial endowments determine whether traders are more inclined to buy or sell. As in ZI-C, traders obey a no-loss constraint in which they will only place orders above their current indifference curve.

The price selection process is predicated on choosing some angle (i.e. relative price) that satisfies the no-loss constraint. For example, with no restriction on quantity, the angle may lie anywhere in $[MRS_c, \pi/2]$ in radians, where MRS_c is the marginal rate of substitution at the trader's current allocation. The step-size of the order in the Edgeworth box, however, is

[5] The other main driver being the trade of extramarginal units in place of intramarginal ones.

restricted. The length of the order vector is set via $r = \sqrt{x^2 + y^2}$, where r is constant for the duration of the market across all traders.[6] This means the price choice space is restricted further, such the lowest price is that of the order vector which lies secant to the trader's current indifference curve on the side of entry.

4 Model

I present an alternative version of zero intelligence in the Edgeworth box. While the flavor of the 1993 and 2004 models remains, the majority of their assumptions are relaxed to consider more generalized environments. The model rests in a two-good (X and Y) Edgeworth box with two types of traders, with rule and trader behavior adjustments as follows.

Traders are induced with constant elasticity of substitution (CES) preferences

$$u(x_i, y_i) = c_\Delta((a_\Delta x_i)^{r_\Delta} + (b_\Delta y_i)^{r_\Delta})^{\frac{1}{r_\Delta}} \tag{1}$$

which, depending on r, can represent other popular preferences such as Cobb Douglas ($r \rightarrow 0$), Leontief ($r \rightarrow \infty$) or perfect substitutes ($r = 1$). Each trader can be either a natural buyer or natural seller. As in GSS, such a distinction is determined via the utility parameters (here c, a, b and r) and initial endowments. Note that Δ denotes the type of trader, with $\Delta = b$ for natural buyers and $\Delta = s$ for natural sellers. Alternatively, a natural buyer (seller) can be described as a trader whose marginal rate of substitution is greater (less) than the competitive equilibrium (CE) price when evaluated at his initial endowment.

Traders randomly enter the market one at a time. When a trader enters, he determines the side he will place an order on by flipping a weighted coin. The weights are the relative areas[7] on either side of the market available for order placement.[8] The trader then uniformly

[6] x and y are the quantity of x and y to be traded in the event the order is crossed.

[7] Area, here, is the size of the rectangle representing all bundles that satisfy a weak improvement in one good and a weak loss in the other (i.e. a proper bid or ask).

[8] This is admittedly a mild improvement in intelligence over the natural choice of an evenly weighted coin flip. First, this choice is made to help avoid runs of entries on the same side of the market (especially in

randomly chooses an (x, y) bundle from the feasible set. As no no-loss constraint is imposed on the trader, this feasible set can be defined as follows for bids and asks:

$$Buy: \quad [X_{current,i}, X_{endow,b} + X_{endow,s}] \times [0, Y_{current,i}] \tag{2}$$

$$Sell: \quad [0, X_{current,i}] \times [Y_{current,i}, Y_{endow,b} + Y_{endow,s}] \tag{3}$$

$X_{current,i}$ is the X holding of trader i in his current allocation, and $X_{endow,b} + X_{endow,s}$ denotes the total X holding of a buyer-seller pair at the inception of a market.

The exchange does not enforce a spread reduction rule, allowing any order choice to be post-able to the orderbook. In the same vein, the orderbook is not reset upon an order crossing, meaning the book lives the length of the market trading period.

5 Simulations

In this section, I present a panel of simulations. Each assumption or rule housed in the model, market, and/or setting is incrementally varied, yielding market outcomes for each potential iteration. The reason for such an expansive investigation is two-fold: (1) it provides a proper test of the model presented in this paper, and (2) it creates the most complete test of the zero intelligence paradigm in a single paper. Below, I give the design for the panel, followed by analysis of each environment at the market level.

5.1 Design

Two defining assumptions of the model are primed for variation. First, the set of admissible orders is determined by the inclusion/exclusion of a (budget) no-loss constraint. Prior research (Gjerstad and Shachat (2021)) has pointed out the importance of the budget con-

certain treatments of the simulation panel to be described in Section 5). Second, I argue that this decision imbues far less control over the capabilities of the traders and the convergence of the market compared to enforcement of an assumption like a no-loss constraint.

straint in funneling ZI-C traders to equilibrium. Second, the selection process is one of the main deviations in this model from Gode, Spear and Sunder. The sequential process of angle (price) and quantity selection is tested against the likely less advantageous lattice choice method.

With regards to the market, Gode and Sunder (1993) made three distinct choices about the rules defining their double auction: (1) orders must be for a single unit,[9] (2) the orderbook is refreshed after any clearing takes place, and (3) trades are priced at that specified in the earlier of the crossing orders. The first two present differences in ZI and the model presented in this paper, and thus are prime variation candidates. The third assumption matches that of this paper, and is also less interesting to test[10]; as such, it is not tested here. A major assumption made in ZI (as well as a vast array of other such models) which could be considered either feature of the institution or choice of the model is the spread reduction rule. I test the (lack of) enforcement of such a rule.

Each of the five rules can be described more explicitly as follows:

Spread Reduction: An order only makes its way to the orderbook if its price reduces the current best bid-ask spread of the market. In other words, the price must be higher than the current best bid (as a bid) or lower than the current best ask.[11] Such a rule creates a funneling effect on prices, for better or worse.

[9] In GSS (2004), this is amended to have the order vector maintain some uniform length, such that the step-size on the market path is fixed. Assuming single unit order size in general equilibrium can be thought of as a halfway point between full dominion over order choice, and being constrained to a set arc of potential orders. Any results found for the single unit restriction can be expected to inflate if a step-size rule was instead enforced.

[10] The natural alternative here is an even split between the crossing prices, however this just adjusts the split of the gains from trade. While it is feasible this could impact trader behavior and potential shading strategies, zero intelligence traders are incapable of such response, making the factor relatively uninteresting to test.

[11] Two ways of enforcing this rule exist: (1) accepting all orders for submission with a spread reducing orders being accepted by the exchange and others being rejected, and (2) applying a prerequisite such that orders can only be sent to the exchange if they already satisfy a reducing rule. In essence, the same orders get to the orderbook through the exchange (or the orders that pass under rule (1) are a subset of those under rule (2)); however, the former leads to fewer orders being placed (at least when traders maintain zero intelligence).

Single Unit Orders: Many early models of double auction dynamics enforced single unit order quantities, and often even only single unit endowments. For the purpose of comparability, I don't enforce the latter, more extreme, restriction. However, the first provides an analog to the distance restriction imposed in Gode, Spear and Sunder (2004), or at least a more palatable or common version of such a restriction. (See again footnote 9).

Order Selection (Lattice/Angle): The order selection process directly impacts the shape of the distribution placed over the price support available to an agent. A sequential choice process in which the (relative) price is chosen first in and of itself can provide varying distributions on the order's price entry. For instance, consider using price (units of Y per unit of X) versus angle/radian as the base unit. A uniform distribution over radians clearly pushes the mass of the distribution over slopes towards lower prices.[12]

Orderbook Reset: Much like the single unit restriction, resetting the orderbook after a transaction occurs is common among the partial equilibrium literature, generally implemented as a simplification tool. *Need a sentence here on funneling and intramarginal unit effects*

No-loss (Budget) Constraint: Under a setting with cost and redemption schedules, a no-loss (budget) constraint implies random price draws never result in losses upon transaction. A natural GE analog requires new orders (if fully cleared) to weakly improve the trader's utility (at the time of placement).

Wholly, these factors combine to create a full factorial (simulated) experimental design with a total of 2^5 treatments. The five main effects and 27 interactions are tested in Section

[12] A quick transformation of random variables shows (taking $r \in R$ as a random draw from the set of radians in $\pi/2$ to π and $s \in S$ as a random slope drawn over $(-\infty, 0]$ that $f_R(r) = 2/\pi$ becomes $f_S(s) = (2/\pi)(\pi/180)\frac{1}{s^2+1}$.

