

The Favored but Flawed Simultaneous Multiple-Round Auction*

Nicholas C. Bedard, Jacob K. Goeree, Philippos Louis, and Jingjing Zhang

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Abstract

We compare the first-price sealed-bid (FPSB) auction and the simultaneous multiple-round auction (SMRA) in an environment based on the upcoming sale of 900 MHz spectrum in Australia. Three bidders compete for five indivisible items. Bidders need to obtain at least two items to achieve profitable scale, i.e. items are complements, and are permitted to obtain at most three items. We find that the FPSB is superior to the SMRA across a range of bidding environments: in terms of efficiency, revenue, protecting bidders from losses due to the exposure problem, and price discovery.

Keywords: *Spectrum auctions, laboratory experiments, exposure problem, market design*

JEL codes: *C78, C92, D47*

*Bedard: Wilfrid Laurier University. Goeree: AGORA Center for Market Design and School of Economics, University of New South Wales, 2052 Sydney, Australia. Louis: Department of Economics, University of Cyprus, CY-1678 Nicosia Cyprus. Zhang: Economics Discipline Group, UTS Business School, University of Technology, Sydney, 2007 Ultimo, Sydney Australia. We would like to thank the Australian Research Council (ARC-DP150104491) for financial support.

1. Introduction

Since pioneered by the US Federal Communication Commission in 1994, the simultaneous multiple-round auction (SMRA) has come to dominate the allocation of radio spectrum, earning hundreds of billions of dollars for treasuries around the world (Goeree & Lien, 2014). Its procedure is simple: all items are put up for sale simultaneously with a separate price associated with each item. Bidders can bid on any subset of items they wish and the auction ends when only when no new bids are made on *any* items. At the end of the auction, bidders win the items they bid highest on and pay the price they bid.

The success of the SMRA has been attributed to, in part, its ability to provide price information to bidders thereby leading to efficient outcomes and competitive revenues for the seller (Cramton, 2006; Ausubel & Cramton, 2004; Milgrom, 2004). The intuition is that bidders will reduce their demands gradually over the rounds of the auction as prices rise until supply equals demand. Thus, the SMRA is akin to Adam Smith’s “invisible hand” or the Walrasian *tatonnement* process that identifies efficient allocations and generates prices to support them in the market. This intuition bears out in theory: when the items for sale are substitutes and bidders bid on subsets of items that provide the highest possible profit (i.e. straight-forward or myopic bidding), prices will be competitive and the allocation will be efficient if the bid increment between rounds is sufficiently small (Milgrom, 2004; Gul & Stacchetti, 1999). Moreover, analysis of the SMRA for allocating spectrum licenses since 1994 suggests that, in practice as well, it has been exceedingly successful, with allocations generally thought to be efficient and revenues high (Cramton, 1997).

Like the classical notions of market forces, the theoretical success of the SMRA to guide the market towards efficient outcomes depends on assumptions that ensure demand responds appropriately to prices. In particular, all items must be substitutes for all bidders. In contrast, when some items are complements for some bidders, it is generally possible to find bidder preferences such that the SMRA fails to generate efficient allocations and competitive prices (Milgrom, 2004). Note that the latter are not guaranteed to exist in the presence of complementarities. In this case, recent research suggests using the *core* as a proper benchmark for “reasonably”

competitive outcomes.¹ More fundamentally, the ascending nature of the auction may preclude the former: as prices rise, a bidder for whom goods are complements becomes interested in only *large* packages, resulting in *increasing* demand.² This raises the broader question, which we begin to answer in this paper, of how to design a practical vehicle for implementing efficient and competitive outcomes when complementarities cannot be ruled out and one cannot rely on prices to guide participants to efficient and stable outcomes.

We conduct an experimental comparison of the SMRA and a first-price sealed-bid (FPSB) auction in which bidders are able to bid on any package of items they wish (subject to a bidding cap). Five indivisible items are available. Bidders can acquire at most three items and the items are complements: bidders need at least two items to earn significant value in the auction. We find that the FPSB is far superior to the SMRA across a range of bidding environments, in terms of efficiency, revenue, price discovery and protecting bidders from losses.³

While the theoretical results indicate that the SMRA *may* fail to generate satisfactory outcomes, they do not suggest how frequent or how costly we should expect these failures to be in the real world. It is posited that, in practice, the same forces in the SMRA that generate competitive prices for substitutable goods will at least mitigate any problems caused by complementarities as well as provide the seller with sufficiently competitive revenues (Cramton, 2006). Indeed, there have been notable spectrum auctions involving complements that appear to have performed quite well, such as the US regional narrowband auction in 1994 (Milgrom, 2000). Nevertheless, we show that the flaws in the SMRA can be significant in an important setting. Our motivation for the experimental framework is, primarily, Australia’s upcoming sale of blocks of spectrum in the 900 MHz band; the environment – a small number of licenses on offer, hard limits on the number of licenses each bidder can win and a small number of bidders

¹Core outcomes are reasonably competitive in the sense that there does not exist a coalition of players (including the seller) that would be better off exiting the market and trading amongst themselves; that is, prices reflect the opportunity costs of the allocation. These constraints are weaker than those imposed by competitive equilibrium. See for example (Milgrom, 2004; Day & Cramton, 2012; Day & Raghavan, 2007; Day & Milgrom, 2008; Goeree & Lien, 2016).

²In the sense that the minimum number of items a bidder is willing to accept is increasing in the price.

³The post-auction financial viability of bidders is essential to the efficient use of spectrum. For example, albeit not related to exposure risk, bankruptcy proceedings of a successful bidder in the 1994 American spectrum auctions precluded the use of valuable spectrum for nearly ten years (Cramton, Kwerel, Gregory, & Skrzypacz, 2011).

– is typical, with similar auctions planned or having taken place in Canada, Denmark, Italy, Austria, Switzerland, Belgium, Greece, the Netherlands and the UK (Klemperer, 2002; Earle & Sosa, 2013). Moreover, industry commentary in Australia and elsewhere has suggested that firms, depending on their current holdings, often need to acquire multiple licenses to achieve profitable scale; that is, the licenses are complements for some bidders. Our result is therefore an important evidentiary critique of the SMRA in an important setting.

Our analysis shows that bidders in the SMRA are highly susceptible to the *exposure problem*: for a bidder whose per-item value increases in the number of items she obtains, bidding up to her value for two items, for example, exposes her to the risk of having to pay a large amount for a single item that she places little value on. In equilibrium, the bidder will reduce her demand too early in the auction, from the perspective of the seller, ending up winning too few items and paying too little for them. In the laboratory, we find that bidders are not adequately cautious of their exposure risk to avoid low or even negative earnings.

To unpack and illustrate the forces at work in the laboratory, we first examine the SMRA in a simple analytical framework.

2. A Theoretical Analysis

2.1. Bidding Environment

Our experiment is designed to replicate an upcoming auction by the Australian Communications and Media Authority (ACMA) for five (paired) nationwide blocks of 2×5 MHz of spectrum in the 900 MHz band. In the ACMA auction, three bidders are expected to compete for the five blocks and each will face a cap of three blocks. Bidders are thought to need at least two blocks to implement any profitable business plans.

The ACMA expects there to be strong complementarities between blocks although it is not clear whether the strongest synergies occur when going from one to two blocks or from two to three blocks. To keep this analysis tractable, we consider two types of bidders: type X bidders who needs exactly two items (i.e. she place zero value on a single item and zero marginal value

on each item above two), and type Y bidders who needs exactly three items (i.e. she place zero value on a obtaining one or two items and zero marginal value on each item above three).

In each auction three bidders compete for five items. We study multiple combinations of these bidder types: auctions with composition XXX , XXY , XYY , or YYY . An X type bidder draws a valuation for any pair of items uniformly from $[0, 1]$ and a Y type bidder draws a valuation for any three items uniformly from $[0, \alpha]$ for $\alpha > 1$. Bidders get zero value if they obtain less than what they need and zero additional value if they obtain more than what they need. We consider only equilibria wherein the same types bid the same way if such an equilibrium exists.

2.2. Bayesian Nash Equilibria

We summarize the structure of the equilibria for the various auctions here while relegating the technical details to the Appendix. For the XXY environment, the first price auction does not admit an analytical solution (bid functions have no Taylor series expansion) and for the XYY environment, the SMRA does not admit an analytical solution. In these cases, numerical approximations are used in what follows. We first discuss a common theoretical benchmark for auction performance.

2.2.1. The Vickrey-Clark-Groves Mechanism

The theoretical benchmark against which auction formats are often compared is the Vickrey-Clark-Groves (VCG) mechanism (Vickrey, 1961). The VCG mechanism always allocates the items efficiently but has been criticized for, among other shortcomings, generating revenues (or prices) that are far too low when items are complementary for at least one bidder. As a result, the VCG mechanism has never been seriously considered for practical applications (Rothkopf2007, AusubelMilgrom2006, see for example). The format of the VCG mechanism is as follows. Bidders are asked to report their values to the seller and, based on these reports, the seller chooses the allocation that maximizes total surplus (i.e. the efficient allocation). For $S \subseteq \{0, 1, 2, 3\}$, let $v(S)$ indicate the maximum surplus that the coalition of players S can

generate (where the seller is player 0). Then VCG profits for bidders $i = 1, 2, 3$ are

$$\pi_i^{\text{VCG}} = v(\bar{S}) - v(\bar{S} \setminus \{i\}) \quad (1)$$

where $\bar{S} = \{0, 1, 2, 3\}$ is the grand coalition and $\bar{S} \setminus \{i\}$ is the grand coalition without bidder i .

The seller's revenue in the VCG auction is

$$R^{\text{VCG}} = V_{\text{opt}} - \sum_{i=1}^3 \pi_i^{\text{VCG}}.$$

Given these payments, it is a dominant strategy for bidders to report their true values in the mechanism.

2.2.2. The First Price Auction

In the first price auction bidders submit one bid for every possible package (i.e. subset) of items, which they bid if and only if they win the package. Since the X type bidders only value a pair of items, we need only consider their bids for one item. Since the Y type bidders only value a package of three, we need only consider her bids for three items. An equilibrium in the first price auction in a particular environment will consist of a bidding function for each of the types present in the environment. The equilibrium calculations are described in Appendix A.1. The right-hand column of Figure 1 displays the equilibrium bid functions for each environment for the first price auction.

