Submitted to *Interfaces*
manuscript (Please, provide the manuscript number!)

Designing Environmental Markets for Trading Catch Shares

Martin Bichler
Technical University of Munich, Germany, bichler@in.tum.de

Douglas Ferrell
Department of Primary Industries, New South Wales, Australia, doug.ferrell@dpi.nsw.gov.au

Vladimir Fux
Technical University of Munich, Germany, vladimir.fux@in.tum.de

Jacob K. Goeree
AGORA Center for Market Design, University of New South Wales, Australia, jacob.goeree@gmail.com

Overfishing is a prime environmental concern. Catch share systems are an effective way to combat overfishing but they introduce economic inefficiencies when the allocation of shares does not align with industry needs. This paper describes the implementation of a market-based approach to reallocating fishing shares in New South Wales (NSW), Australia. The design of the market needed to address several nonstandard requirements, including the possibility of all-or-nothing offers, fair prices, and an endogenously determined subsidy. These features were crucial for the adoption of the reform but also led to computationally challenging allocation and pricing problems. The market operated from May to July 2017 and successfully reallocated shares from inactive fishers to those who were impacted the most by the introduction of catch shares, with significant savings for the taxpayer. It provides a template for the reallocation of catch shares elsewhere and of resource rights in other applications (e.g., water rights, pollution rights, and environmental offsets). We provide a project description, practical examples, and implementation details of the implemented exchange.

Key words: Overfishing, Fishery Management, Catch Shares, Market Design

---

Introduction

Rising demand for seafood and advances in fishing technologies have caused overfishing to become a prime environmental concern (Jackson et al. 2001). More than 30% of the world's fisheries are overexploited and require strict management to restore and maintain sustainability (Worm et al.

2006). Some reports warn that most of the world's commercial fisheries could collapse within decades (Costello et al. 2008).

The most commonly adopted approach to curb overfishing is to introduce catch share quotas, also called individual transferable quotas (Branch 2009). First, a scientifically sound limit (a cap) is set on the amount of fish that can sustainably be extracted from a fishery. Then a percentage of that limit is guaranteed to individual fishermen or groups of fishers. The use of catch share programs has grown considerably since their first implementations in the late 1970s (Costello et al. 2008). There are now over 150 catch share programs worldwide, covering a wide variety of marine and freshwater species (Lynham 2014). The latest empirical studies indicate that these programs are indeed efficient tools in tackling overfishing problem (Melnychuk et al. 2012, Birkenbach et al. 2017).

However, catch share programs can introduce economic inefficiencies when the allocation of shares does not align with fishers' needs. The initial distribution of shares is typically based on historical fisher performance, a practice known as "grandfathering" (Lynham 2014). With time, this initial allocation becomes suboptimal when the industry undergoes structural change, the stock recovers, or quotas are tightened. If the shares are then put into effect, the most effective fisheries might have to reduce their catches to inefficient levels with ample effect on the profitability of these businesses. To restore efficient outcomes, shares need to be reallocated, which raises several complex policy issues as we discuss next (Rosenberg 2017).

Problem Description

There are approximately 1,000 fishers catching around 15,000 tons of fish and prawns in New South Wales (NSW). The potential for overfishing prompted the NSW government to introduce a catch share system, comprised of over 100 different share classes, more than a decade ago. This catch share system provides an instrument to curb future overfishing. A share class comprises multiple catch shares of the same type, which define the quantity of fish that a fisher is allowed to catch. For example, a share held by a business allows one tonne of sardines to be caught in a region. Between

2002 and 2007, catch shares were distributed among fishers in an equitable manner, meaning that all fishers got an equal amount of shares within the share classes in which they were active. However, these shares were not put into effect until 2017 when the NSW government's linkage program required fishers to hold enough shares to justify their catch. The linkage program introduced linkages between shares and catch or effort, to ensure the long-term viability and sustainability of the NSW commercial fishing industry.

The uniform allocation of shares was likely not optimal when the shares were first allocated and it certainly did not match industry needs in 2017. Based on data from the Structural Adjustment Reform Committee report (Cartwright et al. 2015), the NSW fishing industry is characterized by something akin to the familiar 80/20 rule in that a small set of fishers are responsible for most of the total catch. By making shares binding, the linkage program forced the most active fishers to either reduce their catch (which would adversely impact industry profitability) or to obtain additional shares. At the time the linkage program was enforced many of active fishers faced "share deficits" of up to 50% (i.e., they held only half of the shares required to justify their catch). To allow these fishers to reduce their share deficits, the linkage program was supplemented with a "market for catch shares" where active fishers could buy shares from inactive ones.

One might naively assume that fishers can simply trade catch shares among themselves. One impediment to efficient bilateral trade is that fishers are geographically dispersed making it costly for buyers and sellers to match, especially because fishers typically hold a portfolio of shares and, hence, sellers need to match with a variety of buyers. Another impediment is that even the most active and commercially viable fishers are budget constrained and unable to finance the large number of shares needed to cover their share deficits. In recognition of the financial hardship the linkage program imposed on the most active fishers, the NSW government was willing to provide a subsidy of up to A$15 million to "grease the market." But it is unclear how this subsidy should be distributed to achieve efficient and fair outcomes in bilateral-trade settings, where prices for shares within the same share class may vary across different pairs of buyers and sellers. Finally, under bilateral trade, fishers who wish to exit the industry might end up selling only part of their portfolios, leaving them with a nonviable business as well as little proceeds – a possibility known as the "exposure problem."

To reduce transaction costs, ensure a fair and effective use of government subsidies, and avoid undesirable outcomes where selling fishers end up with fragmented portfolios, a centralized clearinghouse is needed. The importance of an efficient exchange can hardly be overstated. Without it, the introduction of the linkage program might have distorted catch levels and endangered the long-term viability of the NSW fishing industry. Indeed, if industry participants and other stakeholders did not believe the share reallocation would be done in a fair, transparent, and efficient manner, they would likely have rejected the introduction of the government's linkage program.

Solutions

One option is for the government to apply a fixed budget to buy back shares, which are then retired or redistributed among active fishers. The possibility of a government "buyout" was considered in the context of the NSW reform, but it was deemed to have too many shortcomings. First, the buyout provides no opportunity for "price discovery" (i.e., there is no guidance as to what should be the (relative) prices for different share classes, because there is no information about the demand of fishers who want to buy). As a result, it is unclear whether the correct shares are bought out (e.g., some shares may have little value to current owners but even less so to others). Second, acquired shares may go underutilized for some time and it is unclear how the government should redistribute them. Allocating these shares "by formula" likely reintroduces inefficiencies because it would rest on imperfect assumptions about industry needs. Third, government buyouts often face legal challenges (Ludicello and Lueders 2016) because the inefficiencies that arise from purchasing shares, as well as from redistributing them, are easily mistaken for favoritism toward specific market participants.

This paper describes implementation details of a market-based approach to the reallocation of fishery shares. By creating a market where inactive fishers ("sellers") and active fishers ("buyers") trade shares, transaction costs are reduced and delays minimized. Moreover, the market allows for price discovery (i.e., fishers can learn about the market value of their shares as reflected by the bids and asks submitted to the market). As a result, market-based exchanges hold the promise to yield more efficient allocations than government buyouts. Finally, government policy embedded in a market instrument typically faces less legal resistance because the outcome is determined by the buy/sell choices of participating fishers.

The desirability of market-based solutions has recently been flagged in the literature (Marszalec 2017), but no specific proposals have been put forward. One likely reason is that the market for fishery shares requires several nonstandard features – standard market forms are inadequate. First, in standard markets (like those used in stock exchanges), different share types would be sold separately, which exposes sellers to the risk of selling some share types but not others (i.e., the "exposure problem"). To avoid such fragmentation, sellers should be allowed to submit "all-or-nothing" asks. Second, all-or-nothing asks raise questions about pricing because a single seller may be matched with many buyers who each express their individual willingness to pay. Using a "discriminatory" price rule where successful buyers pay their own bids seems natural but may cause envy and political stir. Participants feared paying too much or receiving too little for their shares compared to others. On the other hand, a uniform clearing price paid by all successful buyers can lead to paradoxically rejected bids and inefficiencies.

Finally, a subsidy may be required to ensure industry acceptance of the "cap-and-trade" program and to stimulate participation in the market. In particular, the subsidy should aid those fishers for whom the introduction of the linkage program resulted in an immediate deficit of shares. These active and commercially viable fishers were hit hardest by the linkage program, while inactive fishers were not impacted at all. The difficulty is that optimal subsidy levels should be determined based on bids and asks received. Without this market information, it is impossible to know buyers' preferences for shares, the correct market prices, and the costs to active fishers to effectively reduce their share deficits. For this reason, it is not possible to set correct subsidy levels upfront. And if the government simply promised to pay a rebate to the buyers after the exchange, it would expose itself to the possibility of having to pay much more than the planned subsidy. To summarize, even when data on individual share deficits are available, it is not possible to efficiently distribute the subsidy before or after the exchange takes place. Instead subsidies have to be distributed by the market via discounts to market prices.

