A note on disappointment in risk elicitation tasks

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Abstract

The choice of functional form of an agent's utility function is an important step when estimating risk preferences experimentally. We compare estimates of single parameter CRRA preferences to two parameter disappointment aversion (a la Gul (1991)) using techniques and laboratory data in Friedman et al. (2022), making use of a sufficient statistic, L, for prices and probabilities developed in Habib et al. (2017). Evidence suggests minimal or no gains in estimation when amending constant relative risk aversion with an extra parameter for disappointment aversion. The two parameter model does provide small improvements in estimation for subject's with less stable behavior.

Keywords: Disappointment Aversion, Risk Aversion, Experiment, Elicitation, Multiple Price List
JEL Classifications: C91, D81, D89

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1 Introduction

Risk elicitation procedures have become exceedingly common in a variety of settings; from brokers providing risk aversion screening surveys to field experimenters surveying farmers in developing countries on their preferences over new capital lending/purchasing services. The increased dependence on such experimental procedures begs the questions of what elicitation task to use and what decision theoretic model to estimate. While a sibling paper, Friedman et al. (2022), makes great strides in providing context for the first question, this paper aims to give incite for the second.

One alternative to expected utility (EUT) models explored in this paper is the Gul (1991) model of Disappointment Aversion (DA). The model provides a palatable platform for a common non-EUT behavioral phenomenon, disappointment, with only one more parameter than standard EUT. Additionally, the axiomatic style is both tractable and attractive to other theorists. Several other models closely related to disappointment have been proposed, including Bell (1985), Loomes and Sugden (1986) Grant and Kajii (1998), and Delquié and Cillo (2006).

While behavioral alternatives to standard EUT models are attractive among theorists, such an alternative as disappointment aversion has been less prominent in applied cases. Camerer and Ho (1994) Loomes and Segal (1994) Hey and Orme (1994) Importantly, Abdellaoui and Bleichrodt (2007) is the first to experimentally test the merits of Gul's DA model, making use of Wakker and Deneffe's (1996) tradeoff method to elicit choices.[1]. While this result and paper provided the first applied look at disappointment aversion, a less common elicitation process was required, leaving a direct estimation left to be explored.

This note provides an estimation process for Gul's two-parameter disappointment aversion model, tests the process against standard EUT estimates using experimental data, and perform a holdout procedure to investigate . We propose a non-linear least squares estimat-

[1] A two stage elicitation process, with the first stage determining the utility function via a string of lottery indifferences and the second stage finding the level of disappointment aversion via a final certainty equivalent.

ing procedure using a piecewise-linear adjustment for the left-hand-side variable, log choice ratio (i.e. ln x/y). Little gain is found in estimation (for most subjects) in our data, though this process could prove helpful in parametric estimation when more erratic behavior is expected or found. That being said, whether the width of the model is affording the gain in fit or the behavior described in the model is actually driving these disparities in estimation is up to debate.

The article proceeds as follows. Section 2 provides the theoretical and methodological background for the note. Section 3 summarizes the data-generating process while Section 4 presents the findings. Section 5 concludes.

2 Theory and Methods

This notes continues the theoretical setting of Friedman et al. (2022), in which agents make budget decisions over Arrow securities where two states of the world exist, X and Y. These states are mutually exclusive, with each state always having non-zero probability; a bundle of securities (x', y') yields payoff x' if state X occurs and y' if Y occurs, making the expected payoff $\pi_X x + \pi_Y y$. Under expected utility theory (EUT), agents maximize their expected utility $(\pi_X u(x) + \pi_Y u(y))$ over their set of budget-feasible bundles, $p_x x + p_y y = m$.

As described in Habib et al. (2017) and harnessed in Friedman et al. (2022), taking the logarithm of the marginal rate of substitution (after taking first order conditions) yields a sufficient statistic for prices and probabilities, named L:

$$L \equiv ln\ \pi_X - ln\ \pi_Y - p_x + p_y \tag{1}$$

In estimating elicited risk preference parameters for said agents, L is used as the main right-hand-side regressor, a la

$$ln \frac{x}{y} = \frac{-1}{\gamma}[ln\ \pi_X - ln\ \pi_Y - p_x + p_y] = \frac{-1}{\gamma} L \tag{2}$$

The following subsections lay out the theory and estimation process for amending this one-parameter model to a two-parameter disappointment aversion (DA) model, while still using the sufficient statistic L.