5.2, with the version described in Section 4 representing the control/holdout. Each of the 'factors' in the design are named for analysis as follows: spread reduction (SR), single unit (SU), lattice/angle (LA), orderbook reset (OBR), and no loss (NL).

For each of the 32 variations, I run 250 simulations. Each simulation has 3600 market entries across 12 rounds. Eight computerized traders make up each market, half induced to be natural buyers and half as natural sellers.[13] At the beginning of each round, the endowments of each simulated trader is reset, as is the exchange history.

5.2 Analysis

Model Performance

	Mean	St.Dev.
Price	2.27	1.89
$\lvert Price - CE \rvert$	1.45	1.86
Alloc. Eff.	0.66	0.17
Dist. Eff.	0.33	0.13
# Trades	20.13	5.18
RMSE	2.05	8.35
SellerMRS	2.03	0.46
BuyerMRS	3.07	0.63

Table 1: Outcome means for the main model in its intended form: no spread reduction, multiple divisible units, lattice choice, no orderbook reset, and no no-loss constraint.

Table 1 provides an overview of the model, ZI-G, in its intended form (as described in section 4). Each estimate shows the average outcome across the 250 simulations ran for this state of the model. As shown, the model performs well in price space. The average round-average price falls within 0.17 units of the general CE price of 2.44 (though rather imprecisely). Average per-trade deviation in price from CE is relatively tight, with the majority of prices falling between 1 and 4 units. The root mean-squared error is moderately

[13]Buyer endowments are $(x = 3, y = 23)$ with utility parameters $(c = 0.113, a = 0.825, b = 0.175, r = 0.5)$. Seller endowments are $(x = 11, y = 3)$ with utility parameters $(c = 0.099, a = 0.6875, b = 0.3125, r = 0.5)$.

acceptable and reports a similar sentiment.

In allocations, the performance of this model variety varies depending on the statistic. Allocative efficiency, measured as the sum of utility gained relative to the sum of gains in equilibrium, shows mild convergence at 0.66, while distance efficiency, or the proportion of the distance from initial endowment to equilibrium traveled by the market, is quite poor at 0.33.[14] The marginal rate of substitution of the traders' final allocations can provide another viewpoint on convergence in allocation space. Here, the seller (buyer) MRS reported is the MRS of the final allocation of an aggregated seller (buyer) agent who aggregates over all natural sellers (buyers) in the market, so as to appear on a standard Edgeworth box. Table 1 reports the round-end MRS for these aggregated agents. In equilibrium, both measures should equal the general CE price; in this measure the market performs relatively well, with buyers and sellers lying around 0.5 units away on either side. Thus, while the distance efficiency measure suggests the market has a long way to travel, the allocative efficiency and MRS measures suggest the market has moved enough to have realized a majority of the gains from trade.

Panel Investigation

Treatment level averages are reported in Table 2; each entry being the summation of the relevant coefficients from Tables A.1-A.4. For each of the measures representative of convergence or market success, the best performing treatment's mean is bolded in black the worst performing is highlighted in red. SR:SU:OBR markets performed the worst in price measures, with the mix of lattice-choice-single-unit price inflation and lack of price funneling (as the orderbook can't properly age) results in prices well above CE. A no-loss constraint as the lone assumption produced round-average prices just 0.02 units away from CE. Average price deviation and root-mean-squared error were both minimized by an NL market type, though paired with a spread reduction rule now. NL markets not paired with an angle choice

[14]One consolation for these measures is their likely dependence on the length of the market; I report simulations with much longer-lived periods at the end of the section to check this.