Contributions

The recent subsidized share trading market in NSW is a first-of-a-kind market design for the reallocation of catch shares and the largest combinatorial exchange for fishery access rights run to date. The requirements for this market were challenging and likely play a role in other environmental applications (e.g., trading pollution permits, water rights, environmental offsets).

- *Computational challenges in large-scale combinatorial exchanges:*

  Allowing for all-or-nothing offers avoids exposure problems but also introduces several design complexities. First, the allocation or winner-determination problem is an NP-hard combinatorial optimization problem (Bichler et al. 2018). While combinatorial markets have been analyzed in operations research for more than a decade, the size of this market was remarkable. With around 600 participating fishers, 100 share classes, and 1,300 bids in each round, this can be considered a large-scale mathematical programming problem. It was unclear that the allocation problem could be solved to optimality or even near optimality with this input size.

- *Development of new payment rules for combinatorial exchanges:*

  Stakeholders required "fair" prices in the sense of anonymous and linear prices for each share class. This led to fundamentally new economic insights about pricing in combinatorial exchanges (Bichler et al. 2018). Our theoretical analysis of these prices is of interest beyond the fishery market and relevant also to other applications of combinatorial exchanges as they can be found in energy markets or in logistics.

- *Market-based distribution of a subsidy to incentivize participation:*

  The government decided to provide a subsidy to participating fishers in need. This was implemented via a lexicographically ranked objective function, where the subsidy was first used to help active fishers with large share deficits, then active fishers interested in buying but without such a deficit, and finally fishers that wish to exit the industry. All active fishers received a discount on the market price in a share class such that they could buy shares in classes where they would otherwise be restricted by their current share holding compared to their historical catch levels. The distribution of subsidies in the market was also a way to achieve sufficient participation from fishers and reallocate the shares to those in need, but led to new design problems not covered by existing theory.

The software was developed, tested, and revised between early 2015 and 2017 and the market was organized in three rounds between May 1, 2017 and June 30, 2017. In each round buyers and sellers could revise their bids via a web-based bid submission system. To solve allocation and prices, a series of large-scale mathematical programs had to be solved after each round. The branch-and-cut algorithm was executed on a computer cluster. Various modeling approaches and cuts helped to solve the problems to optimality, which cannot generally be expected for a problem of this size.

Impact

The market solved a long-standing policy problem in New South Wales. For over a decade there had been a political struggle between fishers and the government about the correct way to reallocate catch shares. The proposed market design addressed all key concerns and was the decisive factor in finally getting all stakeholders on board.

The market completely reshaped the fishery industry in NSW. Close to 600 fishing businesses registered for the market and placed 740 buy bids, 432 sell offers, and 107 package offers. Importantly, previously underutilized shares were transferred to active fishers, which was the primary policy goal. The market saw 86% of the buy bids of active fishers with a share deficit get matched, reducing 95% of the share deficit in the most important share classes and 75% of the share deficit. In total, 35,954 shares were traded in the market with the expense of A$12.86 million of the available A$15 million subsidy funds. This was done in a fair manner such that the resulting outcome was accepted by the industry and now provides a basis for sustainable fishing in NSW. These cost savings were informed by our evaluation of alternative scenarios, which demonstrated the decreasing returns of higher subsidy expenditures for the government's main objectives.

Lynham (2014) reports that 154 fisheries worldwide have introduced catch share systems, and that the number is growing. The market design we implemented in New South Wales can serve as a template to effectively reallocate fishing shares elsewhere. First, the need for package trading has been motivated in a number of academic articles (Marszalec 2017, Iftekhar and Tisdell 2012, Teytelboym 2018), and it is likely to be a concern in other environmental markets as well. Second, linear and anonymous prices satisfy fairness considerations such as the equal treatment of equals (Bichler et al. 2018), a desirable feature for markets other than in New South Wales. Finally, the possibility to distribute a subsidy within a market can provide important incentives for participation, in markets where this is an issue.

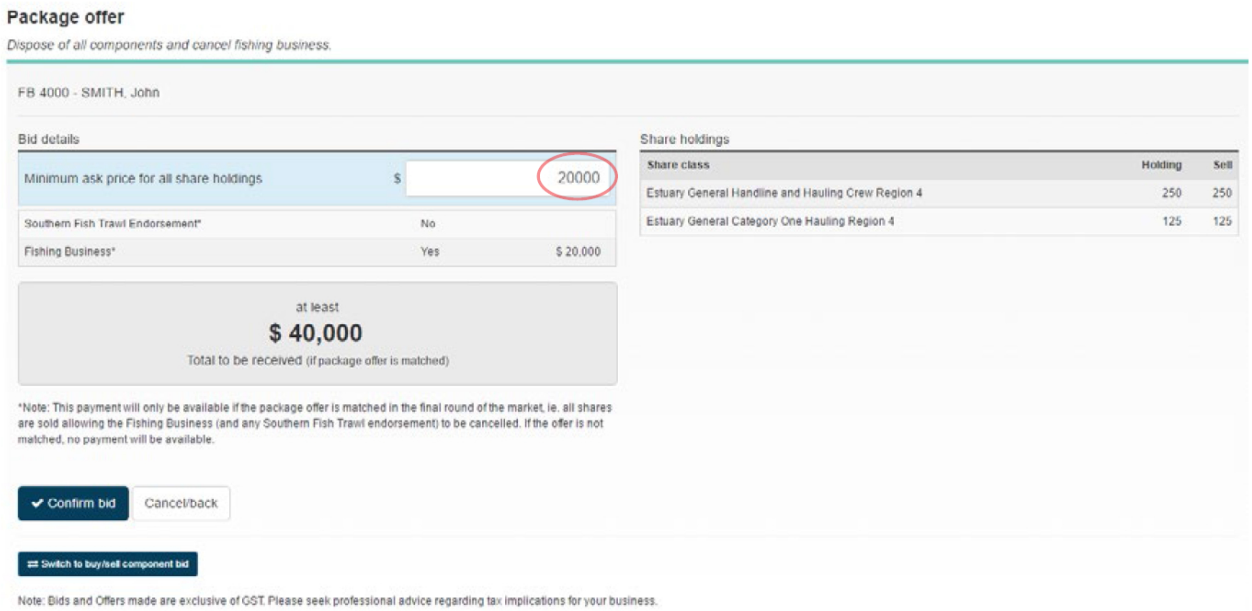
The need for package trading to accommodate synergistic preferences is not unique to the fishing industry. Package trading plays a role in the assignment of airport time slots (Pellegrini et al. 2012), railway slots (Borndörfer et al. 2006), the assignment of time slots at loading docs in retail logistics (Karaenke et al. 2019), pollution rights (Fine et al. 2017), rights on native vegetation offsets (Nemes et al. 2008), and water rights (McAdams 2017). We provide a practical market design with important features for such types of exchanges and as such, the impact of our work extends well beyond the fisheries application discussed in this paper.

User Interface

Let us first describe how bidders submitted bids and the types of bids allowed, which will be useful in the discussion of the mathematical model to solve the allocation problem next.