2.1 Generalizations of EUT

Gul (1991) presents a model with a free parameter $\beta \geq 0$ intended to capture disappointment aversion as a probability distortion in a two state world — people make choices as if maximizing expected utility that assigns extra weight (by a factor of $1+\beta$) to the probability of the less favorable state. In our notation, the unnumbered equations near the top of Gul (1991, p. 678) say that the indifference curve segments have slope

$$-\frac{dy}{dx} = B\frac{\pi_X}{\pi_Y}\frac{u'(x)}{u'(y)} > 0, \tag{3}$$

where $B = (1+\beta)$ if $x < y$ (so X is the less favorable state) and $B = (1+\beta)^{-1}$ if $x > y$ (so Y is less favorable). Thus the indifference curve has a kink on the diagonal $x = y$, with -slope $= \frac{\pi_X}{\pi_Y}(1+\beta)$ on the right and -slope $= \frac{\pi_X}{\pi_Y}(1+\beta)^{-1}$ on the left.

Suppose, as commonly assumed in the subsequent literature, that the underlying Bernoulli function u is CRRA with risk aversion coefficient $\gamma > 0$. The tangency condition $-\frac{dy}{dx} = \frac{p_x}{p_y}$ applies as usual when the optimal choice is interior (not on the diagonal nor at a corner of the budget set). Writing $b = \ln(1+\beta)$, and recalling that in this case $\frac{u'(x)}{u'(y)} = (\frac{x}{y})^{-\gamma}$, we see that disappointment aversion changes equation (2) to

$$\ln\frac{x}{y} = \frac{1}{\gamma}[L - b] \tag{4}$$

when $L > b$ and so $x > y$. By symmetry, when $L < -b$, we again have a tangency but with $x < y$ and with $-b$ replaced by $+b$ in (4). For values of $L \in [-b, b]$, a DA agent will choose at the kink in the indifference curve where the diagonal $x = y$ intersects the budget line. Of course, for extreme values of L and sufficiently small parameters b and γ, corner solutions (on the x or y axis) are also possible. Estimating equations in Appendix A spell this out explicitly. We show below that this extra behavioral complexity does little in way of adding any predictive power to the theory in our dataset relative to the single parameter CRRA case.

2.2 Estimating the DA model

Section 2.1 describes a model which includes the CRRA parameter γ as well as the disappointment aversion parameter $b = \ln(1 + \beta)$ and uses L as an explanatory variable for the log choice ratio $\ln x/y$. Applying the logic of Section 2.1 and the ± 4 truncation noted in the text, we obtain the estimating equations

$$\ln x/y = \begin{cases} 0 & \text{if } L \in (-b, b) & \text{(5a)} \\ \frac{1}{\gamma}[L - b] & \text{if } L \in [b, 4\gamma + b] & \text{(5b)} \\ \frac{1}{\gamma}[L + b] & \text{if } L \in [-4\gamma - b, -b] & \text{(5c)} \\ 4 & \text{if } L > [4\gamma + b] & \text{(5d)} \\ -4 & \text{if } L < -[4\gamma - b] & \text{(5e)} \end{cases}$$

and use NLLS. That is, for a given subject (label suppressed), let $R(L(\pi, p), b, \gamma)$ be the right-hand side of (5a). Then

$$(\hat{b}, \hat{\gamma}) = \text{argmin} \sum_{t=1}^{54} [\tilde{\ln}\ x_t/y_t - R(L_t, b, \gamma)]^2. \tag{6}$$

The tilde in $\tilde{\ln}\ x_t/y_t$ is to remind us that that dependent variable is truncated at ± 4.

3 Data

Laboratory individual choice experiments at LEEPS lab in University of California, Santa Cruz using a novel within-subject design served as the data generating process for this paper (and originally for Friedman et al. (2022) as well). A total of 142 UCSC students participated across 18 sessions. Each subject interacted with 56 individual risk-elicitation trials, each of which was one of six different task types.