2.2.3. The Simultaneous Multiple-round Auction

The simultaneous multi-round auction (SMRA) is modelled using five price clocks (one for each item), each of which ticks upward from zero whenever two or more bidders demand (i.e. bid on) the associated item. Bidders can only decrease the number of items they bid on after the auction starts. Given bidders preferences, each bidder will either bid on her entire demand (i.e. two items for type X , three items for type Y) or on no items; in case of the latter we say the bidder is inactive or has dropped out. If only one bidder demands a particular item, its price clock is paused and this bidder is declared the provisional winner. If other bidders later demand this

item, the price clock restarts and the item becomes provisionally unassigned. When demand on all items is at most one, the auction ends, items are assigned to their provisional winners and the winners pay the prices on the clocks for the items they won.

Since the items within a package are substitutes for the bidders and they can freely switch demand between items throughout the auction, the price clocks will always display the same price. A bid function specifies the price level at which the bidder drops out of the auction; it will depend on the number and types of bidders still bidding in the auction. Beliefs are updated via Bayes rule and according to the equilibrium bid functions when a bidder observes a rival drop out of an auction. An equilibrium in the SMRA in a particular environment will consist of a bidding function for each of the types present in the environment. The equilibrium calculations are described in Appendix A.2. The left-hand column of Figure 1 displays the equilibrium bid functions for each environment for the SMRA. In environments XXX and XXY , as soon as any bidder drops out, the auction ends. In the XYY environment, the auction ends after a type Y bidder drops out but continues after the X type drops out. The third panel in the left-hand column displays the bidding function for the Y type bidder in the SMRA for the case where the X type bidder drops out at price \hat{p} . The dashed curve continues the Y type's bidding function conditional on the X type staying in the auction. In the YYY environment, the auction ends only after two Y type bidders drop out. In equilibrium, one bidder is randomly chosen to abstain from the auction while the remaining bidders compete according to the bid function displayed the forth panel of the left-hand side of Figure 1.

2.3. Comparison of Auctions

Figure 2 displays expected efficiency values as well as expected revenue and payoffs for the bidders for the three mechanisms in all four environments and for $\alpha = 1, \frac{3}{2}$ and 2.

2.3.1. Efficiency

Efficiency is calculated as

$$\text{efficiency}_i = \frac{V_i - V_{\text{random}}}{V_{\text{opt}} - V_{\text{random}}} \times 100\%$$

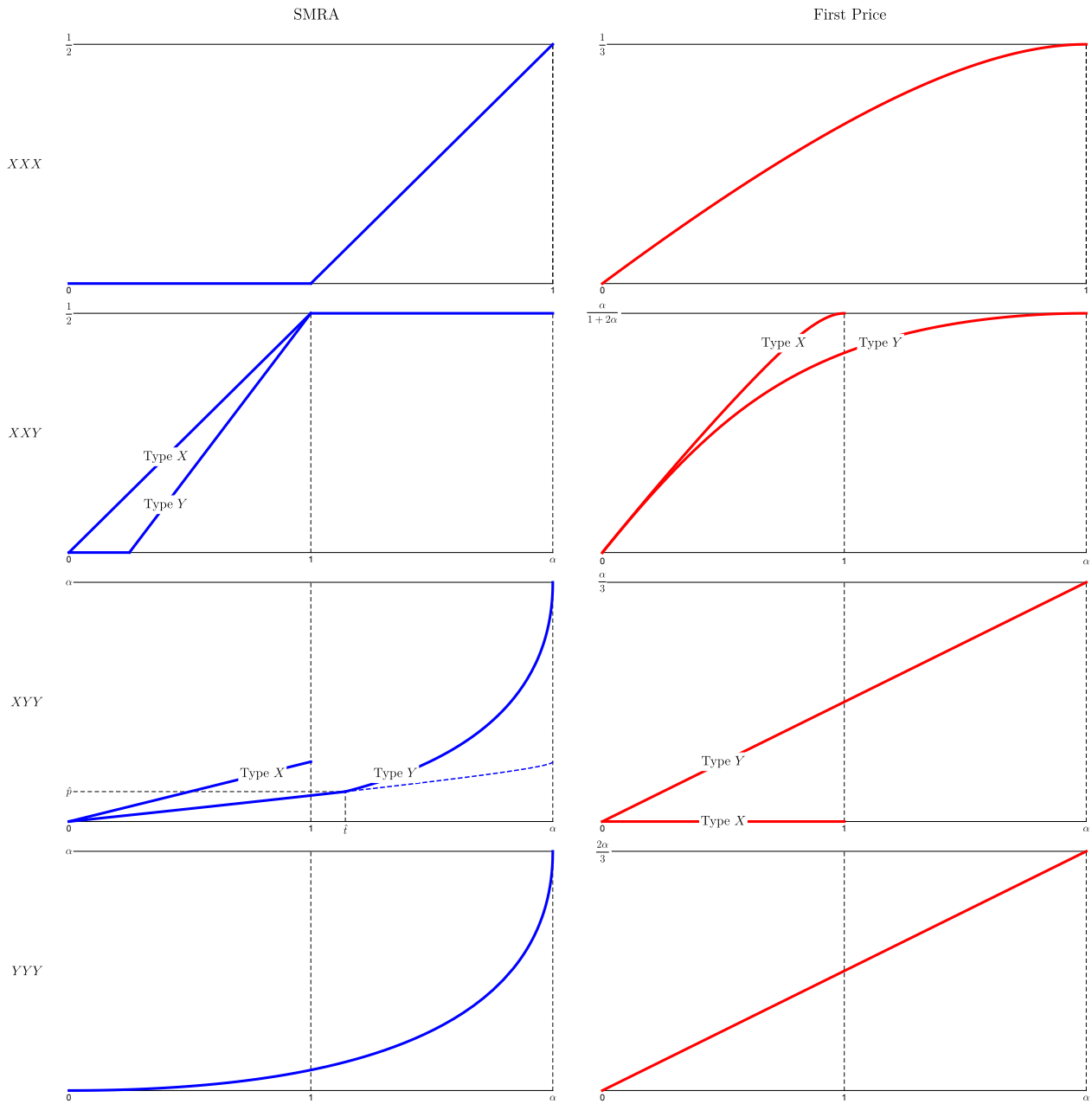


Figure 1: The left column displays the bidding functions of the first price auction by environment. The right column displays equilibrium bidding functions for the SMRA by environment. In the XYY environment, the bidding function for the Y type bidder in the SMRA is depicted when the X type bidder drops out at price \hat{p} . The dashed curve continues the Y type's bidding function conditional on the X type staying in the auction.

where V_i denote the total surplus generated by mechanism $i \in \{\text{SMRA, first price}\}$, V_{opt} the total maximum surplus (generated by the VCG mechanism), and V_{random} the value of randomly assigning all the items to the bidders. This definition has the advantage that it is invariant when bidders' values are multiplied by a common number (i.e. when they are measured in cents rather than dollars) or when a common number is added to all of them. Subtracting surplus generated by randomly assigning all items helps to isolate the added value of mechanisms being studied; it reflects the fact that the relevant alternative to the auction is not the withdrawal of the items from the market but random assignment of all items.⁴

The first price auction is perfectly efficient in all but the XXY environment, where it is at least 98.6% for $\alpha \leq 2$. Meanwhile, the efficiency of the SMRA varies widely between environments, with a low of 15.6% in the XXX environment to a high at least 93.8% in the XXY environment. Both auctions approach perfect efficiency in the XXY environment as α tends to infinity; for $\alpha < 2$, the first price auction is more efficient than the SMRA.

2.3.2. Seller Revenue and Bidder Profit

The seller's *revenue* is the sum of the winning bidders' payments while bidder *profit* is the difference between the value of what bidders won and the payments they made.

As with efficiency, payments in the first price auction closely track those of the VCG; in all but the XXY environment, expected seller revenue and bidders payoffs in the first price auction are equal to those in the VCG auction. In the XXY environment, the seller's expected revenue is higher in the first price auction while bidders' profits are lower; the bidders' thus absorb the loss of efficiency in this environment. Since the VCG mechanism is minimally competitive (i.e. generates the lowest competitive equilibrium revenue for the seller and highest competitive equilibrium payoffs for the bidders), the first price auction can be said to be reasonably competitive in all our environments. The seller's revenue in the SMRA fluctuates around her first price/ VCG revenue between environments, being relatively low in the XXX and YYY environments and high in the XXY and XYY environments; the opposite pattern holds for bidders' profits. Thus, who bears the cost of the inefficiency in the SMRA depends

⁴In fact, radio spectrum was predominantly allocated via lottery prior to the introduction of auctions in 1994 by the FCC (Roth, 2002).

on the environment; it is the seller in the XXX and YYY environments and the buyers in the XXY and XYY environments.

2.3.3. Price Discovery

The use of a multiple-round auction is often justified by appealing to its ability to discover or reveal prices; that is, the process of competitive bidding is expected to determine a set of prices for items and packages of items that are “correct”, in the sense that they are close to what would prevail in a perfectly competitive environment with no uncertainty.⁵ The intuition is that bidders will reduce their demands gradually over the rounds of the auction as prices rise until supply equals demand.

The efficiency and revenue problems with the SMRA are due to the type Y bidder’s *exposure problem*: bidding up to her value for two items exposes her to the risk of having to pay for a single item she places no value on. As a result, the type Y bidder reduces her demand too early, ending up winning two items too infrequently and paying too little for them.

3. Experimental Design

3.1. Bidding Environment

Our experiment is designed to replicate an upcoming auction by the Australian Communications and Media Authority (ACMA) for five (paired) nationwide blocks of 2×5 MHz of spectrum in the 900 MHz band. The lowest of the five blocks is less valuable to bidders due to guard-band issues. We labelled the items A through E and bidders were told that (any combination containing) item A was less valuable than (same-sized combinations containing) other blocks. As above, in the experiment three bidders compete for the five items and we consider two types of bidders: type X bidders whose per-item values peak at two items, and type Y bidders whose per-item values peak at three items. We study multiple combinations of these complementari-

⁵Competitive equilibrium prices are sets of prices for the items at which bidders want to purchase their efficient (i.e. value maximizing) allocation of items.