Buyers and sellers in the market for catch shares had different requirements with respect to what should be expressed by the bids submitted. Some sellers are unprofitable fishers who intend to retire, while others may want to get rid of unprofitable (unused) shares only, still keeping others. Those fishers who want to quit the business need to be able to specify all-or-nothing bids. Such a fisher has only one bid to submit, which includes his endowment in various share classes. The bid has a single ask price for the whole set of shares, and represents the least amount the seller wants to get for the endowment. Selling part of the shares would not be an option because it might render

Figure 1 The Screenshot Illustrates the Submission of a Package Bid



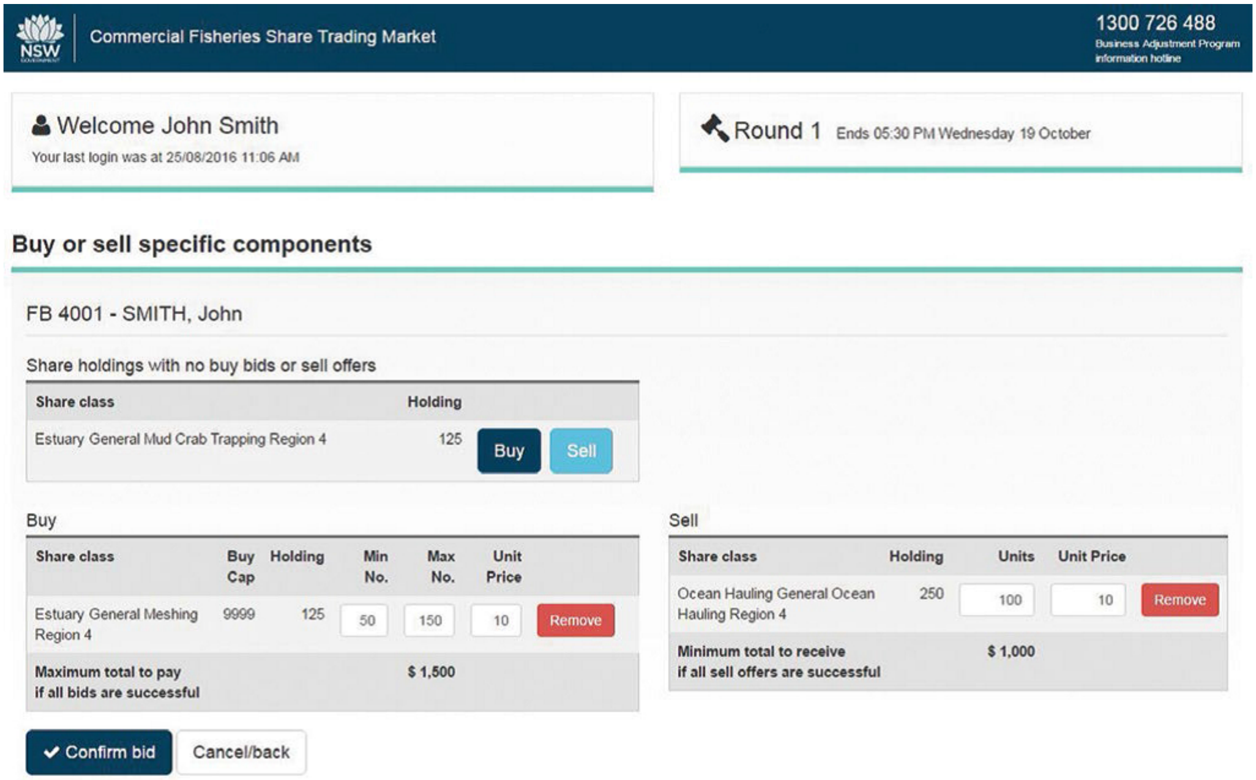
the fisher's operations even less profitable. Also, accepted exit bids lead to an extra payment that is paid out of the subsidy and intended to cover fishing license costs. Figure 1 shows a screen for the submission of a package bid. A fisher who wants to keep his licenses and sell only a part of the endowment can submit a set of independent sell-side bids in individual share classes. For example, the fisher might sell 125 shares of share class A for A\$10 and/or 80 shares of share class B for A\$15.

Buyers want to win shares in one or more share classes. They can submit several bids on multiple shares in a share class. Synergies across share classes were of less concern to buyers; therefore, they were not allowed to submit exclusive-or (XOR) package bids across multiple share classes. Buyers could bid a unit price for a quantity interval. For example, a fisher may want to buy shares from share class A. He needs at least 125 units and at most 250 units and is willing to pay up to A\$3 for each unit in this quantity interval. Figure 2 shows an example of the bid submission page for a buyer who can either buy or sell shares.

Modeling

The reallocation of catch shares based on bids and asks received in the market was facilitated via a sequence of mathematical programs implementing a sequence of lexicographically ordered

Figure 2 The Screenshot Illustrates the Submission of Bids to Buy or Sell Shares in Specific Share Classes



policy goals. This sequence of mathematical programs enforced constraints that determined linear and anonymous prices, respected all-or-nothing bids for packages, and ensured that neither the government nor any of the participants incurred a loss. We next introduce the objectives and constraints in an informal way (the precise mathematical model can be found in the appendix).

Objective Functions

The main objective of the centralized market was to ensure that shares could be reallocated such that existing catch levels remained possible after the introduction of the linkage program. This reallocation had to be based on voluntary trades because the government could not just take shares from inactive fishers and reallocate them to active ones. Even if it could, the government would not be privy to detailed information about fishers' preferences, making it impossible to reallocate efficiently. In contrast, the market can reallocate efficiently because the bids and asks received reflect fishers' preferences.

To alleviate any financial hardship imposed by the linkage program, the government was willing to inject a subsidy of up to A$15 million in the market. As discussed above, this subsidy had to be distributed endogenously based on the bids and asks received. Furthermore, different types of fishers had to be prioritized with regard to how much they should benefit from the subsidy. Active fishers with a share deficit should benefit as much as possible, and any residual budget should be used to help active fishers without a deficit or fishers who wanted to exit the market. This led to four different priorities for the government, which we turned into mathematical programs that we solved in a sequence.

The first priority (P1) was to ensure that the demands of active fishers with a deficit were satisfied as much as possible given the available subsidy. The second priority (P2) focused on active fishers without a share deficit. The third priority (P3) was to use the residual subsidy to support the retiring fishers who wanted to exit the market. The final priority (P4) was to use any leftover subsidy to aid inactive fishers who did not report any catch in the year before.

The mathematical programs implemented the different priorities by maximizing trades for the subset of fishers who were considered by the priority. We ran the programs sequentially. After each optimization, we added the objective function as a constraint in the subsequent optimization to make sure that the objective function value did not deteriorate.

Constraints

A number of constraints ensure a feasible allocation and prices that satisfy the payment rules laid out by the government. *Individual rationality constraints* make sure that no participant will face a loss as the outcome of trade. This means that no winning seller should receive less than his quoted ask price and no winning buyer is paying more than he or she indicated as the maximum price to pay. In similar manner, the government must not spend more than the subsidy it allocated to the market, which we guarantee via budget constraints.

Demand-supply constraints ensure that it is not possible to sell more items than are currently offered (i.e., that in every share class, the total number of units bought does not exceed the total number of units sold).

The prices computed by the optimization formulation are such that each winner pays the same price for a share class (i.e., linear and anonymous prices) and receives the same subsidy if eligible for it. An important argument for linear and anonymous prices in the discussion during the design phase was the perceived fairness of such prices. If two sellers on the exchange sell the same package, but they receive substantially different payments, participants would perceive this as unfair and inequitable. Proportional fairness and equitability of payments was a particularly important design goal for those sellers who exit the market. The subsidy was used to lower the payment by active fishers. That is, all active fishers received the same discount on the market price in a share class, while inactive fishers paid the full market price determined by the mathematical program.

Linear and anonymous prices are also employed on day-ahead energy markets for similar reasons (Van Vyve 2011). Unfortunately, linear and anonymous prices that constitute a competitive equilibrium in a combinatorial exchange are typically not feasible or are feasible only with restrictive assumptions on the value functions (Kelso and Crawford 1982, Gul and Stacchetti 1999, Baldwin and Klemperer 2018). Similar to day-ahead energy markets, we allowed paradoxically rejected losing bids (Meeus et al. 2009). Such buy (sell) bids are losing, even though they were higher (lower) than the market price. For example, a buy bid for two shares for A$20 might be losing, although the market price is only A$8 per share. However, prices were set such that they could not exceed the bid price plus the relative subsidy set for a share class for a winning bidder (i.e., individual rationality constraints were satisfied).

The Multiround Process

There was significant uncertainty of fishers about the value of shares and the government was concerned that the results of a single sealed-bid market might lead to regret among the participants. This was the first time that such an exchange took place and most fishers were inexperienced in any form of electronic trading. Therefore, the market was organized in multiple rounds. If the initial round did not meet a predefined internal goal and objective function values in terms of the priorities, the government could organize up to two more rounds. Bidders did not know a priori, which round would be the last to mitigate gaming behavior.