Within a task type, each trial could vary by price ratio and state probability ratio. The trials were split into blocks which allowed for the control of order effects, as well as the creation of treatments. Within a block, all trials were of the same task type, and were either ordered monotonically or randomly by price or probability ratio. Subjects were filed into one of 24 different treatments based on block ordering, whether they received monotonically order blocks early or late in the session, whether their block trials varied in price ratio or state probability, and whether they received lower or higher probability or price ratios in early or late blocks.

A full characterization of the experimental design and implementation can be found in Friedman et al. (2022).

4 Empirical Findings

We provide a succinct characterization of model performance for both the CRRA and two-parameter DA estimation processes, applied to the large experimental data set described above.

Figure 1 plots for each individual an ordered pair consisting of their γ estimate from a single parameter CRRA model as the horizontal coordinate, and their γ estimate from a two parameter DA model nesting the same single parameter CRRA utility function. For the vast majority of subjects with estimates in the 0.3 to 1 range, we find a close correspondence between γ from the two different models. Figure 2 documents for each subject their estimated b and γ parameters from the DA model.

Figure 3 compares the predictive power of that two parameter model to that of a simple CRRA model or, equivalently, to imposing the restriction $b = 0$. For each subject, we desig-

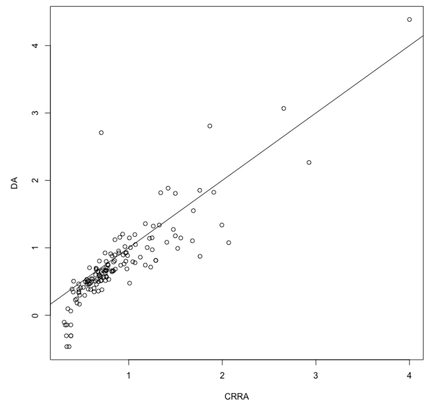

Figure 1: γ estimates from CRRA and DA models.

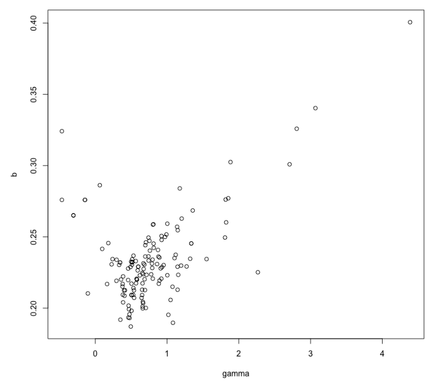

Figure 2: Scatter plot of γ and b estimates from DA model for each subject.

nate one of the observations as the prediction target, estimate both models on the remaining 53 of the 54 observations, predict the target observation, and compute the prediction error for each model. The table reports squared prediction errors summed over all 54 possible prediction targets.

The Figure shows that the majority of our 142 subjects have relatively small prediction errors (SSE < 150) that hardly differ between the two models and so fall almost on top of the diagonal line. The DA model does slightly better than the CRRA model for most (not all) subjects with large SSE's, but (by definition) neither model predicts their behavior very well.

Result. Little predictive accuracy is sacrificed by using the single parameter (γ)

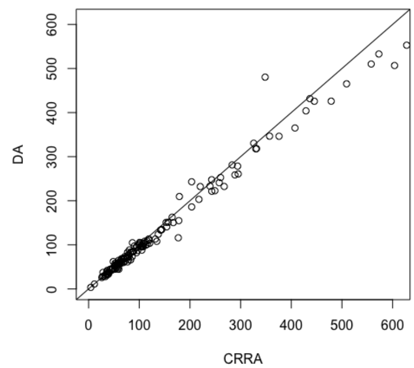

Figure 3: Predictive power comparison. Vertical axis is sum of squared prediction errors (SSE) for the (γ, b) DA model, and horizontal axis is SSE for the CRRA model ($b = 0$).

Constant Relative Risk Aversion model to summarize risk preferences instead of the two parameter (b, γ) Disappointment Aversion model.

5 Conclusion

Estimation in the risk elicitation literature, whether for the sake of study itself or for use as controls in applied research, generally falls into standard EUT procedures. However, the comparison to popular EUT adjustments, such as disappointment aversion, can be informative and thus have garnered substantial interest. That being said, very few entries in the literature have explicitly tested one of the more tractable options, Gul's (1991) disappointment aversion model.