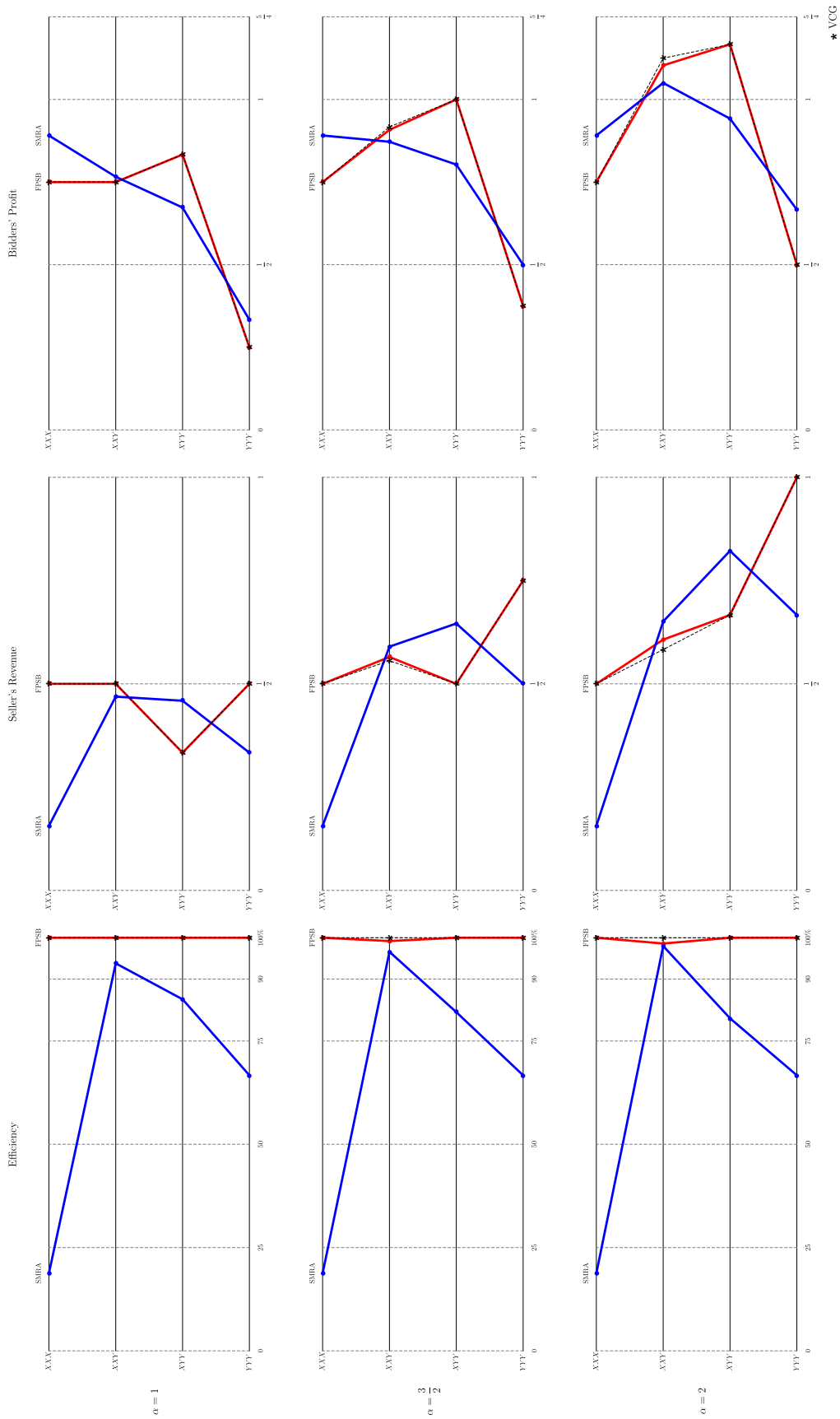


Figure 2: Summary statistics.

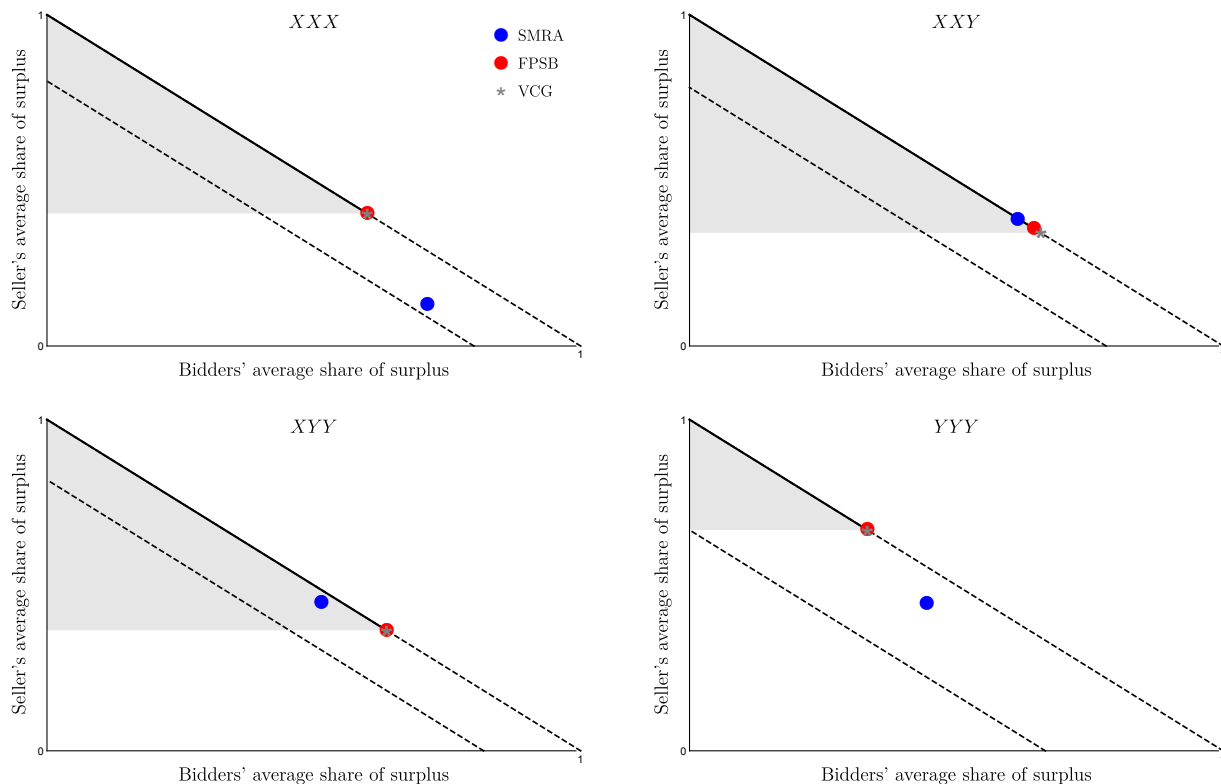


Figure 3: Shares of surplus with $\alpha = 2$. Each panel shows the seller's revenue (y -axis) and average buyer payoff (x -axis) normalized by the maximum surplus. The upper dashed line corresponds to efficient outcomes with the solid segment indicating core outcomes. The lower dashed lines correspond to the random allocation of the items.

ties: two groups each with composition XXX , XXY , XYY , or YYY , for a total of eight groups. Each bidder faces a cap of three items. In the experiment, we eschew any association with radio spectrum, simply telling participants they are to bid on five items that have some value to them. Each group participated in a series of 15 periods.

To generate bidder values, an integer R was drawn uniformly from between 25 and 35 (inclusive). Table 1 describes bidder values depending on draw, type, and whether item A is included. The draws differed across groups and periods, but the same draws were used across treatments (described below). As in the ACMA auction, complementarities between items are realized only if items are consecutive. For example, if a bidder of type X wins items B , C and E , she earns $10 + 3R$ for B and C plus 10 for E rather than $10 + 4R$ for all three. Notice that the increase in value from winning a second item is higher for type X than for type Y . The

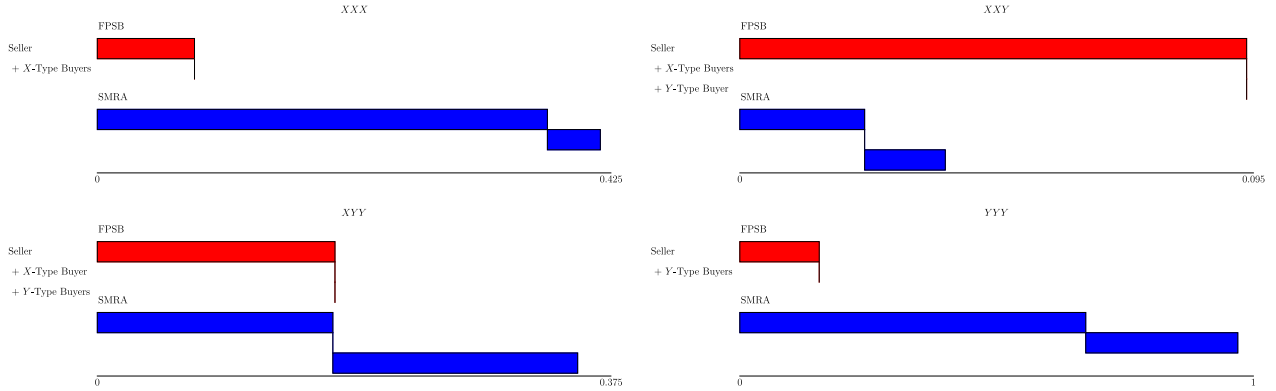


Figure 4: The figure shows the mean distance to the set of core payoffs for the first-price auction and the SMRA for each environment with $\alpha = 2$. The bar graphs are staggered over types to show the distance of each type to their core payoff.

increase in value from winning a third item is higher for type Y than for type X .

In the initial experiments, we describe to bidders how values are generated and the distribution of draws. On the bidding screen, each bidder is shown a table with values as well as the types (but not the values) of the two other bidders in the group. In a series of follow-up experiments for the single stage mechanisms, bidders are only told their own values each periods

# of Consecutive Items	Type X		Type Y	
	With A	Without A	With A	Without A
1	5	10	5	10
2	$10+1.5R$	$10+3R$	$10+0.5R$	$10+R$
3	$10+3.5R$	$10+4R$	$10+3R$	$10+5R$

Table 1: Value specifications for bidders. R is an integer drawn uniformly between 25 and 35 inclusive.