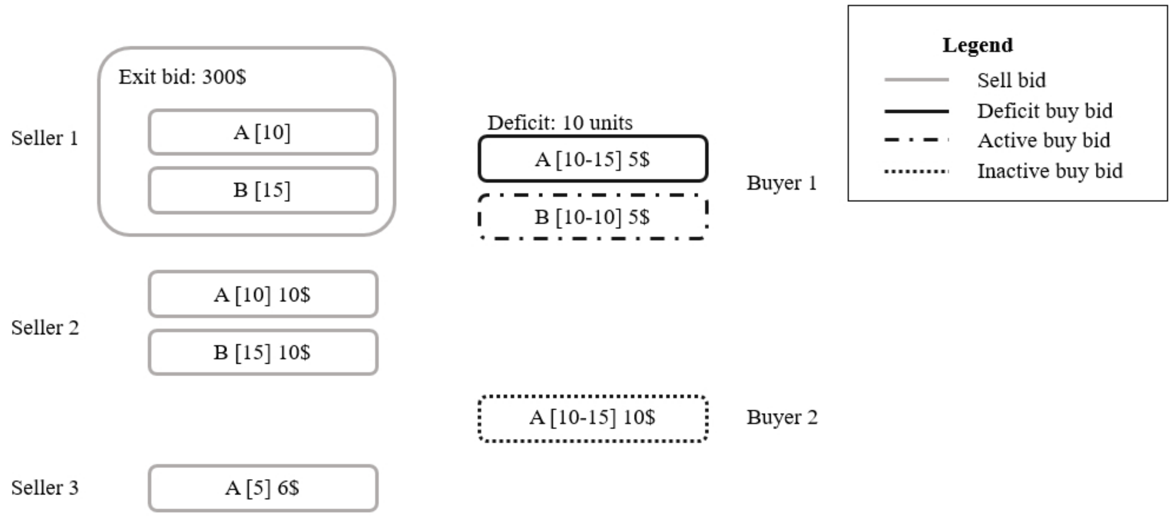
This uncertainty should counteract gaming where some bidders just try to win with a very low bid and learn about the market. We did see a few bids in the first round that were either far higher or far lower than others and thus could either be an attempt to game the market or a result of value uncertainty. However, the bids were relatively stable across all three rounds as the data analysis in section *Results* shows. Overall, the multiple rounds helped to reduce anxiety, set the expectations of fishers, and let them revise their valuation, which was also important for the acceptance of the market.

There were only minimal activity rules across rounds. Bidders needed to have a confirmed bid in a round in order to participate in any subsequent round (if held). Bidders could freely modify their bids within a round (prior to the closing time) or from one round to the next. This could include changing from an exit package offer to individual buy/sell bids, from buy to sell or vice versa, adding bids for new share classes or deleting previous individual bids, or changing quantity or price. Each new bid superseded the previous bid from that shareholder. Bids that were not modified remained valid from one round to the next until the conclusion of the market, and could become successful in a later round even if they were not successful in an earlier round. Stricter activity rules were considered too complex for the large number of inexperienced fishers participating.

After a round closed, allocations, prices, and subsidies were computed for a number of different scenarios – based on predetermined variations in parameters such as total subsidy, subsidy reserved for exit bids, and bounds on the subsidy per share class. An evaluation panel consisting of government and independent experts, with oversight from the projects' independent probity advisor, analyzed the effect of these scenarios on achievement of the market objective in each scenario. The evaluation panel then chose one of the allocations as the final allocation or devised another round. If there was another round, one of the allocations and the associated prices were chosen as a basis for the feedback to the bidders.

Following a round, the government provided individual results to each bidder (i.e., whether bids were successful or unsuccessful and market prices for each share class plus the subsidy that bidder would have received if it was the last round), via a combination of email and a secure website. If there was another round, the government also informed bidders about the next round (e.g., the

Figure 3 **This Example Illustrates an Instance of a Bid**



opening and closing times, bids carried over unless modified, and both share allocations and prices could change in a new round). Once a round was determined to be the final round, the government advised bidders of the final outcomes and next steps for payment.

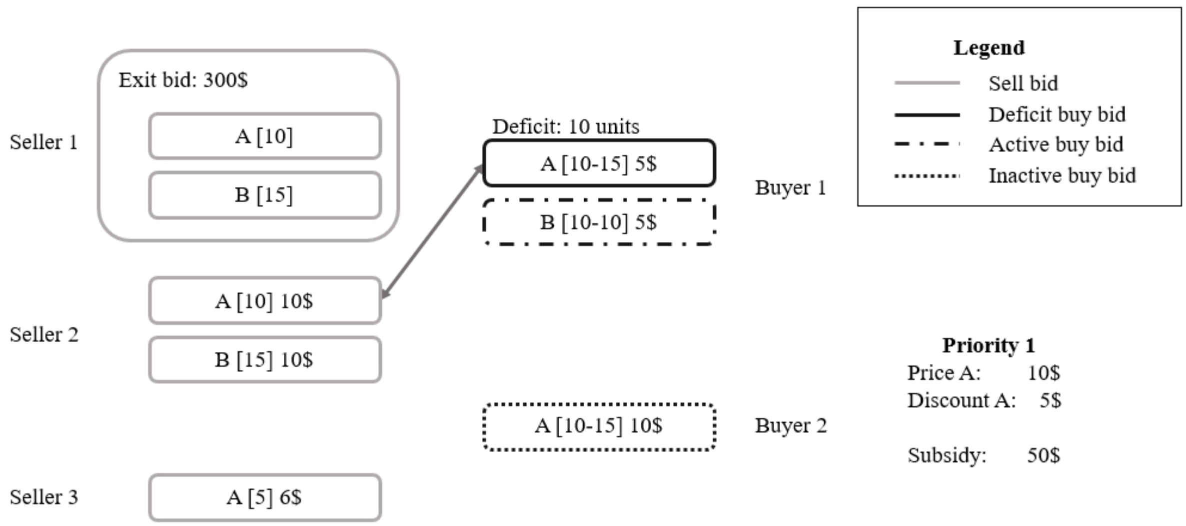
Illustrative Example

Let us illustrate the combinatorial exchange using an example with three sellers (Figure 3). We have three sellers: Seller 1 is providing a package bid with two blocks of shares with a total ask of A$300, and two other sellers submit individual sell bids. On the buy side we have two participants: Buyer 1 has a deficit of 10 units in share class A, is willing to buy from 10 to 15 units, and wants to pay A$5 at maximum; this buyer is also active in share class B, wants to buy 10 units firm in this share class, and quotes the same price. Buyer 2 is also willing to buy shares in A, but was not active in this share class yet, and thus is not eligible for a discount.

If we take a look at the bids from the example in Figure 3, we see that if there is no subsidy, then no trade will happen: buy bids are too low for the sell-side asks. The only match can be observed between Seller 3 and Buyer 2, yet Buyer 2 needs twice as many shares as Seller 3 can provide; thus, they will not trade as well.

In priority P1, the program maximizes volume (price times number of units bought) of deficit shares acquired by buyers. In the example, only Buyer 1 has a deficit of 10 units in share class A.

Figure 4 This Example Illustrates a Priority 1 Computation

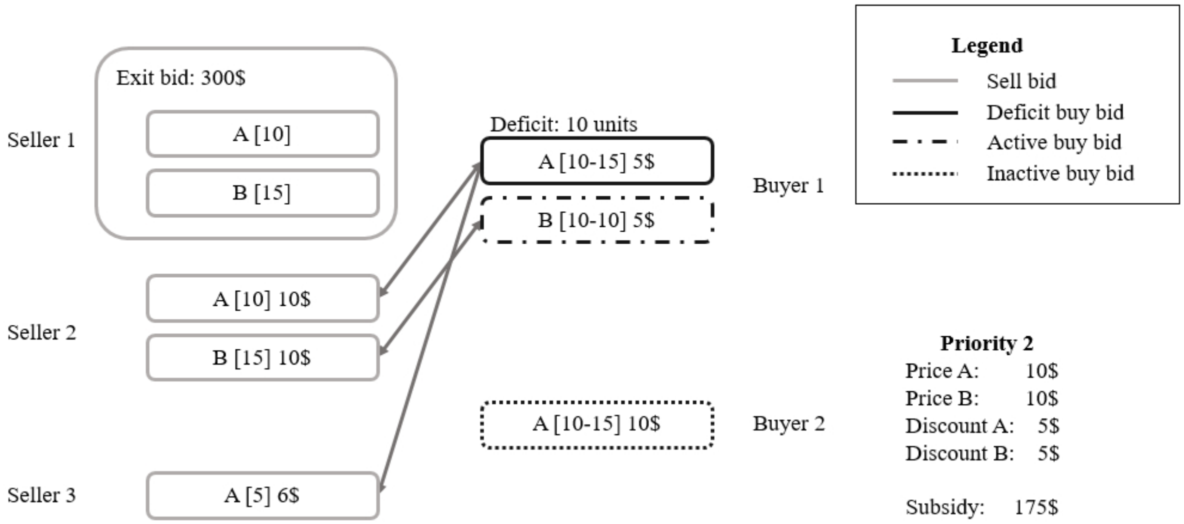


A solution will be to accept the bid of Buyer 1 in class A and that of Seller 2 in the same class. Then, the price for share class A is A\$10 and the government provides a discount of A\$5 per unit to make this trade happen. In total, the government spends A\$50 of the predefined subsidy. The result of priority P1 is shown in Figure 4. For further priorities, Buyer 1 should not receive less than 10 shares in class A.