This paper provides one of the few such exercises. We establish an estimation process for a two-parameter model adapting CRRA preferences to account for disappointment aversion. We then test said process on nearly 8000 choices (elicited in a laboratory setting) from 140 subjects at LEEPS Lab. While we find marginal improvements in estimation for a minority of subjects (those exhibiting less stable behavior), the majority of the data suggests essentially no gain exists when applying the two-parameter model.

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Appendix A Sample scatterplots.

To provide perspective for more formal data analysis, consider the four scatterplots in Figure A.1, each showing all choices in Blocks 2-10 for a single subject. The log allocation ratios for each choice are plotted against the log price - log odds composite variable L; open squares indicate choices in the budget jar task (open circles when there is a no-cash constraint), triangles indicate the budget line task, and plusses indicate the relevant budget dot task. For reference, the solid line graphs what equation (**??**) predicts for a expected utility maximizer with the original Bernoulli function $u(c) = \ln(c)$. The dashed line plots risk-neutral optimal choices, while the dotted line plots the choices of an otherwise risk-neutral person with an interval of disappointment aversion.

The subject in Panel (a) could be described as a noisy CRRA expected utility maximizer with γ close to 1. Panel (b) shows another subject in a fixed probability session who, in 53 of 54 trials, maximized expected value. Panels (c) and (d) show two different subjects in fixed price sessions who seem like noisy disappointment averters in the budget dot task (here BDHL), but who diverge in the other tasks (towards $\gamma = 1$ or $\gamma = 0$, respectively). Our 142 subjects exhibited very diverse behavior; these four examples illustrate only a small slice.