3.2. Mechanisms

We tested four mechanisms:

1. In the first-price sealed-bid (FPSB), bidders place six bids: one for A , one for a single item other than A , one for pair of items including A , one for a pair of items not including

- A , one for a package of three items including A , and one for a package of three items not including A . At most one of the six bids placed by a bidder can become winning. A simple optimization algorithm finds the combination of bids that maximize revenue and the winning bidders pay their bids.
2. In the two-stage FPSB (FPSB-2), bidders place three bids in the first stage: one for a single item, one for pair of items, and one for a package of three items. At most one of the three bids placed by a bidder can become winning. The winners pay their first-stage bids and proceed to the second-stage where they can bid for “not being assigned item A .” In this stage, the lowest bidder is assigned A (by itself or as part of a package, depending on how many items the bidder won in the first stage) and does not pay the second-stage bid. The other bidder(s) pay(s) their second-stage bid(s).
 3. In the simultaneous multi-round auction (SMRA), bidders compete directly for items A through E . A price clock is associated to each of the five items. The price in the first round is five for each item. In each round, bidders indicate whether they demand an item at the price displayed on its clock. For each round and each item, one of the bidders who demands the item is randomly designated the item’s provisional winner. If more than one bidder demands an item, its price increases by 15. If only one bidder demands a particular item, its price clock is paused. If other bidders later demand this item, the price clock restarts and the item is randomly provisionally assigned to one of the new bidders. When demand on all items is one (or less), the auction ends and items are assigned to their provisional winners who pay the price displayed on their clock.
- An activity rule ensures that the auction progresses apace. The sum of items provisionally won by a bidder plus the items she is demanding is called her *activity*. Her activity *limit* in any round is her activity at the end of the previous round, or three if it is the first round. A bidder’s activity cannot exceed her activity limit. Thus, for example, a bidder who fails to bid on any items in round one will be unable to bid in subsequent rounds.
4. In the two-stage SMRA (SMRA-2) bidders first compete for a generic item. A single price clock is associated to the item. In each round, bidders indicate whether they demand zero,

one, two, or three units of the item at the current round price. If the total demand in the round plus total units provisionally assigned for items at the current round price is fewer than five, all bidders are provisionally assigned the quantity they demanded. Otherwise, provisional winners are established in the following way. First, any current provisional winners are reassigned their provisional winnings if the round price has not increased since they were assigned. Second, the bidders demanding items at the current round price are declared provisional winners of the number of goods they demanded in random order until all five units are assigned. The last bidder provisionally assigned items in this process may be assigned fewer items than she demanded. If, at the end of the round, all provisional winners were assigned their items at the current round price, the clock price increases by ten for the next round. Otherwise the round price stays the same. The auction ends after any round with zero new demand, i.e. demand not including provisional winners.

An activity rule ensures that the auction progresses apace. A bidder's *activity* is the maximum of the number of items she is provisionally winning and the quantity she demands. Her activity *limit* in any round is her activity at the end of the previous round, or three if it is the first round. A bidder's activity cannot exceed her activity limit. Thus, for example, if a bidder bids for three items in round one, is provisionally assigned two items, and refrains from bidding in round two, her activity limit in round three is two. Furthermore, bidders cannot demand fewer items than they are provisionally assigned.

The winners pay their first-stage bids and proceed to the second-stage where they can bid for "not being assigned item *A*." The lowest bidder in the second stage is assigned *A* and does not pay her second-stage bid. The other bidder(s) pay(s) their second-stage bid(s).

Table 2 summarizes the four treatments.

3.3. Experimental Procedures

A total of 96 subjects participated in the experiment. Subjects were recruited from University of Technology, Sydney using ORSEE ((Greiner, 2015)). The experiment was programmed and

Treatment	Auction	Bids in 1st Stage	Bids in 2nd Stage	Bidder Information
FPSB	First price sealed bid	6: for 1, 2, and 3 items with or without item A	None	(i) Value distributions and group types; (ii) Own value only
FPSB-2	First price sealed bid	3: for 1, 2, and 3 items	1: to not be assigned A	Value distributions and group types
SMRA	Simultaneous multiple-round ascending	5: for each item	None	(i) Value distributions and group types; (ii) Own value only
SMRA-2	Simultaneous multiple-round ascending	3: for 1, 2, and 3	1: to not be assigned A	Value distributions and group types
All treatments	3 bidders per group; 8 groups; 4 environments			

Table 2: Experimental Design

conducted with the software *z-Tree* ((Fischbacher, 2007)) and MATLAB. Subjects received instructions, answered a quiz and competed in a practice period, before participating in fifteen paid auctions. The experiments lasted from a little over an hour for the FPSB to 2.5 hours for the SMRA-2. Participants were paid the earnings that accumulated over the 15 periods of the experiment if these were positive plus a 10 AUD show-up fee. If their cumulative earnings were negative at the end of the experiment, they were only paid the 10 AUD show-up fee. The conversion rate used in the experiment was 1 Australian dollar (AUD) for every 4 experimental points. The average earnings were 39.95 AUD including a 10 AUD show-up fee.

4. Experimental Results

Figure 5 displays efficiency, seller revenue and bidder profits for all mechanisms by each environment and pooled over all environments. The first row displays results for the one-stage mechanisms when bidders are told the types in their groups and how values are drawn. The second row displays results for the one-stage mechanisms when bidders are told only their values. The third row displays results for the two-stage mechanisms when bidders are told the types in their groups and how values are drawn. Figure 6 displays the distributions of efficiency, seller revenue and bidder profits for both one-stage mechanisms over all environments (see Figure 10

in the appendix for the distributions broken down by group). The first row displays results for the treatments where bidders are told the types in their groups and how values are drawn. The second row displays results for the treatments where bidders are told only their values.

4.1. Efficiency

Result 1 *The FPSB delivers higher efficiency than all other mechanisms on average, followed by the FPSB-2, then by the SMRA and SMRA-2, which are statistically indistinguishable on average. The distribution of efficiency for the FPSB first-order stochastically dominates that of the SMRA.*

Explanation. Table 3 displays the p-values for the Wilcoxon Signed-Rank test where H_0 : mean efficiency $_i$ = mean efficiency $_j$ for $i, j \in \{\text{FPSB}, \text{FPSB-2}, \text{SMRA}, \text{SMRA-2}\}$. Given these, and the results in Figure 5, the mechanisms are ranked $\text{FPSB} \succ^{***} \text{FPSB-2} \succ^{**} \text{SMRA-2} \sim \text{SMRA}$.⁶ The left hand panel of Figure 6 displays the distributions for efficiency for the SMRA are the FPSB. ■

	FPSB	FPSB-2	SMRA	SMRA-2	VCG
FPSB	1	-	-	-	-
FPSB-2	0.0003	1	-	-	-
SMRA	0	0.0048	1	-	-
SMRA-2	0	0.0719	0.3287	1	-
VCG	0	0	0	0	1

Table 3: p-values for the Wilcoxon Signed-Rank test where H_0 : mean efficiency $_i$ = mean efficiency $_j$ for $i, j \in \{\text{FPSB}, \text{FPSB-2}, \text{SMRA}, \text{SMRA-2}\}$.

The top panel of Figure 7 illuminates the shortcomings of the SMRA: these formats often lead to allocations where one or two bidders get a single unit. The bottom panel of Figure 7 displays the degree to which items are sold in non-consecutive packages or unsold in the standard SMRA; evidently, bidders had difficulty coordinating their bids effectively to form packages of consecutive items in the single-stage SMRA.

⁶Here \succ^{***} , \succ^{**} , \succ^* indicates significantly greater at the 1%, 5%, and 10% level respectively, while \sim indicates no statistically significant difference.

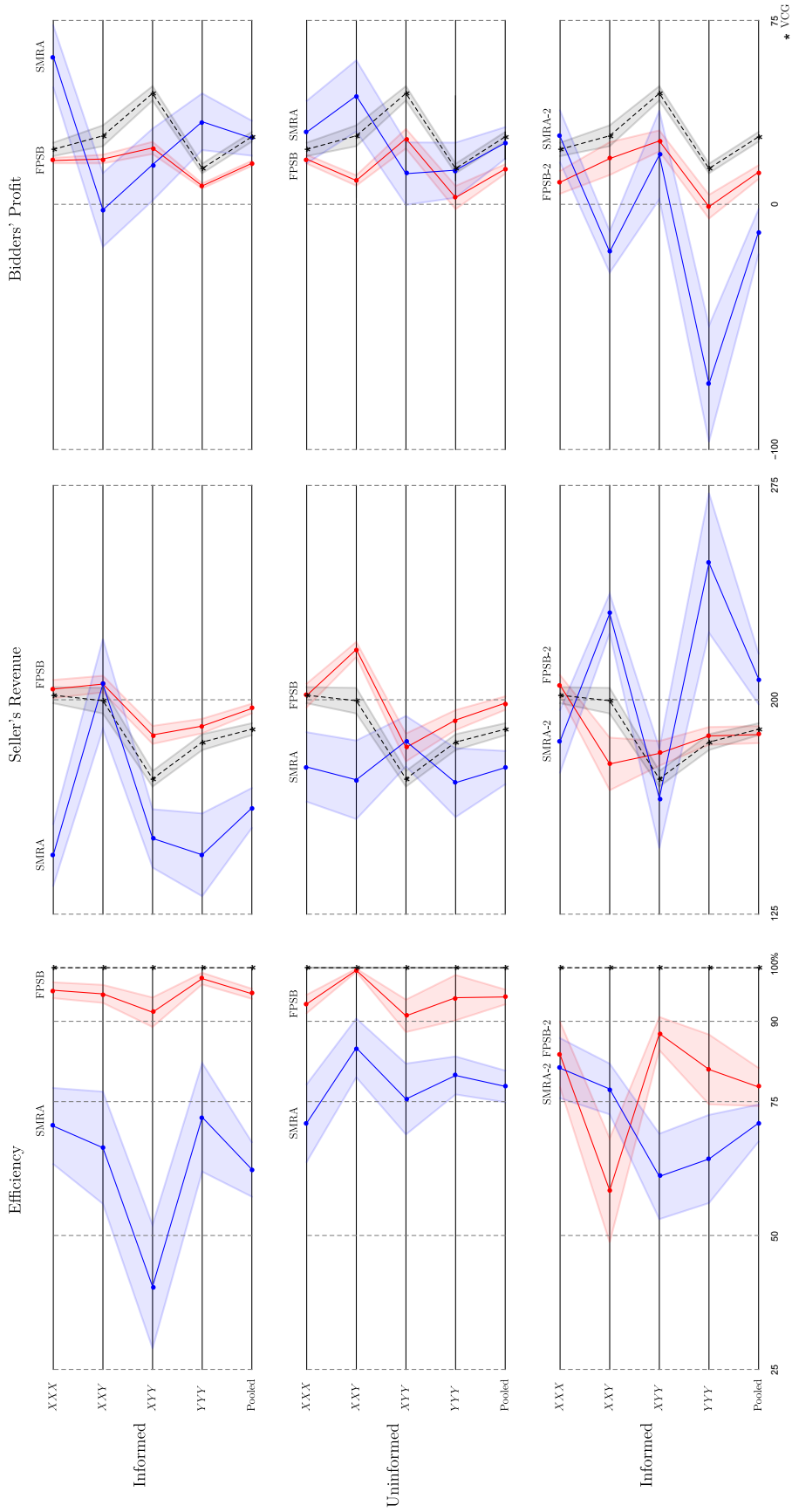


Figure 5: Summary statistics. Observations are pooled over all environments for periods 6 to 15. The first row displays results for the one-stage mechanisms when bidders are told the types in their groups and how values are drawn. The second row displays results for the one-stage mechanisms when bidders are told only their values. The third row displays results for the two-stage mechanisms when bidders are told the types in their groups and how values are drawn.