Priority P2 aims to maximize the volume of trades for bids of active bidders who do not have a share deficit. In our example, we have two active bids, one in share class A and one in share class B, both from Buyer 1. Note, even though Buyer 1 has a deficit of only 10 units, he is willing to buy up to 15 units, which should be taken into account in P2. The best option is again to match Buyer 1 with Seller 2, this time in both share classes and with a single bid of Seller 3 in order to satisfy his demand of 15 units in share class A (see Figure 5).

Note that Seller 2 wants to sell 15 units in share class B, while Buyer 1 demands only 10 units. This means that the government needs to buy out the remaining five units for which the market price needs to be paid. The resulting prices are A\$10 for A and B, discounts of A\$5 for both share classes. The government buys out five units in share class B for the full market price in order to keep the exchange budget-balanced. The subsidy needed for such an allocation is thus A\$175 (25 times A\$5 for discounts and A\$50 for the purchase of the remaining five units in share class B).

Figure 5 This Example Illustrates a Priority 2 Computation



The reform is also meant to consolidate the market by allowing fishers to quit the business and sell all their possessions and fishing licenses. This is the focus of priority P3. In our example, this makes a difference for the sell side: even though it is cheaper to accept bids from Seller 2, the exit bid is preferred in P3 as a policy goal (see Figure 6). Buyer 1 trades with Seller 1 and Seller 3. To efficiently use the subsidy, we charge A\$6 for A and A\$16 per unit of B. Seller 1 gets exactly the A\$300 requested, and the government again needs to buy five shares in share class B. The total subsidy spent is thus A\$205 (A\$15 for discounts in A, A\$110 for discounts in B, and A\$80 for buying out five shares in class B).

Priority P4 focuses on inactive bids, and in our example there is room for improvement in this regard: Buyer 2 can be matched with the bid of Seller 2 in class A. This will increase the subsidy paid to Buyer 1 in class A, and thus the overall subsidy used (see Figure 7). All buy bids are satisfied (Buyer 2 gets only 10 units), and the price for A is A\$10. For B the price increased to approximately A\$13.4 to satisfy the exit bid of A\$300. This means, the discount for class A is A\$5 and for class B A\$8.4. The total subsidy used is now A\$301 (A\$150 discounts for A, A\$84 discounts for B, and A\$67 to buy out five units in class B), and the priorities of the government have been satisfied as far as possible in a lexicographic manner.

Figure 6 This Example Illustrates a Priority 3 Computation

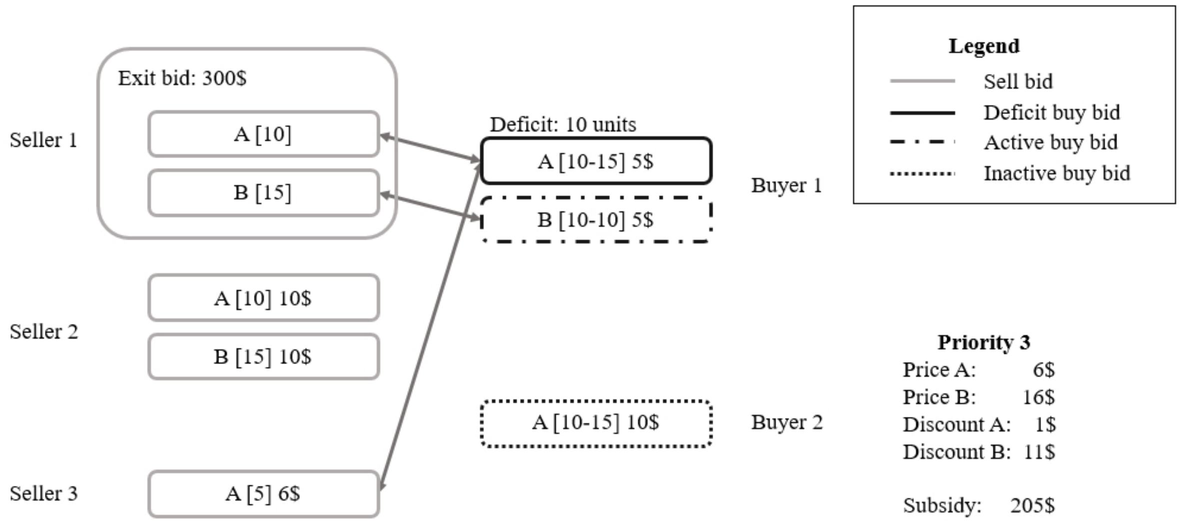
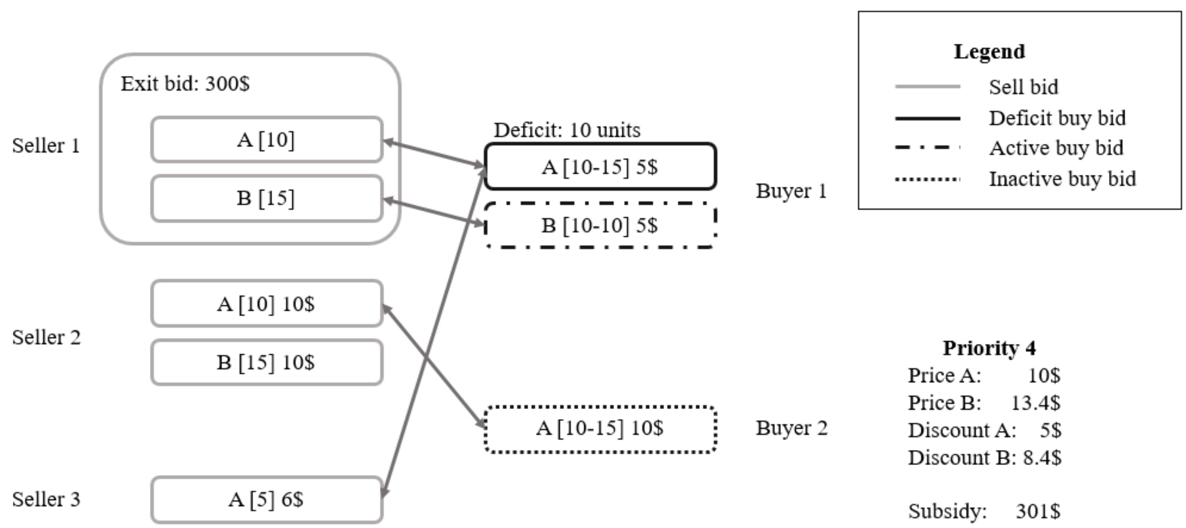


Figure 7 This Example Illustrates a Priority 4 Computation



Results

We first analyze the bidding activity of participants in the market, before we interpret the results and analyze the impact on the market.

Improvements Over the Rounds

Let us briefly discuss the progress of the market, the bidding activity, and the changes across rounds in the market in 2017. Table 1 shows the number of bids submitted in each round. Solving the optimization problems with this number of bids to optimality or near optimality took from 20

Table 1 The Data in the Table Show the Number of Bids for Each of Rounds 1, 2, and 3

	Winning buy	Losing buy	Winning sell	Losing sell	Winning exit	Losing exit
Round 1	158	589	35	386	34	61
Round 2	322	417	90	349	51	50
Round 3	446	294	131	301	62	45

minutes up to several hours in some instances. Actually, the final market could be solved exactly. Interestingly, the number of bids is almost the same in each round, although no activity rules were employed across rounds. Because bidders did not know, which round would be the last, they participated actively from the start. The number of winners was increasing largely because the government decided to increase the total subsidy after each round from A$6 to A$11 million in total based on the bids received.

Most losing buyers increased their bids across rounds and losing sellers typically decreased their bids, while most winners did not change their bids. Of 589 losing buyers in Round 1, 322 increased their bids. The average increase was more than 52%. However, only a few buyers with a very large relative increase are responsible for this large relative number.

Of 417 losing bids in Round 2, 153 were increased by 30% (median), while 212 losing bids remained untouched. The distributions of relative changes and median values have to be taken with a grain of salt, because some buy bids in the initial rounds were low or even close to zero. Once the bid was increased to levels of the market price in the first round, this could lead to a very large relative increase. Note that bids and resulting market prices varied a lot across different share classes. A majority of the share classes traded at prices less than A$200, while a few valuable share classes had share prices of several thousand Australian dollars.