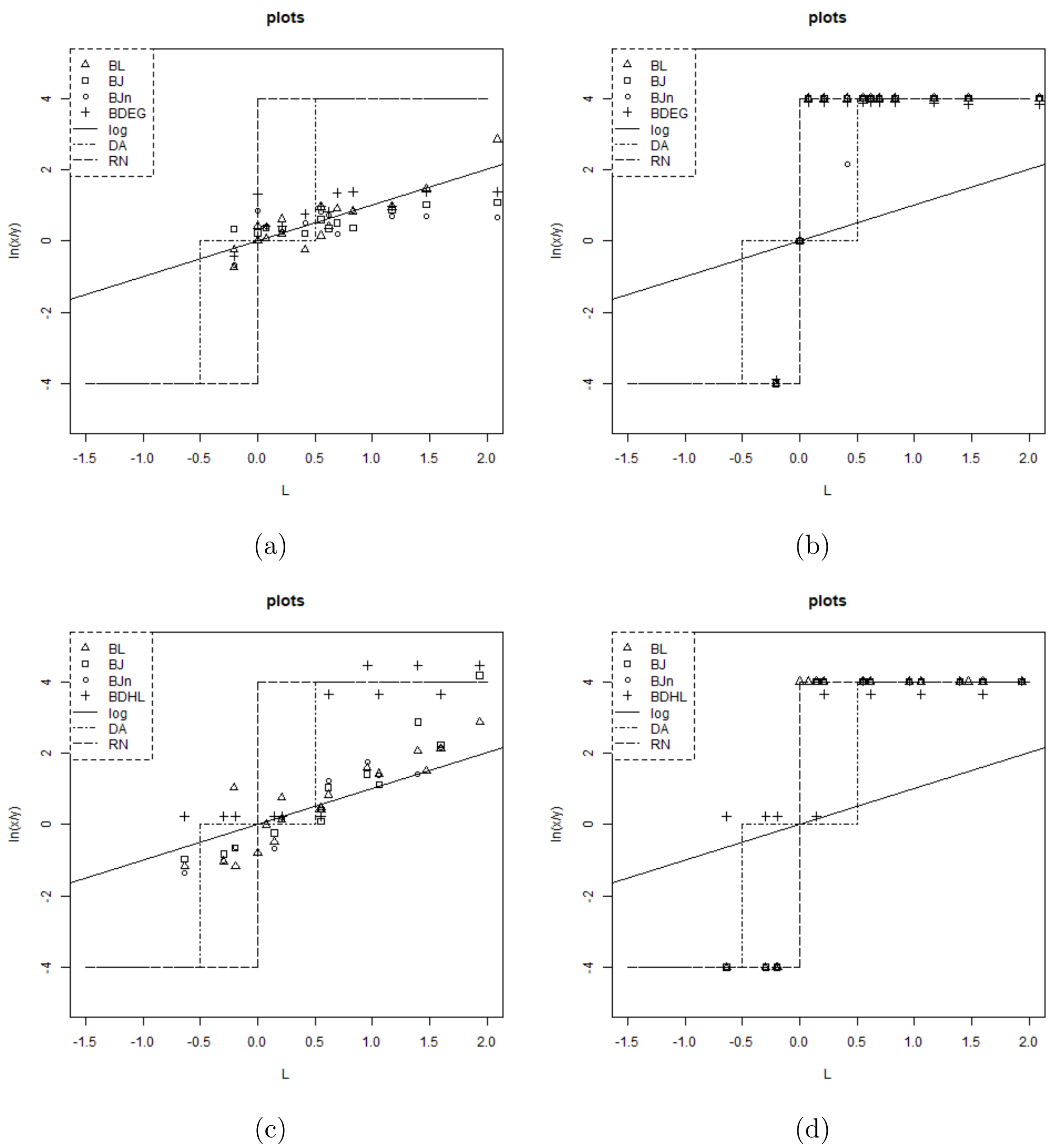


Figure A.1: Choices (truncated at $\ln x/y = \pm 4.0$) for four subjects. Actual choices by elicitation task are plotted with open symbols. Theoretical predictions are plotted with lines, sorted according to $L = \ln \pi_X - \ln \pi_Y - \ln p_x + \ln p_y$; log refers to CRRA model with $\gamma = 1$, RN refers to risk neutral choice ($\gamma = 0$), and DA refers to the disappointment averse model with $b = 0.5$ and $\gamma = 0$.

Appendix B Instructions

Experiment Instructions (VRE)

Welcome! You are about to participate in an experiment in the economics of decision-making. If you listen carefully and make good decisions, you could earn a considerable amount of money that will be paid to you in cash at the end of the experiment.

Please remain silent and do not look at other participants' screens. If you have any questions or need any assistance, please raise your hand and we will come to you. Do not attempt to use the computer for any other purpose than what is explicitly required by the experiment. This means you are not allowed to browse the Internet, check email, etc. If you interrupt the experiment by using your smart phone, talking, laughing, etc., you may be asked to leave and may not be paid. We expect and appreciate your cooperation today.

The Basic Idea

This experiment is composed of dozens of periods in each of which you will choose among lotteries. Each lottery consists of two possible prizes; you get one of them with the stated probability and otherwise get the other prize. For example, if you choose the lottery "4/10 of $2.00, 6/10 of $1.60," then you will win the $2.00 prize if a 10-sided die roll comes up 1,2,3, or 4, and otherwise (if the roll comes up 5,6,7,8,9 or 0) will win $1.60.

The lotteries may differ from period to period. Note that in any given period, the prizes, probabilities and manner in which the lotteries are displayed may be different than in prior or subsequent periods, so in each period you should read the lottery choices carefully.

Your final payment at the end of the experiment will consist of two parts. You are guaranteed a show-up payment of $7.00. Beyond this, you will receive a prize from one of the lotteries that you selected. The conductor will draw a random ball from a bingo cage to determine which period will be paid. If within that period there is more than one choice made, then a dice roll will determine which choice between lotteries (within that period) is selected for payment. Finally, the conductor will roll a 10-sided die to determine which prize you receive from the lottery selected as described.

Display 1

In the example below, you will choose between lottery A, which offers you a payoff of $2 with probability 0.4 (40%) and a payoff of $1.60 with probability 0.6 (60%), and lottery B, which offers you a payoff of $3.85 with probability 0.4 (40%) and a payoff of $0.10 with probability 0.6 (60%).