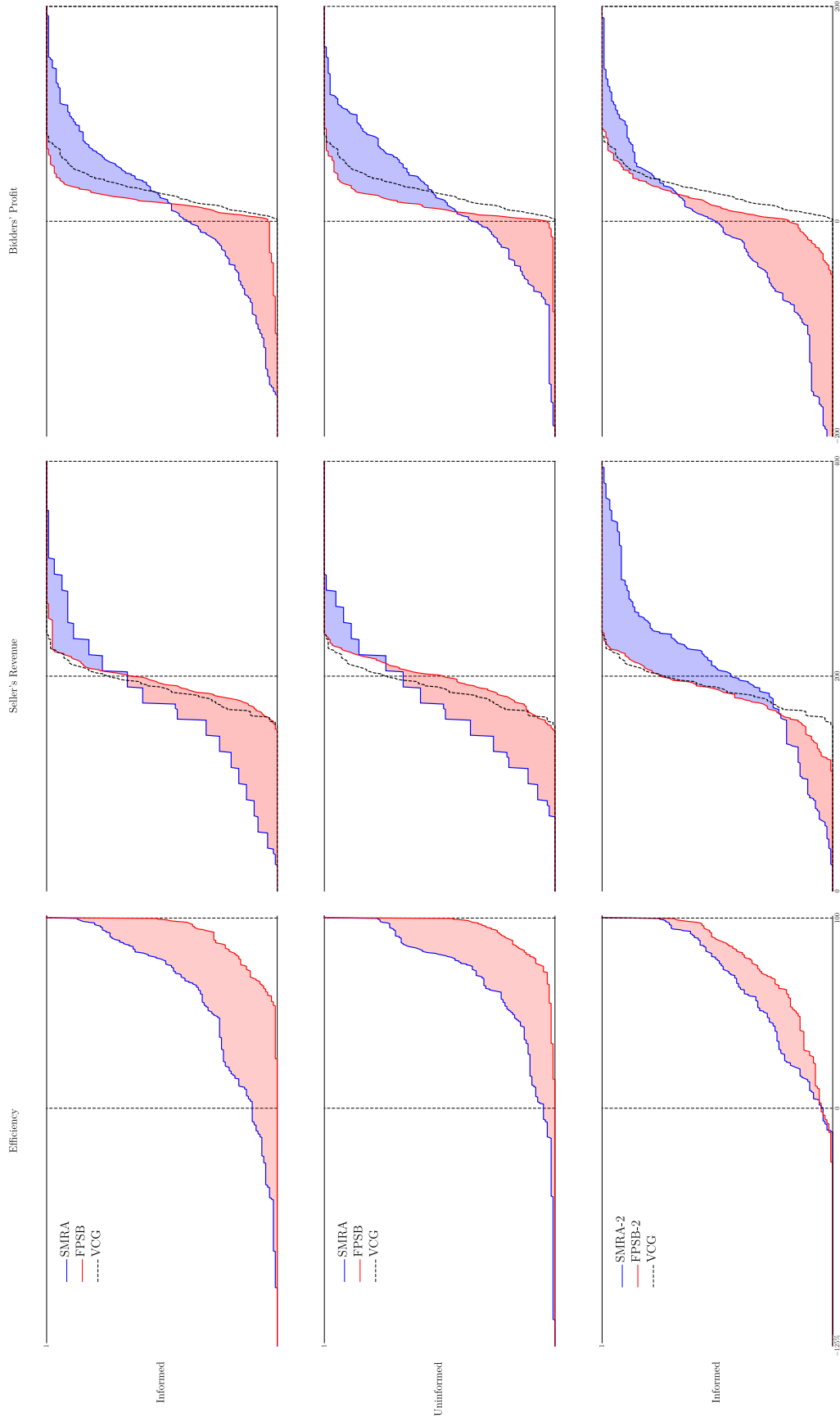


Figure 6: Cumulative distributions of key variables. Observations are pooled over all environments for periods 6 to 15. The first row displays results for the treatments where bidders are told the types in their groups and how values are drawn. The second row displays results for the treatments where bidders are told only their values.

4.2. Seller Revenue and Bidder Profits

Result 2 *The SMRA-2 mechanism delivers higher revenue than all other mechanisms on average, followed by the FPSB, then the FPSB-2, then the SMRA.*

Explanation. Table 4 displays the p-values for the Wilcoxon Signed-Rank test where H_0 : mean revenue $_i$ = mean revenue $_j$ for $i, j \in \{\text{FPSB}, \text{FPSB-2}, \text{SMRA}, \text{SMRA-2}\}$. Given these, and the results in Figure 5, the mechanisms are ranked SMRA-2 \succ^{***} FPSB \succ^{**} FPSB-2 \succ^{***} SMRA. ■

	FPSB	FPSB-2	SMRA	SMRA-2	VCG
FPSB	1	-	-	-	-
FPSB-2	0.0313	1	-	-	-
SMRA	0	0.0002	1	-	-
SMRA-2	0.0077	0.0008	0	1	-
VCG	0.0105	0.9619	0	0.0003	1

Table 4: p-values for the Wilcoxon Signed-Rank test where H_0 : mean revenue $_i$ = mean revenue $_j$ for $i, j \in \{\text{FPSB}, \text{FPSB-2}, \text{SMRA}, \text{SMRA-2}\}$.

Result 3 *The SMRA mechanism delivers higher bidder profits than all other mechanisms on average, followed by the FPSB, then the FPSB-2, then the SMRA-2. Despite high average earnings, the SMRA often delivers negative bidder profits.*

Explanation. Table 5 displays the p-values for the Wilcoxon Signed-Rank test where H_0 : mean revenue $_i$ = mean revenue $_j$ for $i, j \in \{\text{FPSB}, \text{FPSB-2}, \text{SMRA}, \text{SMRA-2}\}$. Given these, and the results in Figure 5, the mechanisms are ranked SMRA \succ^{**} FPSB \sim FPSB-2 \succ^{**} SMRA. The right panel of Figure 6 displays the distributions of bidder profits for the SMRA and the FPSB; nearly 40% of observations are below zero. ■

Note that FPSB yields higher revenue on average than VCG despite its efficiency being less than VCG's 100%. Moreover, this difference in revenue is statistically significant. This comes at a cost to the bidders who make less than under VCG. Importantly, bidders' profits

	FPSB	FPSB-2	SMRA	SMRA-2	VCG
FPSB	1	-	-	-	-
FPSB-2	0.1354	1	-	-	-
SMRA	0.0493	0.0196	1	-	-
SMRA-2	0.0014	0.0241	0.0011	1	-
VCG	0.0001	0	0.7432	0	1

Table 5: p-values for the Wilcoxon Signed-Rank test where H_0 : mean earnings $_i$ = mean earnings $_j$ for $i, j \in \{\text{FPSB}, \text{FSPB-2}, \text{SMRA}, \text{SMRA-2}\}$.

are always positive under FPSB. The reason is that FPSB fully protects bidders from the exposure problem: they can specify a separate bid for each of the (combination of) items they might win and by submitting bids that are less than values, never risk a loss. In contrast, all other mechanisms carry some level of exposure risk. In the two-stage FPSB, for instance, bidders are forced to compete for the number of items prior to knowing whether these items will include the inferior item A . Average bidder profits were positive in the two-stage FPSB, but as can be seen from Figure 5, they were less than in the FPSB as was seller revenue and overall efficiency – this inferior performance is a direct consequence of the exposure risk introduced by the assignment stage. The results for the SMRA are even worse: lower efficiency, lower revenue, and lower bidder profits. In environment XXY , bidders’ profits are negative on average. Again, the reason for this poor performance is exposure risk: in the SMRA, bidders competing aggressively for a package of two or three items may end up winning only one. In addition, there may be fragmentation, i.e. the items a bidder wins may be non-contiguous (see the lower panel of Figure 7).

The two-stage SMRA avoids fragmentation, i.e. items won are contiguous by design, but it has a double exposure problem: bidders who compete aggressively for a package may end up winning only a subset (as in the SMRA) and when competing for the number of items, bidders do not know whether item A will be included or not (as in the two-stage FPSB). For example, in the first stage, a type Y bidder might compete fiercely to win three items but finally give in (at high prices) and settle for two items. In the second stage, the value of what was won may depreciate further if the type Y bidder places the lowest bid. As a result, bidder losses in the

two-stage SMRA are common and substantial. In terms of protecting bidders' from exposure risk, this format is least desirable. This is an important finding as the ACMA plans to use the two-stage format in other applications (e.g. to sell 1800 MHz spectrum).

4.3. Price Discovery

One justification for the use of auctions is that they are *price discovery* mechanisms. Through a competitive bidding process, prices for items and combinations thereof are determined. Ideally, auction prices are competitive equilibrium prices that clear the market (i.e. prices such that auction losers are happy not to be assigned any items and auction winners are happy with their assignment). Notice that in this environment, the prices for items B through E must be identical. Therefore, competitive prices consist of a set of two prices p_A and $p_{-A} = p_i$ for $i \in \{B, C, D, E\}$. As we will see, however, such prices do not always exist, which complicates the notion of what proper price discovery entails.