Similarly, of 386 losing sell-side bids or asks, 188 lowered their bids by 32% in the median while 117 did not change between Round 1 and 2. Between Round 2 and 3, 142 of the 349 losing sell-side bids were decreased by 30% (median) and 166 losing asks remained untouched after round 2. The effect of offers becoming more competitive can be evaluated by applying the parameters from the

Table 2 **The Data in the Table Show the Changes to Bid Prices by Buyers Between Rounds**

Bid type	Round 1 to Round 2	Round 2 to Round 3
Buy bid	$\uparrow$357 $\downarrow$47 $\sim$287	$\uparrow$223 $\downarrow$60 $\sim$413
-Winners	$\uparrow$35 $\downarrow$19 $\sim$103	$\uparrow$70 $\downarrow$42 $\sim$201
-Losers	$\uparrow$322 $\downarrow$28 $\sim$184	$\uparrow$153 $\downarrow$18 $\sim$212

Table 3 **The Data in the Table Show the Changes to Ask Prices by Sellers Between Rounds**

Bid type	Round 1 to Round 2	Round 2 to Round 3
Sell bid	$\uparrow$52 $\downarrow$191 $\sim$141	$\uparrow$21 $\downarrow$153 $\sim$227
-Winners	$\uparrow$6 $\downarrow$3 $\sim$24	$\uparrow$9 $\downarrow$11 $\sim$61
-Losers	$\uparrow$46 $\downarrow$188 $\sim$117	$\uparrow$12 $\downarrow$142 $\sim$166

Table 4 **The Data in the Table Show the Changes to Ask Prices Between Rounds by Sellers with an Exit Bid**

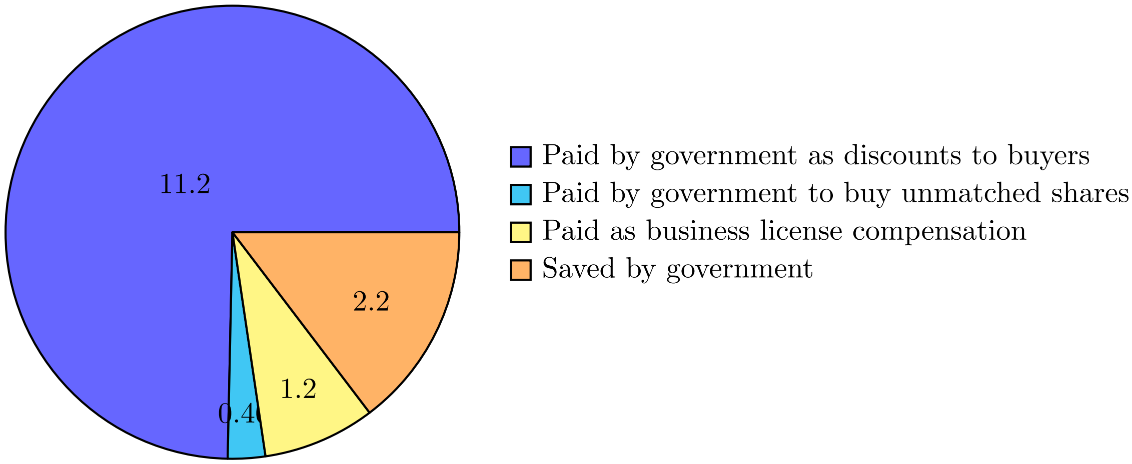
Bid type	Round 1 to Round 2	Round 2 to Round 3
Exit bid	$\uparrow$6 $\downarrow$39 $\sim$45	$\uparrow$10 $\downarrow$34 $\sim$52
-Winners	$\uparrow$4 $\downarrow$1 $\sim$29	$\uparrow$6 $\downarrow$12 $\sim$33
-Losers	$\uparrow$2 $\downarrow$38 $\sim$16	$\uparrow$4 $\downarrow$22 $\sim$19

selected scenario in the final round to all three rounds. The results demonstrate that all priorities increase by at least 30% from round to round, and highlight the value of a multiround process (Bichler et al. 2019).

Outcomes

The main objective of the market was to reallocate shares so that existing catch levels would remain possible after the introduction of the linkage program. Or, stated differently, the main goal of the market was to provide an opportunity to undo the share deficits created by the linkage program. The exchange can be considered a great success in light of this. Close to 600 fishers participated and 86% of the buy bids from active fishers with a deficit were matched. Their overall share deficit was reduced by 95% in the high-priority share classes.

Figure 8 The Pie Chart Shows a Subsidy Distribution in Millions of Australian Dollars (A$)



In total, A$14.8 million was paid to the sellers: the buyers paid A$3.2 million and the remaining A$11.2 million was subsidized by the government. The government spent an additional A$0.4 million to buy out shares of sell-side package bids for which there was no buyer. An extra sum of A$1.2 million was set aside for business license compensation for those fishers who decided to exit. Also, 60% of the package offers were matched and 62 businesses successfully sold all their shares. These exiting fishers received A$10.1 million for their shares. Around A$5.9 million went to sellers who just sold part of their endowment of catch shares.

The government had set aside A$15 million to subsidize the market but ended up spending only A$12.8 million (including business license compensations). This decision was informed by our evaluation of alternative scenarios, including the effects of higher subsidy levels. This showed that the government's main objectives were largely unaffected by higher expenditure levels. This aspect of our approach saved taxpayers several millions.

Because of the linear and anonymous prices, as well as the subsidy, buyers typically paid less than their submitted bid. In total, winning buyers bid A$6.2 million, but only had to pay A$3.2 million. Similarly, the winning sellers asked for A$12.7 million in total, but received payments of A$14.8. The subsidy of A$11.2 millions enabled these trades and provided enough incentives for fishers to participate. These results would not have been possible with a government buyout.

Lessons Learned

Market-based solutions are increasingly being considered for the allocation or reallocation of resource shares (Marszalec 2017). A recent fisheries reform bill in the Faroe Islands, for example, proposes to use auctions to allocate 25% of the country's fishing shares (Government of the Faroe Islands 2017). Initial allocations decided by auction tend to be more efficient than those resulting from grandfathering, but changes in the industry, fish stock, or overall quota will necessitate revisions to the initial share allocation.

This paper describes the implementation of a combinatorial exchange to reallocate fishing rights in New South Wales, Australia. The design of the exchange addressed several nonstandard requirements, including all-or-nothing offers, fair prices, and an endogenously determined subsidy. The exchange was conducted in May and June 2017 and successfully reformed the NSW fishing industry. The exchange may serve as a template for the reallocation of fishing rights in other jurisdictions. Lynham (2014) reports that there are 154 catch share systems worldwide and that the number is growing. Moreover, similar exchanges can be used to reallocate resource rights in other domains (e.g., to trade water rights, pollution rights, or environmental offsets).

An important takeaway from this paper is that operations research and market design have significant potential to address challenging policy problems. And this potential is not limited to the fisheries domain. The recent incentive auction in the United States, which reallocated spectrum from TV stations to telecoms (Leyton-Brown et al. 2017), is a different application. In this case, new and innovative designs were required to address the nonstandard requirements imposed by the market. While there is no "one size fits all" in market design, these examples highlight that template designs that are portable across jurisdictions and domains can successfully be developed.

Acknowledgments

We should like to thank Geoff Allan, Rebecca Chapman, Andrew Goulstone, Scott Hansen, John Ledyard, Simon Miller, Sevaly Sen, and the Honourable Minister Niall Blair for their feedback and support. This paper was a finalist for the INFORMS Daniel H. Wagner Prize on Excellence in Operations Research Practice. We thank all the referees for their valuable input.

References

Baldwin, Elizabeth, Paul Klemperer. 2018. Understanding preferences:'demand types', and the existence of equilibrium with indivisibilities. Tech. rep., Oxford University.

Bichler, M., V. Fux, J. Goeree. 2018. A matter of equality: Linear pricing in combinatorial exchanges. *Information Systems Research* **29** 1024–1043.

Bichler, Martin, Vladimir Fux, Jacob K Goeree. 2019. Designing combinatorial exchanges for the reallocation of resource rights. *Proceedings of the National Academy of Sciences* **116**(3) 786–791.

Birkenbach, Anna M, David J Kaczan, Martin D Smith. 2017. Catch shares slow the race to fish. *Nature* **544**(7649) 223–226.

Borndörfer, Ralf, Martin Grötschel, Sascha Lukac, Kay Mitusch, Thomas Schlechte, Sören Schultz, Andreas Tanner. 2006. An auctioning approach to railway slot allocation. *Competition and Regulation in Network Industries* **1**(2) 163–196.

Branch, Trevor A. 2009. How do individual transferable quotas affect marine ecosystems? *Fish and Fisheries* **10**(1) 39–57.