					Outcome:									
SR	SU	LA	OBR	NL	Price	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.	Dist. Eff.
0	0	0	0	0	2.27 (1.89)	1.45 (1.86)	2.05 (8.35)	14.94 (1.04)	20.13 (5.18)	3.15 (0.68)	2.03 (0.46)	3.07 (0.63)	0.66 (0.17)	0.33 (0.13)
0	0	0	0	1	**2.42 (0.61)**	0.63 (0.36)	0.69 (0.38)	15.29 (0.87)	2.03 (1.19)	3.48 (1.39)	1.32 (0.28)	4.36 (0.68)	0.35 (0.19)	0.26 (0.15)
0	0	0	1	0	3.12 (3.09)	2.26 (3.05)	4.15 (11.78)	14.8 (1.02)	15.02 (3.38)	3.47 (0.84)	1.99 (0.41)	3.11 (0.54)	0.68 (0.15)	0.34 (0.13)
0	0	0	1	1	2.42 (0.64)	0.64 (0.38)	0.69 (0.40)	15.33 (0.89)	1.77 (0.98)	3.66 (1.49)	1.29 (0.27)	4.44 (0.66)	**0.34 (0.19)**	0.25 (0.15)
0	0	1	0	0	1.62 (1.78)	1.66 (1.77)	4.15 (21.39)	38.14 (289.29)	157.09 (9.88)	1.99 (0.20)	2.08 (0.57)	3.08 (0.60)	0.67 (0.17)	0.15 (0.16)
0	0	1	0	1	1.99 (0.16)	0.65 (0.11)	0.77 (0.11)	2.61 (0.21)	30.01 (4.69)	0.50 (0.08)	1.72 (0.11)	3.40 (0.18)	0.83 (0.06)	0.60 (0.06)
0	0	1	1	0	9.54 (155.42)	9.54 (155.42)	**68.02 (1476.74)**	33.78 (210.17)	87.25 (5.77)	1.92 (0.26)	2.03 (0.48)	3.06 (0.53)	0.73 (0.14)	0.16 (0.15)
0	0	1	1	1	1.96 (0.19)	0.80 (0.12)	0.93 (0.13)	2.67 (0.26)	24.39 (3.96)	0.53 (0.11)	1.57 (0.12)	3.65 (0.24)	0.76 (0.08)	0.52 (0.07)
0	1	0	0	0	15.15 (1.53)	12.72 (1.53)	14.06 (1.67)	1.00 (0.00)	29.36 (4.58)	0.92 (0.05)	1.77 (0.24)	3.70 (0.69)	0.64 (0.17)	0.24 (0.08)
0	1	0	0	1	4.07 (0.46)	1.67 (0.43)	1.79 (0.39)	1.00 (0.00)	3.90 (1.55)	1.00 (0.00)	1.23 (0.15)	4.78 (0.40)	0.42 (0.14)	0.24 (0.09)
0	1	0	1	0	14.91 (1.75)	12.55 (1.73)	14.48 (1.99)	1.00 (0.00)	25.87 (3.96)	0.92 (0.05)	1.78 (0.25)	3.67 (0.71)	0.64 (0.18)	0.24 (0.09)
0	1	0	1	1	4.07 (0.47)	1.68 (0.43)	1.80 (0.39)	1.00 (0.00)	3.75 (1.51)	1.00 (0.00)	**1.21 (0.15)**	**4.81 (0.39)**	0.41 (0.14)	0.23 (0.09)
0	1	1	0	0	1.26 (0.22)	1.53 (0.11)	1.77 (0.25)	1.00 (0.00)	94.45 (5.23)	0.89 (0.03)	1.95 (0.22)	3.04 (0.22)	0.87 (0.06)	0.53 (0.08)
0	1	1	0	1	1.94 (0.20)	0.72 (0.11)	0.84 (0.12)	1.00 (0.00)	18.41 (1.80)	1.00 (0.00)	1.96 (0.13)	3.05 (0.17)	0.91 (0.04)	0.71 (0.06)
0	1	1	1	0	1.67 (0.28)	1.85 (0.20)	2.43 (0.51)	1.00 (0.00)	71.05 (4.48)	0.90 (0.04)	1.95 (0.22)	3.05 (0.29)	0.84 (0.08)	0.50 (0.10)
0	1	1	1	1	1.99 (0.20)	0.77 (0.11)	0.90 (0.13)	1.00 (0.00)	17.54 (1.73)	1.00 (0.00)	1.91 (0.13)	3.11 (0.17)	0.90 (0.04)	0.69 (0.06)
1	0	0	0	0	1.97 (1.61)	1.18 (1.58)	1.80 (8.77)	9.01 (1.43)	27.28 (5.19)	2.93 (0.52)	2.06 (0.46)	3.03 (0.56)	0.66 (0.17)	0.36 (0.13)
1	0	0	0	1	2.28 (0.32)	**0.45 (0.20)**	**0.53 (0.22)**	9.60 (2.29)	4.70 (1.51)	3.18 (0.82)	1.79 (0.35)	3.39 (0.59)	0.63 (0.15)	0.48 (0.13)
1	0	0	1	0	3.74 (13.55)	2.81 (13.54)	6.72 (46.79)	11.78 (1.21)	17.35 (3.31)	3.50 (0.76)	2.03 (0.41)	3.04 (0.50)	0.71 (0.15)	0.35 (0.13)
1	0	0	1	1	2.33 (0.36)	0.52 (0.22)	0.61 (0.26)	11.28 (1.88)	3.34 (0.97)	3.77 (1.08)	1.64 (0.31)	3.64 (0.58)	0.61 (0.16)	0.46 (0.13)
1	0	1	0	0	2.41 (8.12)	2.51 (8.12)	11.51 (100.31)	12.04 (123.6)	144.69 (8.52)	1.93 (0.21)	**2.12 (0.58)**	3.06 (0.69)	0.68 (0.17)	**0.13 (0.16)**
1	0	1	0	1	2.04 (0.15)	0.61 (0.11)	0.74 (0.12)	1.37 (0.26)	35.39 (4.50)	0.46 (0.06)	1.82 (0.10)	3.23 (0.14)	0.87 (0.04)	0.65 (0.05)
1	0	1	1	0	7.35 (33.47)	7.34 (33.46)	43.49 (299.96)	19.97 (42.80)	82.36 (4.97)	1.89 (0.27)	2.07 (0.44)	3.02 (0.51)	0.75 (0.13)	0.16 (0.15)
1	0	1	1	1	1.93 (0.17)	0.84 (0.11)	0.96 (0.12)	2.28 (0.26)	26.67 (3.33)	0.50 (0.09)	1.59 (0.10)	3.60 (0.18)	0.78 (0.06)	0.53 (0.06)
1	1	0	0	0	15.20 (1.52)	12.79 (1.52)	14.08 (1.59)	1.00 (0.00)	32.33 (4.27)	0.93 (0.05)	1.79 (0.23)	3.65 (0.68)	0.66 (0.17)	0.24 (0.08)
1	1	0	0	1	4.63 (0.37)	2.21 (0.35)	2.27 (0.30)	1.00 (0.00)	4.49 (1.57)	1.00 (0.00)	1.34 (0.15)	4.54 (0.40)	0.52 (0.14)	0.29 (0.09)
1	1	0	1	0	**15.27 (1.73)**	**12.92 (1.71)**	14.97 (1.98)	1.00 (0.00)	28.19 (3.84)	0.92 (0.05)	1.79 (0.25)	3.64 (0.72)	0.65 (0.18)	0.24 (0.09)
1	1	0	1	1	4.53 (0.40)	2.13 (0.36)	2.21 (0.30)	1.00 (0.00)	4.41 (1.55)	1.00 (0.00)	1.32 (0.15)	4.58 (0.40)	0.50 (0.14)	0.29 (0.09)
1	1	1	0	0	1.42 (0.25)	1.66 (0.15)	2.04 (0.37)	1.00 (0.00)	87.40 (4.70)	0.90 (0.03)	1.97 (0.22)	3.03 (0.24)	0.86 (0.06)	0.52 (0.09)
1	1	1	0	1	1.91 (0.20)	0.74 (0.12)	0.86 (0.12)	1.00 (0.00)	19.90 (1.76)	1.00 (0.00)	2.05 (0.13)	**2.93 (0.15)**	**0.94 (0.03)**	**0.75 (0.06)**
1	1	1	1	0	1.73 (0.29)	1.90 (0.21)	2.53 (0.52)	1.00 (0.00)	72.48 (4.29)	0.90 (0.04)	1.96 (0.22)	3.03 (0.29)	0.85 (0.07)	0.50 (0.09)
1	1	1	1	1	1.99 (0.20)	0.78 (0.11)	0.91 (0.12)	1.00 (0.00)	18.78 (1.67)	1.00 (0.00)	2.00 (0.13)	3.00 (0.16)	0.92 (0.03)	0.73 (0.06)

Table 2: Outcome averages by treatment. Observations are at the round-average or round-end level. The left panel shows the assumptions enforced. Black bolded estimates are the 'best' in the column, while red are the 'worst'.

method seemed to perform poorly in allocation measures, likely due to higher variation in prices and thus more frequent deviation from the equilibrium path. Markets that are fully restricted, aside from orderbook resetting, lead the pack in both measures of efficiency, as well as buyer MRS. GSS markets outperform ZI-G markets in both measures of efficiency and both measures of price variation, with MRS splits being with 0.1 of each other and the ZI-G average price mildly outperforming GSS prices (though insignificantly).

	Dependent variable:									
	Price	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.	Dist. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Spread Red.	0.012	0.012	−0.842	−3.765***	0.484***	−0.025***	0.095***	−0.181***	0.057***	0.043***
	(0.185)	(0.185)	(1.736)	(0.436)	(0.128)	(0.005)	(0.002)	(0.004)	(0.001)	(0.001)
Single Unit	2.641***	2.168***	−4.386**	−12.432***	−9.196***	−1.341***	−0.074***	0.217***	0.051***	0.076***
	(0.185)	(0.185)	(1.736)	(0.436)	(0.128)	(0.005)	(0.002)	(0.004)	(0.001)	(0.001)
Lattice/Angle	−3.489***	−2.175***	3.739**	0.677	47.743***	−1.086***	0.271***	−0.692***	0.255***	0.186***
	(0.185)	(0.185)	(1.736)	(0.436)	(0.128)	(0.005)	(0.002)	(0.004)	(0.001)	(0.001)
OB Reset	1.004***	1.017***	6.656***	0.555	−13.208***	0.100***	−0.054***	0.069***	−0.006***	−0.019***
	(0.185)	(0.185)	(1.736)	(0.436)	(0.128)	(0.005)	(0.002)	(0.004)	(0.001)	(0.001)
No Loss	−3.497***	−4.423***	−11.915***	−5.878***	−48.301***	−0.257***	−0.349***	0.573***	−0.054***	0.169***
	(0.185)	(0.185)	(1.736)	(0.436)	(0.128)	(0.005)	(0.002)	(0.004)	(0.001)	(0.001)
Round	−0.010	−0.009	−0.100	0.053	−0.002	−0.001	0.00004	0.0002	−0.00004	−0.0001
	(0.027)	(0.027)	(0.252)	(0.063)	(0.019)	(0.001)	(0.0003)	(0.001)	(0.0002)	(0.0002)
Constant	6.142***	4.967***	11.083***	17.294***	49.120***	2.935***	1.841***	3.496***	0.543***	0.169***
	(0.286)	(0.285)	(2.684)	(0.674)	(0.198)	(0.008)	(0.003)	(0.006)	(0.002)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000	96,000
R^2	0.010	0.009	0.001	0.011	0.756	0.532	0.320	0.394	0.417	0.389
Adjusted R^2	0.010	0.009	0.001	0.011	0.756	0.532	0.320	0.394	0.417	0.389
Note:										*p<0.1; **p<0.05; ***p<0.01

Table 3: Main effects or first differences, with a round time control added. The independent variables are treatment arm indicators.