○ 4/10 of $2.00, 6/10 of $1.60	○ 4/10 of $3.85, 6/10 of $0.10

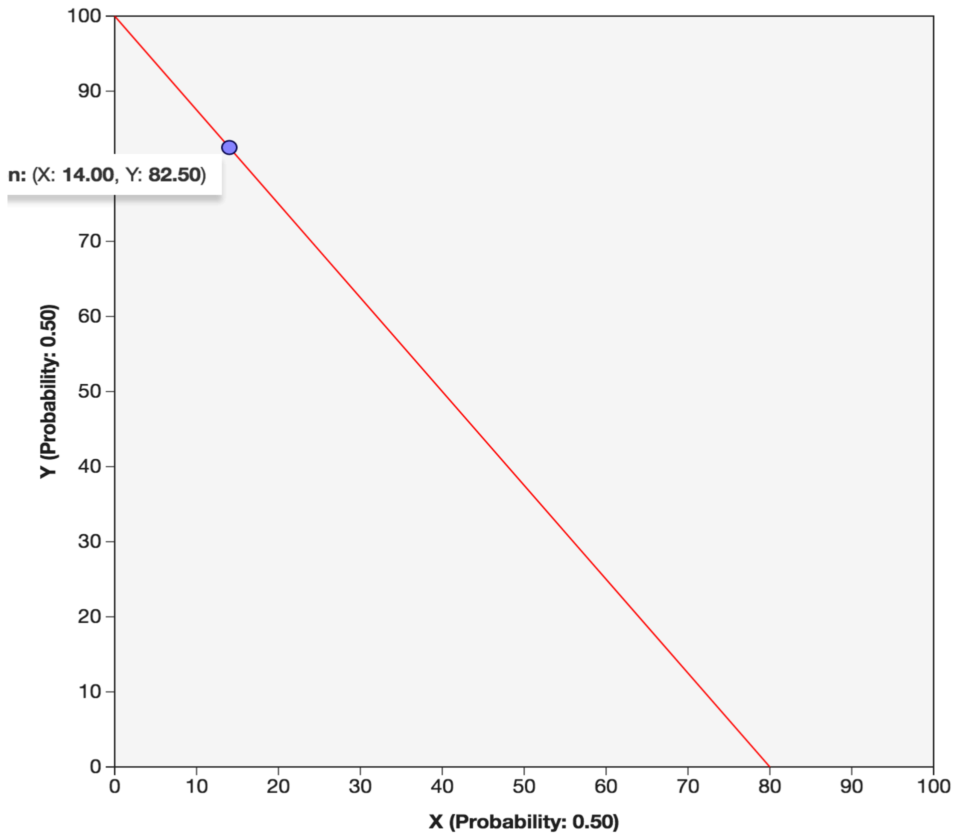
By clicking the radio button, you chose lottery A or lottery B for potential payment.

In some periods there may be 6 lines that look like the one above, except that the probabilities are different; in each line you choose between two lotteries. If such a period is selected for payment, by bingo cage, then a 6-sided dice roll will determine which of the six rows from this period is selected for payment. The lottery you chose in that row will then be played out by means of a roll of a 10-sided die.

For example, if you chose the lottery B (right), and if this line is selected for payment, a 10-sided die will be rolled. If the number showing is 1 to 4 then you are paid $3.85, otherwise (if the number is 5-10), then you are paid $0.10.