Cartwright, Ian, Mary Lack, Sevaly Sen. 2015. Final share linkage recommendations. Tech. rep., Structural Adjustment Review Committee: `https://www.dpi.nsw.gov.au/__data/assets/pdf_file/0004/656806/SARC-Final-Recommendations-on-Share-Linkage_final.pdf`.

Costello, Christopher, Steven D. Gaines, John Lynham. 2008. Can catch shares prevent fisheries collapse? *Science* **321**(5896) 1678–1681. doi:10.1126/science.1159478.

Fine, Leslie, Jacob K Goeree, Tak Ishikida, John O Ledyard. 2017. ACE: A combinatorial market mechanism. Martin Bichler, Jacob K Goeree, eds., *Handbook of Spectrum Auction Design*. Cambridge University Press, 874–903.

Government of the Faroe Islands. 2017. Faroe islands fisheries reform. `http://www.government.fo/news/news/new-fisheries-reform-introduced-to-parliament/`. Accessed: 2018-10-30.

Gul, F., E. Stacchetti. 1999. Walrasian equilibrium with gross substitutes. *Journal of Economic Theory* **87**(1) 95–124.

Iftekhar, M.S., J.G. Tisdell. 2012. Comparison of simultaneous and combinatorial auction designs in fisheries quota market. *Marine Policy* **36**(2) 446–453.

Jackson, Jeremy BC, Michael X Kirby, Wolfgang H Berger, Karen A Bjorndal, Louis W Botsford, Bruce J et al. Bourque. 2001. Historical overfishing and the recent collapse of coastal ecosystems. *Science* **293**(5530) 629–637.

Karaenke, Paul, Martin Bichler, Stefan Minner. 2019. Coordination is hard: Electronic market mechanisms for increased efficiency in transportation logistics. *Management Science* **forthcoming**.

Kelso, A. S., V. P. Crawford. 1982. Job matching, coalition formation , and gross substitute. *Econometrica* **50**(6) 1483–1504.

Leyton-Brown, Kevin, Paul Milgrom, Ilya Segal. 2017. Economics and computer science of a radio spectrum reallocation. *Proceedings of the National Academy of Sciences* **114**(28) 7202–7209.

Ludicello, S., B. Lueders. 2016. A survey of litigation over catch shares and groundfish management in the pacafic coast and northeast muiltspecies fisheries. *Environmental Law* **46**(1) 157–208.

Lynham, John. 2014. How have catch shares been allocated? *Marine Policy* **44** 42–48.

Marszalec, Daniel. 2017. Auctions for quota: A primer and perspectives for the future. *Fisheries Research* (203) 84–92.

McAdams, David. 2017. Smart watershed markets: The case of the central platte groundwater exchange .

Meeus, L., K. Verhaegen, R. Belmans. 2009. Block order restrictions in combinatorial electric energy auctions. *European Journal of Operational Research* **196** 1202–1206.

Melnychuk, Michael C, Timothy E Essington, Trevor A Branch, Selina S Heppell, Olaf P Jensen, Jason S Link, Steven JD Martell, Ana M Parma, John G Pope, Anthony DM Smith. 2012. Can catch share fisheries better track management targets? *Fish and Fisheries* **13**(3) 267–290.

Nemes, V., C. R. Plott, G. Stoneham. 2008. Electronic bushbroker exchange: designing a combinatorial double auction for native vegetation offsets. *Available at SSRN 1212202* .

Pellegrini, Paola, Lorenzo Castelli, Raffaele Pesenti. 2012. Secondary trading of airport slots as a combinatorial exchange. *Transportation Research Part E: Logistics and Transportation Review* **48**(5) 1009 – 1022.

Rosenberg, Andrew A. 2017. Marine conservation: The race to fish slows down. *Nature* **544** 165–166.

Teytelboym, Alexander. 2018. Natural capital market design. Tech. rep., Oxford University.

Van Vyve, Mathieu. 2011. Linear prices for non-convex electricity markets: models and algorithms. *Université catholique de Louvain, Center for Operations Research and Econometrics, Tech. Rep* .

Worm, Boris, Edward B Barbier, Nicola Beaumont, J Emmett Duffy, Carl Folke, Benjamin S Halpern, Jeremy BC Jackson, Heike K Lotze, Fiorenza Micheli, Stephen R Palumbi, et al. 2006. Impacts of biodiversity loss on ocean ecosystem services. *Science* **314**(5800) 787–790.

Appendix. Mathematical Model

This appendix provides a formal description of the optimization models, which were solved after each round to determine the allocation and prices. Let $\mathcal{B}$ denote the set of all buy bids, which can be divided into two disjoints sets of active buy bids $\mathcal{B}^{\mathcal{A}}$ and inactive buy bids $\mathcal{B}^{\mathcal{I}}$ with $\mathcal{B}^{\mathcal{A}} \cap \mathcal{B}^{\mathcal{I}} = \emptyset$. Each bid $j \in \mathcal{B}$ concerns only one share class $l \in \mathcal{L}$ and is denoted by the tuple $< l, {b_j}^l, \underline{\beta}_j^l, \overline{\beta}_j^l >$, with ${b_j}^l$ the per-unit price and $\underline{\beta}_j^l$ and $\overline{\beta}_j^l$ the lower and the upper bounds on the number of units demanded. Associated with each active buy-side bid $j \in \mathcal{B}^{\mathcal{A}}$ is the deficit value d_j^l, which is the buyer's shortage of shares in share class l and is determined prior to the market.

Similarly, let $\mathcal{S}$ denote the set of sell-side bids and $\mathcal{S}^E$ the set of exit bids (package bids). An individual bid $i \in \mathcal{S}$ in a share class $l \in \mathcal{L}$ consists of a quantity q_i^l and a per-unit price s_i^l. An exit bid $i \in \mathcal{S}^E$ consists of a package vector $q_i = \{q_i^l\}_{l \in \mathcal{L}}$, which contains the number of units owned by a retiring fisher in each share class (some elements of the vector may be zeros) and the total price s_i. If an exit bid was accepted, the seller also received a compensation, p^E, for relinquishing the business license. This compensation was paid from the subsidy the government provided to the market.

To each bid we assign an allocation variable that describes whether the bid was successful and in case of a buy bid, how many units are allocated. For an individual sell (exit) bid, this is a binary variable σ_i^l (σ_i). For a buy bid it is an integer variable β_j^l, which is zero or lies within the quantity bounds specified by the buyer, $[\underline{\beta}_j^l, \overline{\beta}_j^l]$. To ensure this, we introduce the binary variable ζ_j^l, which is 1 if and only if the corresponding β_j^l is positive:

$$\beta_j^l \leq \overline{\beta}_j^l \zeta_j^l \qquad \forall j \in \mathcal{B}, l \in \mathcal{L} \qquad \text{(BQty1)}$$

$$\underline{\beta}_j^l \zeta_j^l \leq \beta_j^l \qquad \forall j \in \mathcal{B}, l \in \mathcal{L} \tag{BQty2}$$

Objectives

The government's lexicographically ordered policy goals were implemented by maximizing several priorities consecutively. We next describe these priorities as an objective function for a mixed-integer linear program. Priority P1 concerned buy bids with a deficit:

$$\max \sum_{j \in \mathcal{B}^{\mathcal{A}}} \sum_{l \in \mathcal{L}} \kappa_j^l b_j^l \tag{P1}$$

In addition, the constraints that apply to all priorities (described below), this priority requires $\kappa_j^l \leq \beta_j^l, \kappa_j^l \leq d_j^l \; \forall j \in \mathcal{B}^{\mathcal{A}}, l \in \mathcal{L}$. In words, the goal is to maximize the volume of units bought up by active fishers up to their deficit levels, which is implemented by the variable $\kappa_j^l \leq d_j^l$. Optimizing priority P1 results in an allocation $(\{\beta_j^{l,P1}\}_{j \in \mathcal{B}^{\mathcal{A}}})$, which then becomes a constraint for priority P2 to ensure that later priorities do not adversely affect the main policy goal P1 (i.e., reduce deficits for active fishers).

Priority P2 considers all active buyers:

$$\max \sum_{j \in \mathcal{B}^{\mathcal{A}}} \sum_{l \in \mathcal{L}} (\beta_j^l b_j^l), \tag{P2}$$

where we require $\beta_j^l \geq \beta_j^{l,P1} \; \forall j \in \mathcal{B}^{\mathcal{A}} : d_j^l > 0$ (Transition 1).

Priority P3 maximizes the number of accepted package bids by bidders who wish to leave the market:

$$\max \sum_{i \in \mathcal{S}^E} \sigma_i, \tag{P3}$$

This priority requires $\beta_j^l \geq \beta_j^{l,P2} \; \forall j \in \mathcal{B}^{\mathcal{A}}$ (Transition 2), which implies Transition 1.