To illustrate this, consider the valuations in Table 6 that were used in one of the XXX groups where the numbers in bold indicate the best allocation for a total surplus of 217. For

# items	Bidder 1			Bidder 2			Bidder 3		
	1	2	3	1	2	3	1	2	3
including A	5	47.5	97.5	5	49	101	5	61	129
not including A	10	85	110	10	88	114	10	112	146

Table 6: Example of bidders' valuations in the XXX environment.

the XXX environment, competitive equilibrium prices do always exist. For the example in Table 6 the following prices clear the market: $p_A = 13$ and $p_{-A} = 42.5$. It is readily verified that at these prices it is optimal for bidder 1 to demand nothing, for bidder 2 to demand two items without A , and for bidder 3 to demand three items including A . Competitive equilibrium prices are typically not unique, e.g. $p_A = 17$ and $p_{-A} = 44$ also clear the market. Indeed, any combination of prices (p_A, p_{-A}) satisfying $13 \leq p_A \leq 17$ and $42.5 \leq p_{-A} \leq 44$ are competitive equilibrium prices.

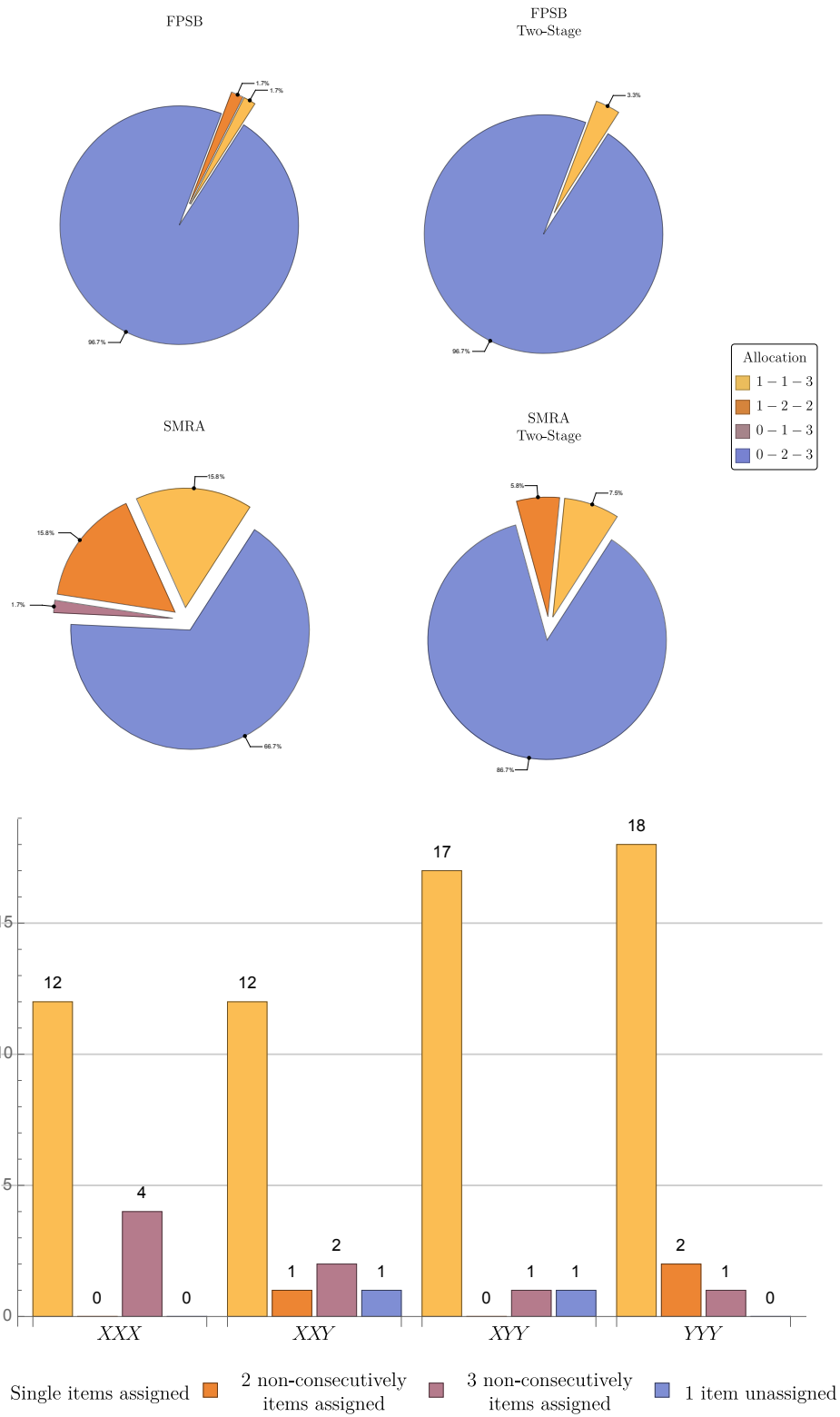


Figure 7: Observed outcomes in the different mechanisms (top panel) and fragmentation in the SMRA (bottom panel).

This example might suggest that the proper definition of price discovery is for the auction to produce prices somewhere in the range of competitive equilibrium prices. But this definition is too narrow since competitive equilibrium prices do not necessarily exist in environments with complementarities (e.g. with one or more Y types). Consider, for instance, the valuations in Table 7, which were used in one of the YYY groups. Now the inequalities defining competitive equilibrium prices have no solution, e.g. $p_A + p_{-A} \leq 23$ conflicts with $3p_{-A} \geq 135$.

# items	Bidder 1			Bidder 2			Bidder 3		
	1	2	3	1	2	3	1	2	3
including A	5	24.5	97	5	22.5	85	5	23	88
not including A	10	39	155	10	35	135	10	36	140

Table 7: Example of bidders' valuations in the YYY environment.

Because competitive equilibrium prices do not always exist in the presence of complementarities, attention has turned to the *core* as a proper benchmark for “reasonably” competitive outcomes.⁷ The core is defined by combinations of seller and buyers' payoffs that satisfy certain stability constraints. The intuition is that auction payoffs are in the core when no coalition of bidders and/or seller can all do better than their auction payoffs. If we index the seller by $i = 0$ and the three bidders by $i = 1, 2, 3$ then the possible coalitions are the non-empty elements of the powerset of $\{0, 1, 2, 3\}$.⁸ A vector of payoffs $\{\pi_0, \pi_1, \pi_2, \pi_3\}$ is in the core if

$$\sum_{i \in S} \pi_i \geq v(S)$$

for all $S \subseteq \{0, 1, 2, 3\}$ where π_i is the auction profit for coalition member $i \in S$, and $v(S)$ is the maximum surplus that coalition S can generate. Competitive equilibrium prices, when they exist, always produce core payoffs. But while competitive equilibrium prices may not exist, the

⁷See for example (Milgrom, 2004; Day & Cramton, 2012; Day & Raghavan, 2007; Day & Milgrom, 2008), each of which study mechanisms to achieve core outcomes in complete information environments. (Goeree & Lien, 2016) note that, when bidders values are private information, if the VCG outcome is not in the core, no core-selecting auction exists.

⁸They are $\{0\}, \{1\}, \{2\}, \{3\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}$, and the grand coalition $S = \{0, 1, 2, 3\}$.

core is always non-empty in auction applications. As such it seems the right benchmark for competitive outcomes in settings with complementarities.

Of course, simply because the core is non-empty does not mean that it is easy for a particular auction format to discover prices that lead to core payoffs. For the environments considered in our experiment, the VCG auction produces core outcomes.⁹ In fact, the VCG outcome corresponds to the point in the core that assigns the lowest revenue to the seller and the highest profits to the bidders. In this format, truthful bidding is a (weakly) dominant strategy and the outcomes are fully efficient. For the example of Table 6, the bidders payoffs are (0, 3, 31) and the seller's revenue is 183. Note that these payoffs correspond to the lowest competitive equilibrium prices: $p_A = 13$ and $p_{-A} = 42.5$.

Figure 8 shows core payoffs for each of the four environments: XXX , XXY , XYX , and YYY . To produce a two-dimensional graph, the sum of bidders' profits is shown on the horizontal axis and the seller's revenue is shown on the vertical axis. All payoffs are normalized by the maximum surplus, $v(\bar{S})$, and the negatively sloped dashed line corresponds to all possible divisions of the maximum surplus among the bidders and the seller. The subset of core constraints that dictate individual rationality (i.e. $\pi_i \geq 0$ for $i = 0, 1, 2, 3$) imply that the core is part of the positive orthant. The other core constraints set a minimum revenue for the seller, here given by R^{VCG} . So in each of the four panels of Figure 8, the core corresponds to the solid segment that runs from the VCG payoff point to (0,1).

The grey triangles in each of the panels reflect alternatives to the VCG outcome that might interest a seller. These alternatives are not all fully efficient but do yield higher seller revenue than the VCG auction and generate positive profits for the bidders. As such they reflect a trade-off between efficiency and revenue that sellers typically face (e.g. in the use of reserve prices). The results for the 1-stage mechanisms are represented by disks while the 2-stage mechanisms are triangles; the markers for the first price auctions are red and for the SMRA are blue.

Note that the FPSB does a remarkable job at price discovery: the red points are always

⁹This is not necessarily the case in the presence of complementarities. In our example of Section 2, VCG prices are often below core prices.