Panel Main Effects

While a full factorial analysis is informative of the incremental response to variation in the market's design, the main effect of relaxing or enforcing an assumption may be hard to discern.[15] Tables 3 and 4 show such impacts for a variety of variables in prices, efficiencies and measures of volatility.

In first differences, allocative and distance efficiencies show quite different trends, though

[15] See appendix A for such an analysis.

with a unifying mechanism. Spread reduction and single unit assumptions show small, yet significant improvements in allocative efficiency while orderbook resetting and no-loss constraints lead to mild reductions. Angle choice leads to a rather large improvement, likely driven by the more CE-localized trade prices and increased trade count. Distance efficiency on the other hand sees substantial improvements across the board. Such a mild response in one measure and improvement in the other points towards a more equitable redistribution of gains from trade in utility-terms so as to bring the path traveled by the market closer to the equilibrium path.

Prices are less consistently impacted by the enforcement or relaxation of the tested assumptions. Spread reduction inclusion sees zero adjustment in average price; Figure B.1 confirms the distributions are nearly identical. The other four assumptions are split in their direction of change. Single unit order and orderbook reset constraints both inflate prices on average, though with much different responses in their distributions. A massive redistribution, or flattening, occurs with single-unit-order constrained markets, while orderbook resetting markets see a small shift right in price (as shown in Figure B.1). Markets with traders who either use an angle choice process for their orders or follow a no-loss constraint report nearly identical adjustments in average round-average price, over halving the estimates from above 6 units of y per unit of x, to relative prices just over 0.2 units above the CE prediction. Distributional changes, however, are quite different between these two sets of markets. Figure B.1 shows the mass of the distribution funnelling quite close to the mean for angle-choice markets, however the support for the distribution remains unchanged. No-loss constrained markets also show tighter mass near the CE-price price. The massive reduction in the size of the support is perhaps the more remarkable result when giving traders the intelligence to always obey their own preferences.

A check of second order treatment pair impacts (without concern for higher order pairings), as shown in Table 4, reveals stark contrasts for single treatment adjustments within pairs (i.e. the interaction effects). To begin, the average price deviation within SU markets is shown to have been largely weighed down by those which also employed an angle choice

	Dependent variable:									
	Price	$\lvert Price - CE \rvert$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.	Dist. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Spread Red. (SR)	0.094	0.079	0.385	−9.550***	0.674***	−0.119***	0.124***	−0.247***	0.076***	0.054***
	(0.411)	(0.411)	(3.880)	(0.973)	(0.128)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
Single Unit (SU)	9.628***	8.365***	6.356	−20.844***	−6.632***	−2.702***	−0.347***	0.708***	0.015***	−0.033***
	(0.411)	(0.411)	(3.880)	(0.973)	(0.128)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
Lattice/Angle (LA)	−1.551***	−0.422	14.188***	8.769***	101.625***	−1.663***	0.006	−0.003	0.089***	−0.099***
	(0.411)	(0.411)	(3.880)	(0.973)	(0.128)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
OB Reset (OBR)	2.356***	2.321***	15.721***	0.213	−21.573***	0.260***	−0.030***	0.010	0.034***	0.006***
	(0.412)	(0.412)	(3.888)	(0.973)	(0.128)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
No Loss (NL)	−3.084***	−3.813***	−7.387*	−7.879***	−37.885***	−0.188***	−0.659***	1.103***	−0.204***	0.047***
	(0.414)	(0.414)	(3.906)	(0.973)	(0.128)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
SR:SU	0.322	0.336	1.996	7.529***	−0.055	0.065***	−0.081***	0.164***	−0.053***	−0.045***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SR:LA	−0.379	−0.359	−2.644	−2.856***	−4.096***	0.021***	−0.092***	0.234***	−0.080***	−0.059***
	(0.368)	(0.368)	(3.472)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SR:OBR	−0.247	−0.248	−3.609	2.212**	0.767***	0.086***	−0.027***	0.032***	−0.001	−0.003**
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SR:NL	0.195	0.183	1.941	4.685***	3.002***	0.004	0.144***	−0.301***	0.096***	0.085***
	(0.368)	(0.368)	(3.472)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SU:LA	−9.076***	−7.882***	−20.961***	−1.353	−28.563***	2.159***	0.336***	−0.898***	0.152***	0.356***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SU:OBR	−1.852***	−1.887***	−12.634***	−1.110	14.377***	−0.202***	0.077***	−0.097***	−0.013***	0.011***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
SU:NL	−3.310***	−2.915***	10.276***	11.756***	9.112***	0.686***	0.217***	−0.155***	−0.014***	−0.106***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
LA:OBR	1.393***	1.416***	11.134***	0.022	−20.287***	−0.212***	−0.033***	0.032***	−0.012***	−0.024***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
LA:NL	4.243***	3.368***	−8.305**	−11.998***	−54.817***	−0.828***	0.322***	−0.751***	0.271***	0.298***
	(0.368)	(0.368)	(3.472)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
OBR:NL	−2.011***	−1.902***	−13.090***	−0.440	21.871***	0.011	−0.065***	0.152***	−0.053***	−0.034***
	(0.368)	(0.368)	(3.471)	(0.870)	(0.115)	(0.007)	(0.004)	(0.006)	(0.002)	(0.001)
Round	−0.010	−0.010	−0.100	0.053	−0.002	−0.001	0.00003	0.0002	−0.00004	−0.0001
	(0.027)	(0.027)	(0.251)	(0.063)	(0.008)	(0.001)	(0.0003)	(0.0005)	(0.0001)	(0.0001)
Constant	3.424***	2.463***	2.020	19.406***	34.448***	3.392***	2.039***	3.102***	0.617***	0.289***
	(0.407)	(0.406)	(3.836)	(0.962)	(0.127)	(0.008)	(0.004)	(0.007)	(0.002)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000	96,000
R^2	0.019	0.016	0.002	0.016	0.951	0.791	0.437	0.572	0.584	0.719
Adjusted R^2	0.019	0.016	0.002	0.016	0.951	0.791	0.437	0.572	0.584	0.719
Note:										*p<0.1; **p<0.05; ***p<0.01

Table 4: Treatment effects including second order interactions. Treatment labels are color-coded to correspond to rule orientation. Market-oriented rules are cooler colors (i.e., SR, SU, and OBR) and behavior-oriented rules are warmer colors (LA and NL).

process (LA=1), implying the combination of a set step size and a price choice distribution heavily skewed to favor lower prices (and thus closer to the equilibrium price) greatly abates the pattern of large price deviations seen in non-angle-choice SU markets. A similar story can be seen with buyer and seller MRS in SU markets. Pairing the distance restriction that is SU with a choice restriction on prices provides significant guidance for the ZI traders. Within SU markets, those with LA=1 entirely revers and even improve the negative impact SU has on MRS spreads. In fact, pairing SU with either of the behaviorally-oriented rules (LA and NL) improves any measure associated with allocation space.[16] The mirror to these adjustments show similar trends as well, with LA markets finding SU and NL pairings making up most of the benefit found in LA estimates from Table 3. In fact, LA market impacts on efficiency relative to non LA markets reverse in some cases when accounting for interaction effects.

Perhaps more generally, within treatment types as defined by a single rule, splitting the markets by a rule of a different type (i.e. splitting markets that are characterized by a market-oriented rule by a behaviorally-oriented rule) shows far better performance in those with both rules enforced as opposed to one. This is less true for pairings of the same type (e.g. LA:NL or SU:OBR). A finer investigation into the different treatments (looking at the full five factor combination) can be found in Appendix A.