Priority P4 just maximizes the total volume of shares bought of *all* fishers, including inactive fishers:

$$\max \sum_{j \in \mathcal{B}} \sum_{l \in \mathcal{L}} (\beta_j^l b_j^l), \tag{P4}$$

$$\beta_j^l \geq \beta_j^{l,P2} \qquad \forall j \in \mathcal{B}^{\mathcal{A}} \tag{Transition 2}$$

$$\sigma_i \geq \sigma_i^{P3} \qquad \forall i \in \mathcal{S}^E \tag{Transition 3}$$

which requires the following constraints:

$$\beta_j^l \geq \beta_j^{l,P2} \qquad \forall j \in \mathcal{B}^{\mathcal{A}} \tag{Transition 2}$$

$$\sigma_i \geq \sigma_i^{P3} \qquad \forall i \in \mathcal{S}^E \tag{Transition 3}$$

Priorities P1-P4 are solved consecutively. We used a "warm start" for priorities $k = 2, 3, 4$ by feeding the solver the solution of priority $k - 1$ (this solution is feasible but not necessarily optimal for the new objective).

Constraints that Apply to All Priorities

The demand-supply constraint ensures that in every share class the total number of units bought does not exceed the total number of units sold:

$$\theta^l + \sum_{j\in\mathcal{B}} \beta_j^l = \sum_{i\in\mathcal{S}^E} q_i^l \sigma_i + \sum_{i\in\mathcal{S}} q_i^l \sigma_i^l, \qquad \forall l \in \mathcal{L} \qquad \text{(Demand-Supply)}$$

Here θ^l is an integer-valued variable that captures "artificial" demand by the government to facilitate trades when sell-side asks are not exactly matched by buy-side bids. This variable is positive only if it raises the objective function (i.e., one of the priorities P1-P4). The government pays the market price for all units bought to guarantee anonymity and a balanced budget. Shares bought by the government were terminated after the market was conducted and the amount paid by the government came out of the total subsidy budget.

An important requirement is that no participant incurs a loss (i.e., no winning seller receives less than the ask price and no winning buyer pay more than the bid price). Hence, for winning buyers an important requirement is that no participant incurs a loss (i.e., no winning seller receives less than the ask price and no winning buyer pays more than the bid price). Hence, for winning buyers

$$\rho^l \le b_j^l, \qquad \forall j \in \mathcal{B} : \beta_j^l > 0, l \in \mathcal{L} \qquad \text{(IRB}')$$

where the continuous variable ρ^l represents the share price in share class l. For active fishers, the government pays a discount, δ^l, to the market price, which changes this constraint to

$$\rho^l \le b_j^l + \delta^l. \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : \beta_j^l > 0, l \in \mathcal{L} \qquad \text{(IRBA}')$$

These constraints can be linearized using "big-M" constraints.

Similarly, for winning sellers

$$s_i \sigma_i \le \sum_{l\in\mathcal{L}} q_i^l \rho_i^l \qquad \forall l \in \mathcal{L}, \forall i \in \mathcal{S}^E : \qquad \text{(IRP)}$$

$$s_i^l \sigma_i^l \le \rho^l \qquad \forall i \in \mathcal{S} \qquad \text{(IRS)}$$

To encourage participation, the government provided a subsidy Δ, part of which was used to buy out the business licenses of those fishers who wished to exit the market:

$$\sum_{i\in\mathcal{S}^E} p^E \sigma_i \le \Delta^E \qquad \text{(Exit subsidy)}$$

The residual subsidy $\Delta^R = \Delta - \Delta^E$ was applied to provide discounts, δ^l, to active buyers and pay for the shares bought by the government:

$$\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{B}^A}\beta_j^l\delta^l + \sum_{l\in\mathcal{L}}(\rho^l-\delta^l)\theta^l \le \Delta^R \qquad \text{(Subsidy)}$$

This constraint contains the product of continuous and integer variables. To linearize it would require the introduction of a large number of additional binary variables and constraints. Instead, we rewrote the constraint such that only inactive bids are linearized (there were fewer inactive than active fishers).

Price and Payment Computation

The formulation in the previous subsections determined an allocation that satisfies the lexicographic objective function of the government. The resulting prices are not necessarily unique. The government decided to minimize the variance of prices in different share classes as a secondary objective. For this, the variables $\sigma_i^{l*}, \sigma_i^*, \beta_j^{l*}$ become fixed parameters, but prices ρ^l remain variables. A quadratic optimization minimizes the differences in prices across share classes l.

$$\min \sum_{l\in\mathcal{L}}(\rho^l)^2 \qquad \text{(Prices)}$$

$$\text{s.t. } s_i \le \sum_{l\in\mathcal{L}} q_i^l\rho^l \qquad \forall l\in\mathcal{L}, \forall i\in\mathcal{S}^E : \sigma_i^* = 1 \qquad \text{(IRP)}$$

$$s_i^l \le \rho^l \qquad \forall i\in\mathcal{S} : \sigma_i^{l*} = 1 \qquad \text{(IRS)}$$

$$\rho^l \le b_j^l + \delta^l \qquad \forall j\in\mathcal{B}^{\mathcal{A}} : \beta_j^{l*} > 0, l\in\mathcal{L} \qquad \text{(IRBA)}$$

$$\rho^l \le b_j^l \qquad \forall j\in\mathcal{B}^{\mathcal{I}} : \beta_j^{l*} > 0, l\in\mathcal{L} \qquad \text{(IRBI)}$$

$$\sum_{i\in\mathcal{S}^E}\sum_{l\in\mathcal{L}} q_i^l\sigma_i^*\rho^l + \sum_{i\in\mathcal{S}}\sum_{l\in\mathcal{L}} q_i^l\sigma_i^{l*}\rho^l - \sum_{j\in\mathcal{B}^I}\beta_j^{l*}\rho^l - \sum_{j\in\mathcal{B}^A}\beta_j^{l*}(\rho^l-\delta^l) + \sum_{i\in\mathcal{S}^E} p^E\sigma_i^* \le \Delta \qquad \text{(Subsidy)}$$

$$\underline{\delta}^l\rho^l \le \delta^l \le \overline{\delta}^l\rho^l \qquad l\in\mathcal{L} \qquad \text{(Subsidy bounds)}$$

$$\rho^l, \delta^l \in \mathbb{Z}_{\ge 0} \qquad \forall j\in\mathcal{B}, l\in\mathcal{L} \qquad \text{(Int)}$$

Finally, after unique prices were determined, the subsidy by the government was minimized. For this, the variables ρ^{l^*}, the resulting prices for each share class from the previous model, were fixed, and the following final optimization problem was solved.

$$\min \sum_{j \in \mathcal{B}^{\mathcal{A}}} \beta_j^{l^*} \delta^l \tag{Subsidy}$$

$$\text{s.t. } \rho^{l^*} \leq b_j^l + \delta^l \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : \beta_j^{l^*} > 0, l \in \mathcal{L} \tag{IRBA}$$

$$\underline{\delta}^l \rho^{l^*} \leq \delta^l \leq \overline{\delta}^l \rho^{l^*} \qquad l \in \mathcal{L} \tag{Subsidy bounds}$$

$$\delta^l \in \mathbb{Z}_{\geq 0} \qquad \forall l \in \mathcal{L} \tag{Int}$$

Implementation

Different model formulations and solver implementations with various problem sizes were tested during the development phase, before we decided on the model we described above. The final model was tested with up to 2,000 bids on artificially generated bid data, which was significantly more than what we experienced in the field. These problems could typically be solved to near optimality and many also to optimality. Near optimality describes integrality gaps of of less than 6%. Only very rarely did we experience larger integrality gaps after half an hour for very large problem sizes. The differences in the allocation of a near-optimal solution and the optimal solution were typically very small or did not exist. However, proving the optimal solution could sometimes take a disproportionally long time. Depending on the subsidy provided, problems became harder to solve such that the computations could take several hours. However, on average we used less than 30 minutes for each optimization run. Parameter tuning in the branch-and-cut solver helped to solve the problem sizes. For example, for priority P3 we modified the branching priorities for package bid variables to branch first on low ask bids. The specific model formulation presented in this section played a significant role in keeping the problems tractable.

The final round of the market included 1,280 bids leading to a mathematical program of around 6,000 variables and 10,230 constraints. We were able to solve the problem to optimality with standard branch-and-cut solvers on an Intel i7-7600U CPU with 2.80 GHz, two cores and 16 GB memory. We do not report a detailed analysis of the runtimes as they depend very much on the specific parameters of the problem. However, with current solver technology, problems of this size can be solved to optimality or near optimality.