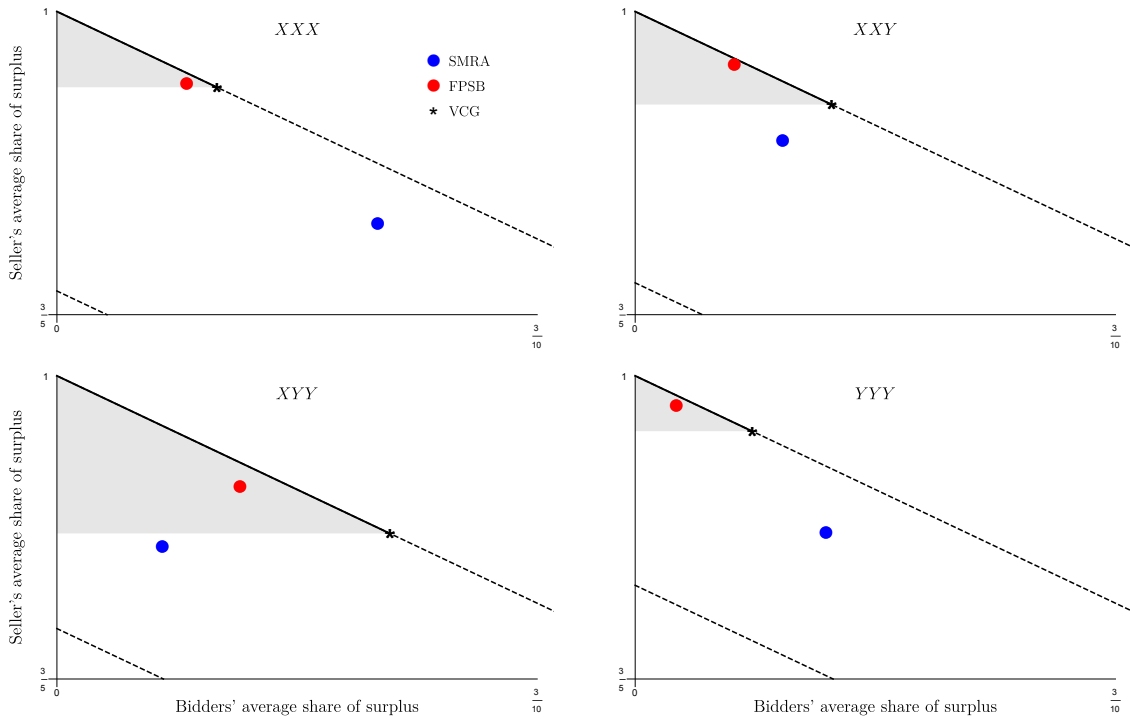


Figure 8: Each panel shows the seller's revenue (y -axis) and average buyer payoff (x -axis) normalized by total surplus and averaged over the last ten periods. The upper dashed line corresponds to efficient outcomes with the solid segment indicating core outcomes. The lower dashed lines correspond to random allocations of the items. These figures use observations from both information treatments for the one-stage mechanisms.

close to fully efficient outcomes while providing more than VCG revenues for the seller. The SMRA formats on the other hand consistently either under perform the VCG from the seller's perspective or generate losses for the bidders (i.e. are outside the grey triangles in the figures).

5. Conclusion

Ascending auctions are typically motivated by their ability to produce correct prices, i.e. competitive prices that clear the market. This motivation, however, rests in the assumption that goods are substitutes. With substitutes, bidders will reduce their demands over the rounds of the auction as prices rise until supply equals demand. This logic breaks down, however, when goods are complements, as is the case in many spectrum auctions. At high prices, all that

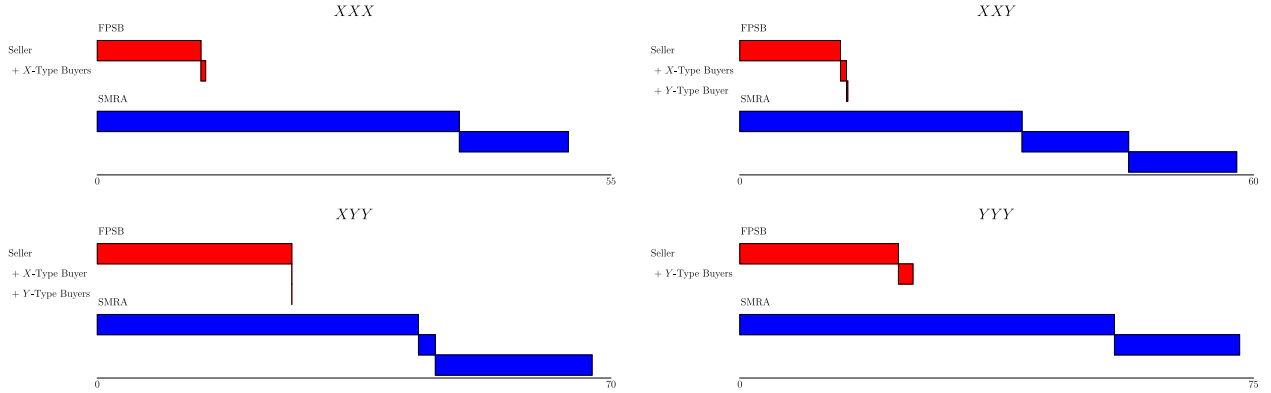


Figure 9: The figure shows the mean distance to the set of core payoffs for the (one-stage) FPSB and SMRA treatments for each environment. Data is pooled over periods 6 to 15 and over both information treatments. The bar graphs are staggered over types to show the distance of each type to their core payoff.

bidders may be interested in are *large* packages. Moreover, ascending auctions typically assign provisional winners each round, which exposes bidders to the risk of winning a subset of their desired package. As is evident from Figure 9, this exposure problem biases price discovery and leads to payoffs that are far from core (i.e. minimally competitive) prices.

The FPSB protects bidders from the exposure problem as it lets them place bids for every possible package they may win (six bids in total). As a result, efficiency is high, revenue is high, bidders make no losses and payoffs are virtually always in the core. In the presence of complementarities, FPSB is a better choice than ascending formats in this important environment.

A. Bayes Nash Equilibrium Calculations

A.1. The First Price Auction

Since the X -type bidders only value a pair of item, we need only consider their bids for two items; denote the equilibrium bid function for X -type bidder $b : [0, 1] \rightarrow \mathbb{R}_+$ and let $\phi(b) = b^{-1}(b)$ be its inverse for $b \in [0, \bar{b}]$ with the upper bound \bar{b} to be determined. Since the the Y -type bidders only value a package of three items, we need only consider her bids for three items; denote

equilibrium bid function for type Y $B : [0, \alpha] \rightarrow \mathbb{R}_+$ for valuation and let $\Phi(b) = B^{-1}(b)$ be its inverse on $b \in [0, \bar{b}]$. As will be confirmed below for each environment, assume for now that the bidding functions are strictly increasing and their inverse functions are therefore well defined.

A.1.1. XXX Environment

Exactly two bidders will be awarded their desired packages in the auction. Therefore, each bidder wins if and only if she bids higher than the lowest of her two rivals. Supposing her rivals play according to their equilibrium strategies, let $\pi_X(b, w)$ denote the expected payoff of a bidder with valuation w when she bids b . Payoffs are

$$\pi_X(b, w) = (w - b)(1 - (1 - \phi(b))^2)$$

Equilibrium requires that $\frac{\partial}{\partial b}\pi_X(b, w) = 0$ when evaluated at the equilibrium strategies. This gives us the differential equation:

$$- (1 - (1 - \phi(b))^2) + 2(1 - \phi(b))\phi'(b) = -(2 - w)w - 2(1 - w)(b(w) - w)/b'(w) = 0$$

together with terminal condition $b(1) = \bar{b}$. This has the solution $b(w) = \frac{w(3-2w)}{3(2-w)}$.

A.1.2. XXY Environment

Exactly two bidders will be awarded their desired packages in the auction. Therefore, each bidder wins if and only if she bids higher than the lowest of her two rivals. Supposing her rivals play according to their equilibrium strategies, let $\pi_i(b, w)$ denote the expected payoff of the type i bidder with valuation w when she bids b . Payoffs are

$$\pi_X(b, w) = (w - b)(1 - (1 - \phi(b))(1 - \Phi(b)/\alpha))$$

$$\pi_Y(b, W) = (W - b)(1 - (1 - \phi(b))^2)$$

Equilibrium requires that $\frac{\partial}{\partial b}\pi_i(b, w) = 0$ when evaluated at the equilibrium strategies. This gives us two differential equations to satisfy:

$$(1 - \phi(b))\left(1 - \frac{\Phi(b)}{\alpha}\right) - (1 - \phi(b))\left(\left(1 - \frac{\Phi(b)}{\alpha}\right)\phi'(b) + (1 - \phi(b))\frac{\Phi'(b)}{\alpha}\right) - 1 = 0 \quad (2)$$

$$(1 - \phi(b))^2 + 2(1 - \phi(b))(\Phi(b) - b)\phi'(b) - 1 = 0 \quad (3)$$

together with the terminal conditions $\phi(\bar{b}) = 1$ and $\Phi(\bar{b}) = \alpha$. We can solve equations (2) and (3) for $\Phi(b)$ as a function of $\phi(b)$ and b only:

$$\Phi(b) = \frac{\phi(b)((\alpha + b)\phi(b) - 2b(1 + \alpha))}{2(1 - \phi(b))(\phi(b) - b)} \quad (4)$$

We need $\Phi(\bar{b}) = \alpha$; then (4) implies $\bar{b} = \frac{\alpha}{2+2\alpha}$. Substituting this back into (2) or (3), we arrive at a single differential equation

$$\phi'(b) = \frac{(\phi(b) - b)(2 - \phi(b))\phi(b)}{(\alpha - b)(\phi(b)^2 + 2b\phi(b)) - 2b^2}. \quad (5)$$

Unfortunately, (5) does not admit a (clean) analytical solution but its numeric solution is simple to generate.

A.1.3. *XYX* Environment

For any set of bids, the seller will allocate two items to the X type bidder and three items to the highest Y type bidder. Therefore, $b(w) \equiv 0$ and a Y type bidder wins only if she out bids the other Y type bidder. Supposing her rivals play according to their equilibrium strategies, let $\pi_Y(b, W)$ denote the expected payoff of the type Y bidder with valuation W when she bids b . Payoffs are

$$\pi_Y(b, W) = (W - b)\Phi(b)$$

Equilibrium requires that $\frac{\partial}{\partial b}\pi_Y(b, w) = 0$ whenever $b > 0$ evaluated at the equilibrium strategies. This gives us the differential equation

$$-\Phi(b) + (W - b)\Phi'(b) = -W + (W - B(W))/B'(W) = 0 \quad (6)$$

together with the terminal conditions $B(\alpha) = \bar{b}$. This has solution $B(W) = \frac{W}{2}$.

A.1.4. *YYY Environment*

The seller will allocated three items to the bidder submitting the highest bid. Therefore, a type Y bidder wins if she out bids both of her rivals. Supposing her rivals play according to their equilibrium strategies, let $\pi_Y(b, W)$ denote the expected payoff of the type Y bidder with valuation W when she bids b . Payoffs are

$$\pi_Y(b, W) = (W - b) \frac{\Phi(b)^2}{\alpha^2}$$

Equilibrium requires that $\frac{\partial}{\partial b} \pi_Y(b, w) = 0$ whenever $b > 0$ when evaluated at the equilibrium strategies. After multiplying by $\frac{\alpha^2}{W}$, this gives us the differential equation

$$-\Phi(b)^2 + 2(W - b)\Phi(b)\Phi'(b) = -W + 2(W - B(W))/B'(W) = 0 \quad (7)$$

together with the terminal conditions $B(\alpha) = \bar{b}$. This has solution $B(W) = \frac{2W}{3}$.