5.3 A Further Look at Prices and Efficiencies

As alluded to in the primary analysis, the five rules defining this paper's panel investigation can be partitioned into two categories: market-oriented rules and behaviorally-oriented rules. Section 5.2 suggests evidence of heterogeneous impacts on key price and efficiency measures between the two categories. Figure 1 presents a heatmap of efficiencies across the 32 treatments, with one axis reporting the enforced market rules and the other axis showing the

[16]Pairing NL with SU, on average, proves to be a detriment to price estimates when not further parsing the markets into smaller treatment groups. Comparatively, the pairing with LA proves to be more beneficial in the allocation measures, despite both pairings yielding positive changes.

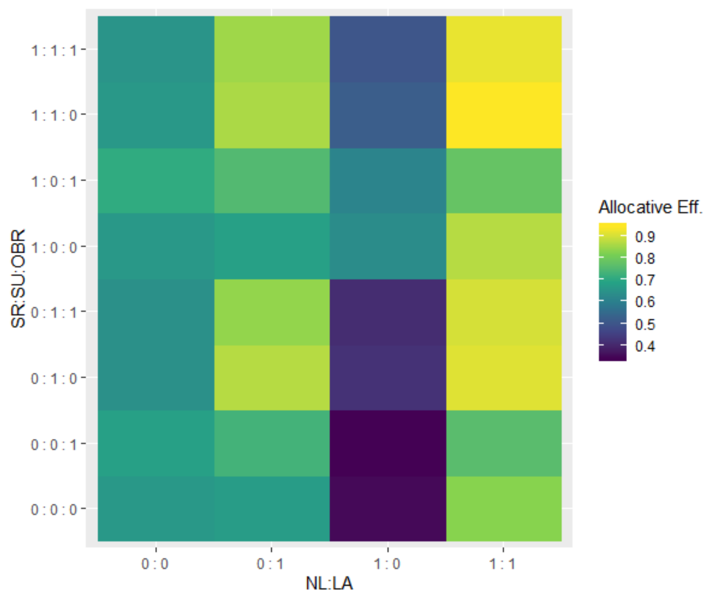

Figure 1: A heatmap of allocative efficiencies plotted over the SR:SU:OBR×NL:LA pairings. The SR:SU:OBR axis marks denote the indicator values for each of the three market-oriented rules. Similarly, NL:LA axis marks show the indicator values for the behaviorally-oriented rules.

enforced behavioral rules. The figure serves as a re-imagination of the estimates presented in the allocative efficiency column of Table 2.

Clearly, column by column comparison reveals far more heterogeneous hues as compared to a row based adjustment. A one step move on the NL:LA axis reveals large consequences in equilibration, with within-row deviations over 0.3 in all but two rows (1:0:0 and 1:0:1). Row to row adjustments are far more tame, though two pairs of row clusters are apparent. It turns out these clusters are exactly partitioned by the inclusion of SU as an enforced rule. Within row, SU=0 rows show muted hue adjustment across columns[17], while SU=1 rows show adjustments of close to 0.5 when moving from the 1:0 to 1:1 column.

Such heterogeneity across rule type and within rule may persist past just allocative efficiency; Figure 2 presents bivariate densities over allocative efficiency and round-average price for each rule switch. A few trends appear upon inspection. Firstly, the density shape and location match remarkably closely within rule type (with the top row showing market

[17]SU=0 rows which also satisfy SR=0 exhibit a large difference in the 1:0 column, but are rather homogeneous in the other three columns.

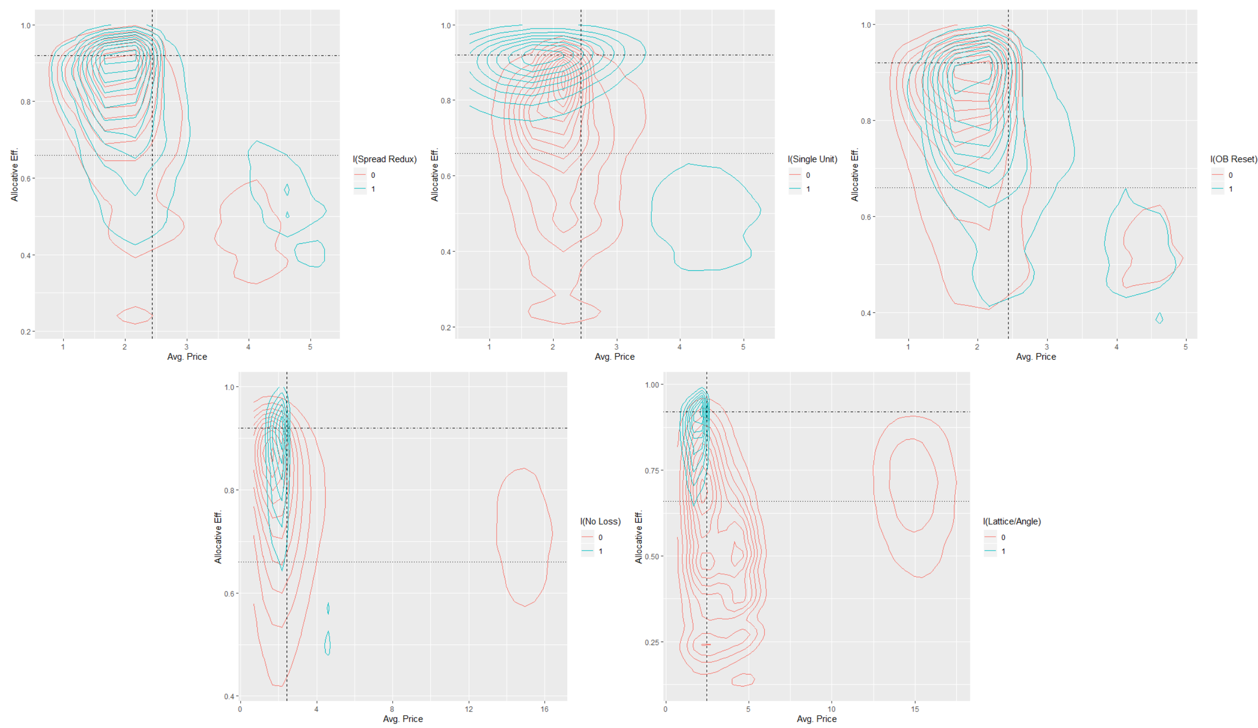

Figure 2: Bivariate densities over average price and allocative efficiency. Densities are separated based on inclusion or exclusion of each rule. The upper horizontal dotted line shows the allocative efficiency of the 1:1:1:1:1 markets; the lower horizontal dotted line shows the allocative efficiency of the 0:0:0:0:0 markets; the vertical dotted line shows the competitive equilibrium price at endowment.

rules and the bottom showing behavioral rules). Secondly, within market rules, only SU reveals stark differences in distributional shape, with the allocative efficiency axis providing the majority of the transformation. The behavioral rules, however, both show markedly large reductions in distributional footprint. Both NL and LA show condensed supports on both axes, of approximately the same size. Thirdly, only NL and LA reflect the ability to fit (almost) entirely within the bounds of the two main ZI model's efficiency estimates and stay relatively close to the equilibrium price throughout the density. Clearly, on their own, enforcement of the behavioral rules proves particularly heavy-handed in the guidance of market performance.