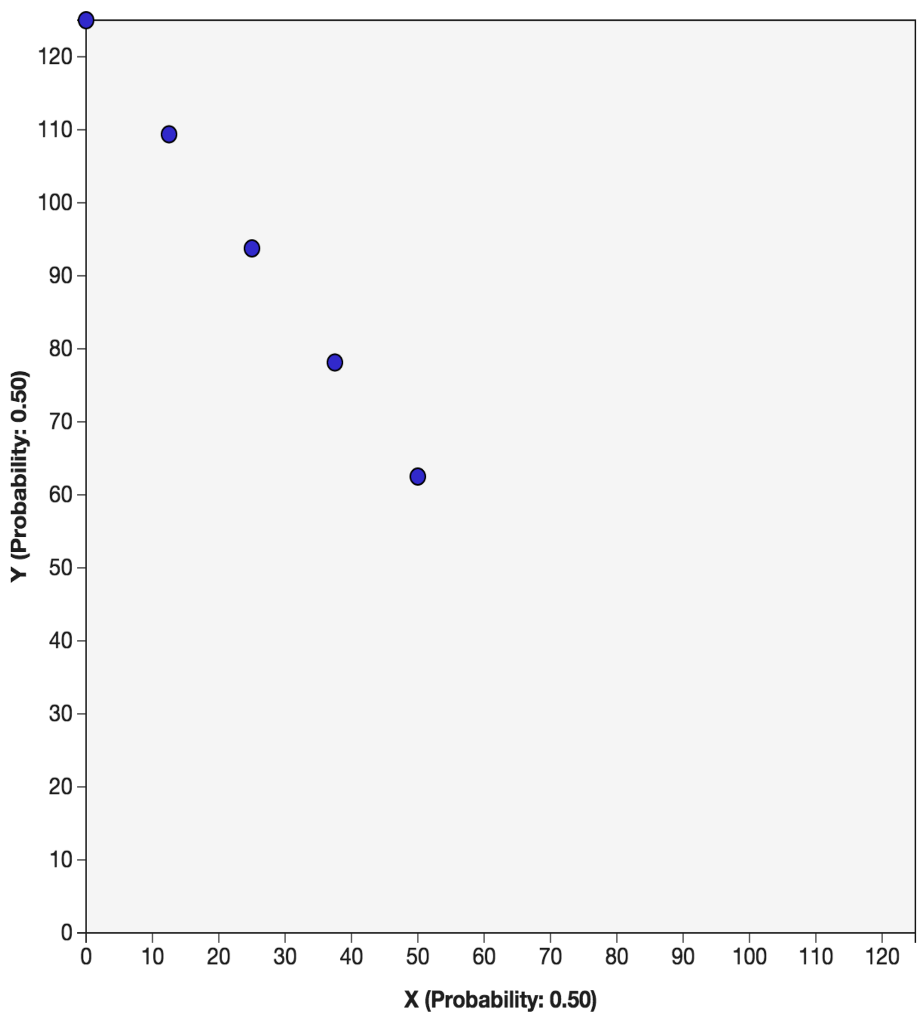
Display 2

This display offers you a range of possible lotteries with fixed probabilities for two possible states. In the Figure below, the two states are called X and Y, and each has probability 0.5; the state (X or Y) will be determined by the roll of a 10-sided die. You can pick any point on the line by clicking on it, and your payoff will be its x-coordinate if the state is X and will be its y-coordinate if the state is Y. In the Figure, the player chose the point (14, 82.5) on the line, so if this period is one that is randomly selected for payment, she would get 14 points in state X or 82.5 points in state Y.



Display 3

In this display you have several lotteries to choose from. In the Figure below you have 5 available lotteries, as indicated by the 5 dots. If this period is selected for payment, you get the x-coordinate of the point you chose if state X occurs or the y-coordinate if state Y occurs; the state (X or Y) will be determined by the roll of a 10-sided die.



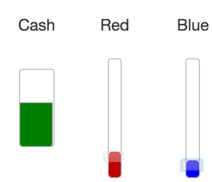
Display 4

This display allows you to allocate across a range of assets that determine your lottery prizes. Each period there are two possible states, called Red or Blue. There are three assets: there is an asset for each state that pays off only in that state (e.g. Blue asset only pays off if the Blue state occurs), and there is cash. If that period is selected for payment, your payment depends on whether the state turns out to be Red or Blue, and how much you hold of each asset.

How do you allocate across assets? You start with 100% to cash, and can purchase Red and/or Blue assets using the sliders. In the Figure below, the person started with a full cash jar (level shown in green) and used the sliders to partially fill the Red and Blue jars (thus partially depleting the Cash jar). Each unit of the Red asset in this example costs 0.444 units of cash, while the price of one unit of the Blue asset costs 0.556. Since the Red asset is cheaper, the person can fill the Red jar higher than the Blue jar.

The text below the jars summarizes how choices determine possible prizes; the column titled "Total" shows the prize amount -- totaled across all three assets -- in each possible state. If this period is selected for payment, a 10-sided dice roll will determine which state occurs. In the figure below, given the asset allocation in the example, the player would stand to get 60.56 points if the state turns out to be Red, or 51.56 points if the state turns out to be Blue.

Please note the prices of each asset, and the probabilities of each state; these may change from period to period. In some periods the submit button may not respond until all cash has been allocated across the Red and/or Blue assets.



Probability	Price		Cash	Red	Blue	Total	Probability
0.5	0.444444444	Payoff if Red	**33.56**	**27**		**60.56**	0.5
0.5	0.555555556	Payoff if Blue	**33.56**		**18**	**51.56**	0.5

Are there any questions? Now you may begin making your choices. Please do not talk with anyone while we are doing this; raise your hand if you have a question.