To target prices more directly, without reference to efficiency, Figure 3 reports binned averages for trade price over time within period.[18] By definition, the floors of each of the

[18]All periods within a run where treated equally and without regard for order as no information is carried

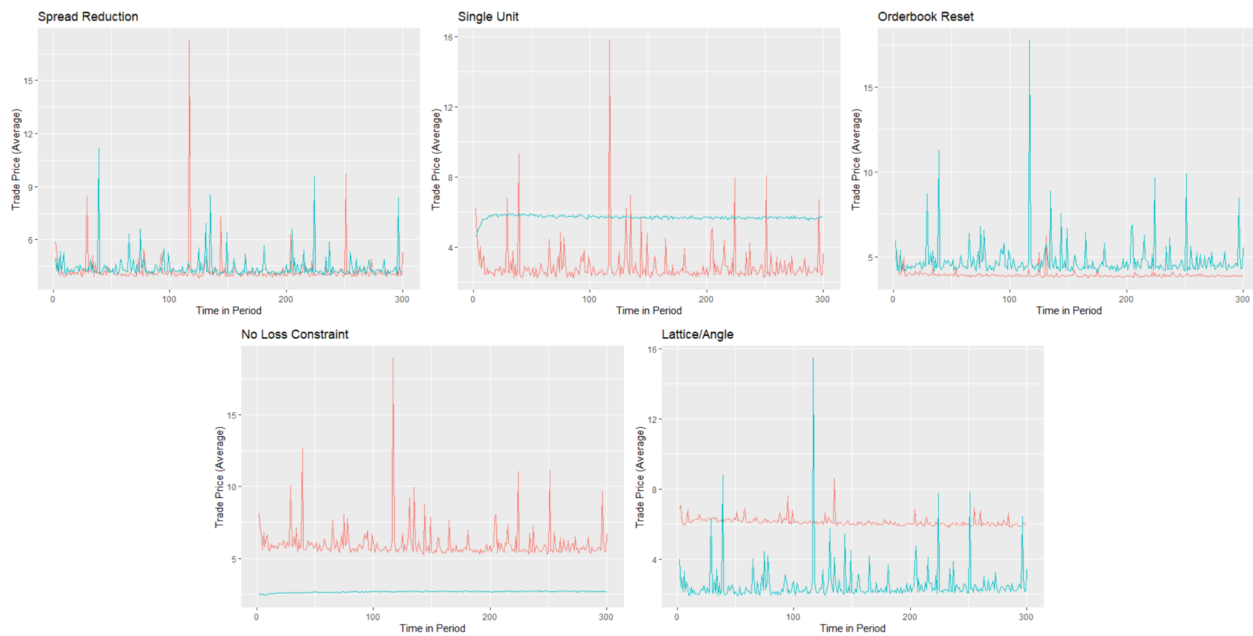

Figure 3: Time series of binned averages at the one 'second' level. Averages are taken at each second marker over the 3000 rounds in each treatment. Then, these are averaged at each second marker across within the half of the treatments that satisfy Rule=0 or Rule=1. The blue line shows Rule=1 binned averages and the salmon line shows Rule=0 binned averages.

time series plotted reveal a slope of zero, as no learning takes place in ZI markets. Volatility in trade price and the level of the trend's floor may vary however, and indeed do so in the figure. Much like Figure 2, SR and OBR report little of interest. Both show matching floors (well above CE price, but equivalent regardless of rule indicator value), though OBR markets do show far more price variation than non-OBR markets. SU markets show stabler binned averages than non-SU markets, but much further from the CE price. Behaviorally-oriented rules show similar floor levels to each other, with Rule=1 markets have a floor far closer to the CE price. Where NL and LA deviate is in price variation, with NL=0 and LA=1 markets revealing higher volatility and NL=1 and LA=0 markets appearing more stable.

over between periods and ZI agents do not have the capacity to learn across rounds (in any treatment included in this study).

6 Conclusion

Understanding both the implications of the rules implied by a market institution and the underlying behavior defining trader actions in said market are crucial tasks in economic research. The main issue plaguing such an endeavor is the entanglement between the two. One way to isolate the first study is to place traders with no strategic behavior in the market, so as to let market outcomes be only guided by the rules of the institution. Gode and Sunder (1993) proposed such a model and test in a partial equilibrium setting, and then brought the model to a general equilibrium via the Edgeworth box (Gode, Spear and Sunder (2004)). The proposed zero intelligence traders however either abided by potentially influential behavioral assumptions (such as a no-loss constraint, or an order choice process giving more weight to less aggressive prices), or participated in markets with rules that may guide the allocation path. This paper provides (1) a more generalized, lower-'zero' version of zero intelligence in an edgeworth box, and (2) tests the assumptions made in this model and those mentioned.

Agents participate as two-way traders in a CDA, where there are two goods and two types of traders (thus constructing an Edgeworth box). Despite being induced with CES preferences, and predisposed to prefer one order type to another based on the curvature of said preferences, traders enter the market (and particular side) randomly. Orders are uniformly drawn from a fine lattice placed over the set of all bundles satisfying the properties of a transaction occurring (a gain in one good and loss in the other), with no regard for gains or losses in utility from trade.

Empirically, I test the major assumptions made in ZI models, as well as those made in the models' respective CDA markets, via a novel, expansive simulation procedure. All combinations of the five rule variants either relaxed or enforced are simulated with traders from this paper's model. Each variation was simulated 250 times with each run containing 3600 entries, yielding a data set of 28.8 million market entries and order placements across 96,000 trading periods in 8000 simulated markets. First differences show an improvement across the board when imposing one of the five rules, as well as a reorganization of gains from trade resulting in systematic improvements in distance efficiency. An interactions model reports the incremental impact of these assumptions in the full factorial design.

Compared to the traders and setting of Gode, Spear and Sunder (2004), this model is shown to provide a much lower level of zero intelligence in a much less constrained version of general equilibrium (despite also residing in an Edgeworth box). Allocative and distance efficiencies are 0.28 and 0.42 units lower than those found in Gode, Spear and Sunder, while average price is slightly (though insignificantly) closer to CE and price volatility is twice as large in the lower-zero model. When paired with another price-funneling-prone assumption, both spread reduction and no-loss rules provide large improvements in market performance. Interestingly, markets with either {pairs of enforced rules} or {at most one relaxed assumption} exhibit the most equilibrating tendencies. Rather unsurprisingly, markets in which traders' order choice behavior is dictated by an angle-choice process or a no-loss tendency benefit immensely in market performance from the added intelligence.

Hopefully, this project displays the benefits of and need for modelling ventures into the wilderness of bounded rationality via the minimal intelligence gate. Many subfields of economics outside of markets could benefit from such practice. Additionally, investigations in more complex, general equilibrium settings are needed and should be pursued moving forward. The world doesn't usually operate in stylized environments with well-known supply and demand schedules or traders content with the exchange on one indivisible unit of a single good.

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Appendix A Regression Table Continued

	Dependent variable:								
	Price	$\lvert Price - CE \rvert$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Spread Reduction (SR)	−0.298	−0.267	−0.249	−5.930***	7.145***	−0.220***	0.029***	−0.035***	0.007**
	(0.732)	(0.732)	(6.918)	(1.738)	(0.108)	(0.013)	(0.008)	(0.012)	(0.003)
Single Unit (SU)	12.885***	11.272***	12.008*	−13.942***	9.230***	−2.235***	−0.264***	0.636***	−0.014***
	(0.732)	(0.732)	(6.918)	(1.738)	(0.108)	(0.013)	(0.008)	(0.012)	(0.003)
Lattice/Angle (LA)	−0.643	0.208	2.092	23.201***	136.953***	−1.161***	0.042***	0.013	0.012***
	(0.732)	(0.732)	(6.918)	(1.738)	(0.108)	(0.013)	(0.008)	(0.012)	(0.003)
OB Reset (OBR)	0.857	0.817	2.098	−0.139	−5.112***	0.314***	−0.048***	0.041***	0.028***
	(0.732)	(0.732)	(6.918)	(1.738)	(0.108)	(0.013)	(0.008)	(0.012)	(0.003)
No Loss (NL)	0.155	−0.816	−1.368	0.352	−18.100***	0.324***	−0.715***	1.292***	−0.303***
	(0.748)	(0.748)	(7.069)	(1.738)	(0.108)	(0.013)	(0.008)	(0.013)	(0.003)
⋮									
Constant	2.268***	1.447***	2.054	14.942***	20.132***	3.155***	2.035***	3.066***	0.656***
	(0.518)	(0.517)	(4.892)	(1.229)	(0.076)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000
R^2	0.024	0.020	0.003	0.019	0.989	0.826	0.464	0.596	0.607
Adjusted R^2	0.023	0.020	0.002	0.019	0.989	0.826	0.464	0.596	0.607
Note:									*p<0.1; **p<0.05; ***p<0.01

Table A.1: Interaction regression results. First order effects are reported here, the rest are in tables to follow.

Tables A.1-A.3 present the treatment analysis of the full factorial design (along with Appendix A). Each of the five assumptions/rules being tested is given an indicator, $I(rule)$, with a value of 1 representing the presence of the constraint in the simulations. The estimation process is represented the following interaction design:

$$Y_i = \alpha + \sum_{i \in Rules} \beta_i I(i=1) + \sum_{i \in Rules} \sum_{j \in Rules_{/\{i\}}} \beta_{ij} I(i=1) I(j=1) + \cdots + \beta_{ijklm} \prod_{i \in Rules} I(i=1) \quad (4)$$

The main effects of the model, i.e. first summation from 4, is provided in Table A.1.[19] Prices seem to be more sensitive to interactions of the main assumptions, as opposed to having only one included. Round-average price, price deviation from CE, and RMSE all

[19]By construction, the constants report the estimates from Table 1.

show insignificant effects for all assumptions aside from a constraint imposing single unit orders. Collapsing the lattice of possible bundles to a subset of bundles along the line $q = 1$ naturally increases likelihood of higher prices, yielding the massive increase in price and price variation when only the SU assumption is imposed. As might be anticipated, imposing a spread reduction (while leaving all other assumptions unimposed) seems to funnel activity in the market, leading to a very tight MRS spread ($\sim$0.10 on either side of the CE price) and a mild, but significant improvement in allocative efficiency. From the control, swapping to an angle choice process has a similar impact on activity, though with a slightly larger efficiency gain and no improvement in final buyer allocations. Resetting the orderbook in an otherwise unconstrained market sees the largest gain in efficiency, though a larger spread in final MRS, likely pointing to less even gains across traders with a few seeing larger gains. Adding a no-loss constraint on its own seems to be harmful to the success of the market, however this is likely due to its interaction with the lattice choice method.Forcing bundles to be chosen above the indifference curve naturally imposes higher likelihoods for prices less likely to result in trades; as is reflecting in the 90% reduction in trade count.

The second order interactions are reported in Table A.2. Much like Table A.1, prices see little adjustment (of any significance) aside from a few interesting interactions. Swapping to an angle choice procedure while SU is imposed (and all other assumptions relaxed) essentially reverses the massive inflation in prices seen in SU estimates from Table A.1. Weak utility improvement imposition in a single unit order market provides the same regression in prices. Resetting the orderbook in a lattice choice market (with no other constraints) doesn't allow for funneling of prices, leading to price inflation. Adding a second constraint systematically reduces round-average trade counts across the board, with the lone exceptions both involving orderbook resetting.

Allocation adjustments seem to be the main beneficiary of imposing a second assumption. All significant estimates for seller MRS being positive, paired with all-but-one significant estimates being negative implies convergence in allocation space. A few act as recoveries, with the damage of orderbook resetting, the no-loss constraint and single unit orders being reclaimed by inclusions of a second constraint. Angle choice and spread reduction restrictions are especially effective in progressing NL markets to a more successful final allocation. Similarly, the vast majority of interactions reflect in an increase in efficiency. Larger improvements are reflective of reversals for NL markets mostly, while smaller improvements are most often continuations of efficiency gain in SR and LA markets.

	Dependent variable:								
	Price	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
⋮									
SR:SU	0.347	0.336	0.267	5.930**	−4.172***	0.226***	−0.009	−0.017	0.006
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
SR:LA	1.082	1.117	7.618	−20.169***	−19.542***	0.159***	0.011	0.020	0.003
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
SR:OBR	0.911	0.808	2.823	2.910	−4.815***	0.256***	0.015	−0.028	0.017***
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
SR:NL	0.156	0.091	0.091	0.237	−4.473***	−0.083***	0.441***	−0.938***	0.269***
	(1.046)	(1.046)	(9.891)	(2.458)	(0.153)	(0.018)	(0.011)	(0.018)	(0.005)
SU:LA	−13.252***	−11.392***	−14.385	−23.201***	−71.869***	1.134***	0.141***	−0.674***	0.212***
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
SU:OBR	−1.097	−0.987	−1.680	0.139	1.625***	−0.315***	0.054***	−0.071***	−0.025***
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
SU:NL	−11.234***	−10.236***	−10.908	−0.352	−7.365***	−0.244***	0.174***	−0.219***	0.086***
	(1.047)	(1.047)	(9.899)	(2.458)	(0.153)	(0.018)	(0.011)	(0.018)	(0.005)
LA:OBR	7.055***	7.068***	61.778***	−4.222*	−64.725***	−0.388***	0.004	−0.065***	0.038***
	(1.035)	(1.035)	(9.784)	(2.458)	(0.153)	(0.018)	(0.011)	(0.017)	(0.005)
LA:NL	0.213	−0.186	−2.006	−35.880***	−108.977***	−1.817***	0.355***	−0.976***	0.464***
	(1.047)	(1.046)	(9.893)	(2.458)	(0.153)	(0.018)	(0.011)	(0.018)	(0.005)
OBR:NL	−0.855	−0.812	−2.095	0.178	4.852***	−0.131***	0.014	0.038**	−0.045***
	(1.059)	(1.059)	(10.009)	(2.458)	(0.153)	(0.019)	(0.011)	(0.018)	(0.005)
⋮									
Constant	2.268***	1.447***	2.054	14.942***	20.132***	3.155***	2.035***	3.066***	0.656***
	(0.518)	(0.517)	(4.892)	(1.229)	(0.076)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000
R^2	0.024	0.020	0.003	0.019	0.989	0.826	0.464	0.596	0.607
Adjusted R^2	0.023	0.020	0.002	0.019	0.989	0.826	0.464	0.596	0.607
Note:									*p<0.1; **p<0.05; ***p<0.01

Table A.2: Interaction regression results for second order interactions. This is a continuation of the regression estimates in Table 2.

Third and fourth order interactions are reported in Appendix A. Tertiary interactions (Table A.4) show mostly decays in market success. Most, if not all, of the improvements seen in Table A.2 are reversed when adding a third assumption to the market (assuming the remaining two assumptions are relaxed). As most of the measures have one or two seemingly negatively-associated assumptions, and each assumption is enforced in six of the ten tertiary interactions, a systematic mild decay is not overly surprising. Quaternary interactions are reported in Table **??**.

	Dependent variable:								
	Price	$\lvert Price - CE \rvert$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
⋮									
SR:SU:LA:OBR:NL	−3.235	−3.403	−33.996	10.169	4.865***	0.170***	−0.072**	0.099**	−0.004
	(2.937)	(2.937)	(27.764)	(6.952)	(0.431)	(0.052)	(0.031)	(0.049)	(0.014)
Constant	2.268***	1.447***	2.054	14.942***	20.132***	3.155***	2.035***	3.066***	0.656***
	(0.518)	(0.517)	(4.892)	(1.229)	(0.076)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000
R^2	0.024	0.020	0.003	0.019	0.989	0.826	0.464	0.596	0.607
Adjusted R^2	0.023	0.020	0.002	0.019	0.989	0.826	0.464	0.596	0.607
Note:									*p<0.1; **p<0.05; ***p<0.01

Table A.3: Interaction regression results for fifth order interaction. This is a continuation of the regression estimates in Table 2.

Table A.3, which reports the quinary interaction, provides an interesting connection to the literature. As the GSS model enforces all five assumptions, flipping the signs in Table A.3 allows the coefficients to represent the average impact of relaxing a single assumption in their model. A slight tightening (∼0.17 reduction) of the MRS spread, along with minimal change in allocative efficiency on average results from a relaxation of one of the five assumptions. As Table 2 will show however, individual comparisons reveal relaxing the angle choice provides most of this variation.

Appendix B Round-Average Price Densities

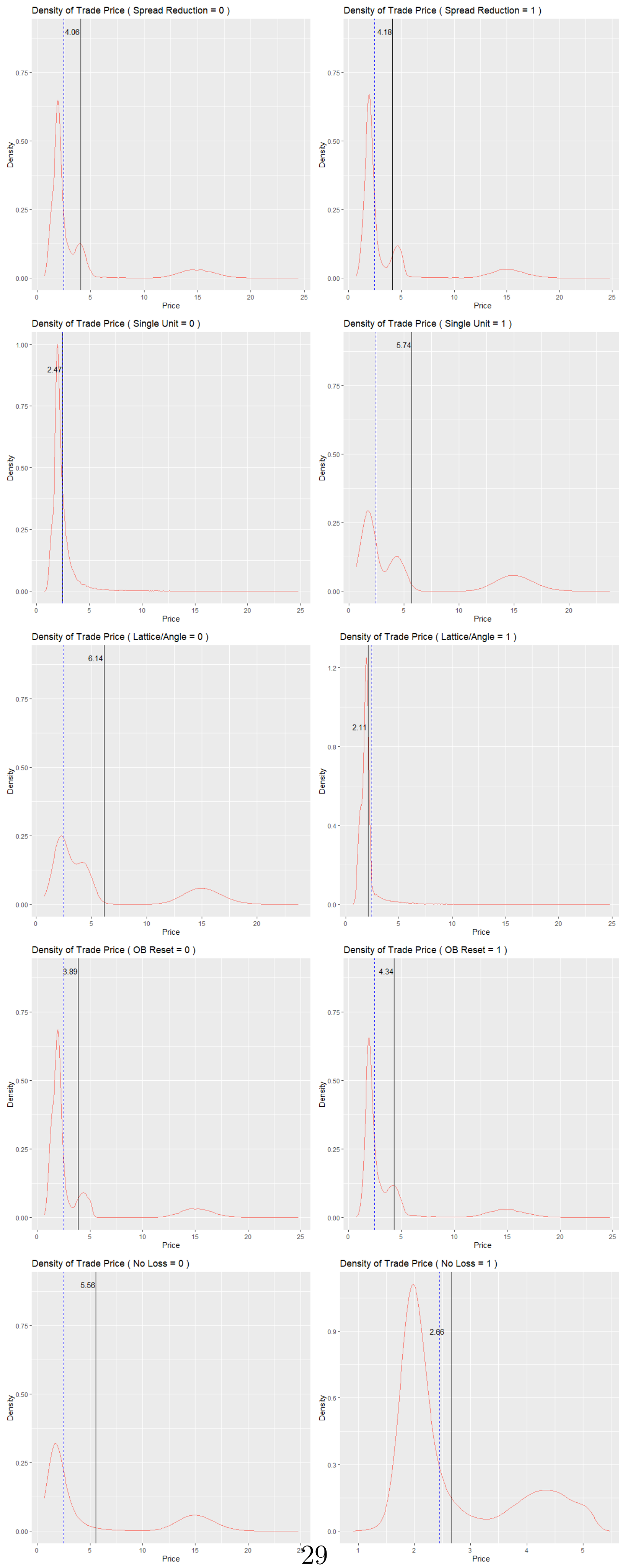

Figure B.1: Round-average price densities. Blue line is CE price and black line is subset average.

	Dependent variable:								
	Price	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
⋮									
SR:SU:LA	−0.972	−1.059	−7.362	20.169***	9.522***	−0.163***	−0.017	0.019	−0.018***
	(1.464)	(1.464)	(13.837)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SR:SU:OBR	−0.599	−0.503	−2.349	−2.910	4.158***	−0.258***	−0.026*	0.048*	−0.028***
	(1.464)	(1.464)	(13.837)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SR:SU:NL	0.352	0.381	0.376	−0.237	2.091***	0.077***	−0.350***	0.757***	−0.192***
	(1.473)	(1.473)	(13.920)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SR:LA:OBR	−3.887***	−3.856***	−34.719**	9.380***	12.323***	−0.230***	−0.022	0.003	−0.010
	(1.464)	(1.464)	(13.837)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SR:LA:NL	−0.888	−0.983	−7.494	24.618***	22.250***	0.101***	−0.374***	0.789***	−0.236***
	(1.472)	(1.472)	(13.914)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SR:OBR:NL	−0.861	−0.745	−2.747	−1.269	3.711***	0.153***	−0.130***	0.206***	−0.018***
	(1.481)	(1.481)	(13.998)	(3.476)	(0.216)	(0.026)	(0.016)	(0.025)	(0.007)
SU:LA:OBR	−6.404***	−6.580***	−61.530***	4.222	44.816***	0.394***	−0.008	0.100***	−0.065***
	(1.464)	(1.464)	(13.837)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SU:LA:NL	11.544***	10.423***	13.354	35.880***	58.409***	1.844***	0.190***	−0.090***	−0.203***
	(1.472)	(1.472)	(13.919)	(3.476)	(0.216)	(0.026)	(0.015)	(0.025)	(0.007)
SU:OBR:NL	1.092	0.992	1.689	−0.178	−1.509***	0.132***	−0.035**	0.031	0.028***
	(1.482)	(1.482)	(14.009)	(3.476)	(0.216)	(0.026)	(0.016)	(0.025)	(0.007)
LA:OBR:NL	−7.090***	−6.924***	−61.623***	4.236	59.364***	0.238***	−0.120***	0.244***	−0.092***
	(1.481)	(1.481)	(13.998)	(3.476)	(0.216)	(0.026)	(0.016)	(0.025)	(0.007)
SR:SU:LA:OBR	3.474*	3.474*	34.067*	−9.380*	−3.183***	0.228***	0.029	−0.019	0.028***
	(2.070)	(2.070)	(19.568)	(4.916)	(0.305)	(0.036)	(0.022)	(0.035)	(0.010)
SR:SU:LA:NL	0.197	0.399	6.771	−24.618***	−11.336***	−0.098***	0.363***	−0.714***	0.185***
	(2.076)	(2.076)	(19.628)	(4.916)	(0.305)	(0.037)	(0.022)	(0.035)	(0.010)
SR:SU:OBR:NL	0.454	0.348	2.205	1.269	−2.985***	−0.150***	0.139***	−0.228***	0.029***
	(2.083)	(2.083)	(19.694)	(4.916)	(0.305)	(0.037)	(0.022)	(0.035)	(0.010)
SR:LA:OBR:NL	3.757*	3.869*	34.709*	−10.169**	−14.317***	−0.169***	0.057***	−0.073**	−0.014
	(2.082)	(2.082)	(19.683)	(4.916)	(0.305)	(0.037)	(0.022)	(0.035)	(0.010)
SU:LA:OBR:NL	6.501***	6.474***	61.427***	−4.236	−40.185***	−0.244***	0.097***	−0.258***	0.121***
	(2.083)	(2.083)	(19.691)	(4.916)	(0.305)	(0.037)	(0.022)	(0.035)	(0.010)
Constant	2.268***	1.447***	2.054	14.942***	20.132***	3.155***	2.035***	3.066***	0.656***
	(0.518)	(0.517)	(4.892)	(1.229)	(0.076)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,424	95,424	95,424	96,000	96,000	95,424	95,422	95,424	96,000
R^2	0.024	0.020	0.003	0.019	0.989	0.826	0.464	0.596	0.607
Adjusted R^2	0.023	0.020	0.002	0.019	0.989	0.826	0.464	0.596	0.607
Note:									*p<0.1; **p<0.05; ***p<0.01

Table A.4: Interaction regression results for third order interaction. This is a continuation of the regression estimates in Table 2.