A.2. The Simultaneous Multiple-round Auction

A.2.1. *XXX environment*

Once any bidder stops bidding the auction ends; therefore, bidding functions depend only on the price level and the bidder's draw. A bidder wins if she outbids the lowest bid of her rivals.

Let $b : [0, 1] \rightarrow \mathbb{R}_+$ denote a bidder's equilibrium bidding function and let $\phi(b) = b^{-1}(b)$ be its inverse for $b \in [0, \bar{b}]$ with the upper bound \bar{b} to be determined. Supposing her rivals play according to their equilibrium strategies, let $\pi_X(b, w)$ denote her expected payoff when she bids b and her draw is $w \in [0, 1]$. Equilibrium payoffs are

$$\hat{\pi}_X(b, w) = 2 \int_0^{\phi(b)} \int_y^1 (W - 2b(y)) dz dy - b(1 - \phi(b))^2. \quad (8)$$

The last term arises when the bidder drops out first at $p = b$ and is forced to purchase one good. Equilibrium requires that $\frac{\partial}{\partial b} \hat{\pi}_X(b, w) = 0$ whenever $b > 0$ when evaluated at the equilibrium

strategies. This gives us the differential equation

$$2\phi'(b)(1 - \phi(b))(w - 2b) - (1 - \phi(b))^2 + 2b(1 - \phi(b)) = \frac{(1 - w)}{b'(w)}(2(w - b(w)) - b'(w)(1 - w)) = 0$$

together with the terminal conditions $b(1) = \bar{b}$. This gives

$$b(w) = \begin{cases} 0 & \text{if } 0 \leq w < \frac{1}{2} \\ 2w - 1 & \text{if } \frac{1}{2} \leq w \leq 1 \end{cases} .$$

A.2.2. *XXY* environment

Once any bidder stops bidding the auction ends; therefore, bidding functions depend only on the price level and the bidder's draw.

For a type *X* bidder with draw w , it is a dominant strategy to bid on two items if $p \leq \frac{w}{2}$ and otherwise to stop bidding on any items.¹⁰

Let B denote the *Y* type's equilibrium bidding function and let $\pi_Y(b, w)$ denote her expected payoff when she bids B and her draw is $w \in [0, \alpha]$. Given the *X*-types' strategy

$$\hat{\pi}_Y(b, W) = 2 \int_0^{2b} \int_w^1 \left(W - 3\frac{w}{2} \right) dz dw - b(1 - 2b)^2. \quad (9)$$

The last term arises when the *Y* type drops out at $p = b$ and is forced to purchase one item. Equilibrium requires that $\frac{\partial}{\partial b} \hat{\pi}_Y(b, w) = 0$ when evaluated at $b = B(W)$ whenever $B(W) > 0$ and $\frac{\partial}{\partial b} \pi_Y(b, w) \leq 0$ when evaluated at $b = B(W)$ whenever $B(W) = 0$. Since

$$\frac{\partial}{\partial b} \pi_Y(B(W), W) = \left(4(W - 2B(W)) - (1 - 2B(W)) \right) (1 - 2B(W)) = (4W - 1 - 6B(W))(1 - 2B(W)) \geq 0$$

if and only if $W \geq \frac{1}{2}$, we have

$$B(W) = \begin{cases} 0 & \text{if } 0 \leq W < \frac{1}{4} \\ \frac{1}{3}(2W - \frac{1}{2}) & \text{if } \frac{1}{4} \leq W \leq \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq W \leq \alpha \end{cases} .$$

The second panel of the left hand side of Figure 1 plots this bid function and the type *X* bid

¹⁰The auction for a type *X* bidder is mathematically identical to a second price sealed bid auction; a type *X* bidder's dominant strategy is to bid her valuation.

function.

A.2.3. *XY* environment

For a type X bidder with draw w , it is a dominant strategy to bid on two items if $p \leq \frac{w}{2}$ and otherwise to stop bidding on any items.

The auction ends only after a Y type drops out; therefore, a bidding functions for the Y type bidder will one her draw, the price level, and who remains in the auction – i.e. whether or not the X type bidder had dropped out. A Y can win if the type X bidder drops out *then* the rival type Y bidder drops out, or if the rival type Y bidder drops out while the type X type is still actively bidding.

Proceeding by backward induction, let $B^Y(W, p)$ denote the price level in equilibrium at which they type Y bidder drops out when her draw is W and the X type bidder has dropped out at the price level p and define $\Phi^Y(b, p)$ such that $B^Y(\Phi^Y(b, p), p) = b$. Supposing her rivals play according to their equilibrium strategies, let $\pi_Y^Y(b, W)$ denote a Y type bidder's expected payoff when she bids drops out at price level b and her draw is $W \in [0, \alpha]$. Equilibrium payoffs are

$$\pi_Y^Y(b, p, W) = \int_0^{\Phi^Y(b, p)} \left(W - 3B^Y(V, p) \right) \frac{dV}{\alpha} - 2b \left(1 - \frac{\Phi(b, p)}{\alpha} \right) \quad (10)$$

The last term arises when the bidder drops out at $p = b$ and is forced to purchase two items. Equilibrium requires that $\frac{\partial}{\partial b} \pi_Y^Y(b, p, W) = 0$ whenever $b > 0$ when evaluated at the equilibrium strategies. This gives us the differential equation

$$\frac{\partial \Phi^Y(b, p)}{\partial b} (W - 3b) - 2 \left(1 - \frac{\Phi(b, p)}{\alpha} \right) + \frac{2b}{\alpha} \frac{\partial \Phi^Y(b, p)}{\partial b} = \frac{1}{\frac{\partial B^Y(W, p)}{\partial W}} \left(W - B^Y(W, p) \right) - 2 \left(1 - \frac{W}{\alpha} \right)$$

together with the terminal conditions $B(\alpha) = \bar{b}$. This gives

$$B^Y(W, p) = W - 2\sqrt{\alpha - W} \left(\sqrt{\alpha - p} - \sqrt{\alpha - W} \right).$$

Expected equilibrium profits for at Y type bidder with a draw of W in this stage – i.e. supposing

that the X type bidder dropped out at p – are

$$\pi_Y^Y(p, W) = \pi_Y^Y(B^Y(W, p), p, W) = \frac{(W - p)^2}{2(\alpha - p)} - 2p$$

Let $B^{XY}(W)$ denote the price level in equilibrium at which they type Y bidder drops out when her draw is W and neither rival has dropped out and define $\Phi^{XY}(b)$ such that $B^Y(\Phi^{XY}(b)) = b$. Supposing her rivals play according to their equilibrium strategies, let $\pi_Y^{XY}(b, w)$ denote a Y type bidder's expected payoff when she bids drops out at price level b , neither rival has dropped out and her draw is $W \in [0, \alpha]$. Payoffs are

$$\pi_Y^{XY}(b, p, W) = \int_0^{\Phi^{XY}(b)} \int_{2b}^1 (W - 3B^{XY}(V)) dy \frac{dV}{\alpha} - \int_{\Phi^{XY}(b, \frac{y}{2})}^{\alpha} \int_{2b}^1 \pi_Y^Y\left(\Phi^{XY}\left(b, \frac{y}{2}\right), \frac{y}{2}, W\right) dv \frac{dV}{\alpha} \quad (11)$$

Equilibrium requires that $\frac{\partial}{\partial b} \pi_Y^{XY}(b, p, W) = 0$ whenever $b > 0$ when evaluated at the equilibrium strategies. After some manipulation, this gives us the differential equation

$$(W - 3B^{XY}(W))(1 - 2B^{XY}(W)) - 4 \frac{\partial B^{XY}(W)}{\partial W} B^{XY}(W)(\alpha - W) = 0$$

together with the terminal conditions $B(\alpha) = \bar{b}$. This equation has no simple analytical solution. Its numeric solution is display in the fourth panel of the left-hand side of Figure 1 for the case where the X type bidder drops out at price \hat{p} .

A.2.4. YYY environment

The auction ends only after two Y types drop out; therefore, a bidding functions for a Y type bidder will depend both on her draw, and how many bidders remains in the auction.

Proceeding by backward induction, let $B^Y(W)$ denote the price level in equilibrium at which they type Y bidder drops out when her draw is W and only one Y type bidder remains active in the auction. Define $\Phi^Y(b)$ such that $B^Y(\Phi^Y(b)) = b$. This is strategically identical to the stage in the XYX environment after the X type has dropped out. Therefore, as derived above,

$$B^Y(W, p) = W - 2\sqrt{\alpha - W} \left(\sqrt{\alpha - p} - \sqrt{\alpha - W} \right).$$

and expected equilibrium profits for a Y type bidder with a draw of W in this stage – i.e. supposing that first bidder dropped out at price p – are

$$\pi_Y^Y(p, W) = \pi_Y^Y(B^Y(W), p, W) = \frac{(W - p)^2}{2(\alpha - p)} - 2p$$

Let $B^{YY}(W, p)$ denote the price level in equilibrium at which the type Y bidder drops out when her draw is W and neither rival has dropped out and define $\Phi^{YY}(b)$ such that $B^{YY}(\Phi^{YY}(b)) = b$. Supposing her rivals play according to their equilibrium strategies, let $\pi_Y^{XY}(b, W)$ denote a Y type bidder's expected payoff when she bids drops out at price level b , neither rival has dropped out and her draw is $W \in [0, \alpha]$. Payoffs are

$$\pi_Y^Y(b, p, W) = \int_0^{\Phi^{YY}(b, p)} \int_V^W \pi_Y^Y(B^{YY}(V), W) \frac{dZ}{\alpha} \frac{dV}{\alpha} \quad (12)$$

Equilibrium requires that $\frac{\partial}{\partial b} \pi_Y^{XY}(b, p, W) = 0$ whenever $b > 0$ when evaluated at the equilibrium strategies. After some manipulation, this gives us the equation

$$-2(\alpha - W)B(W) = 0.$$

But this is negative whenever $B(W) > 0$. Thus, there is no symmetric equilibrium (in pure strategies) wherein all three Y type bidders bid above zero in the auction. Instead, we assume one bidder randomly drops out at any price $p \geq 0$. The remaining two bidders play the equilibrium strategy $B^Y(W, p)$ defined above.

B. Extra Data Figures

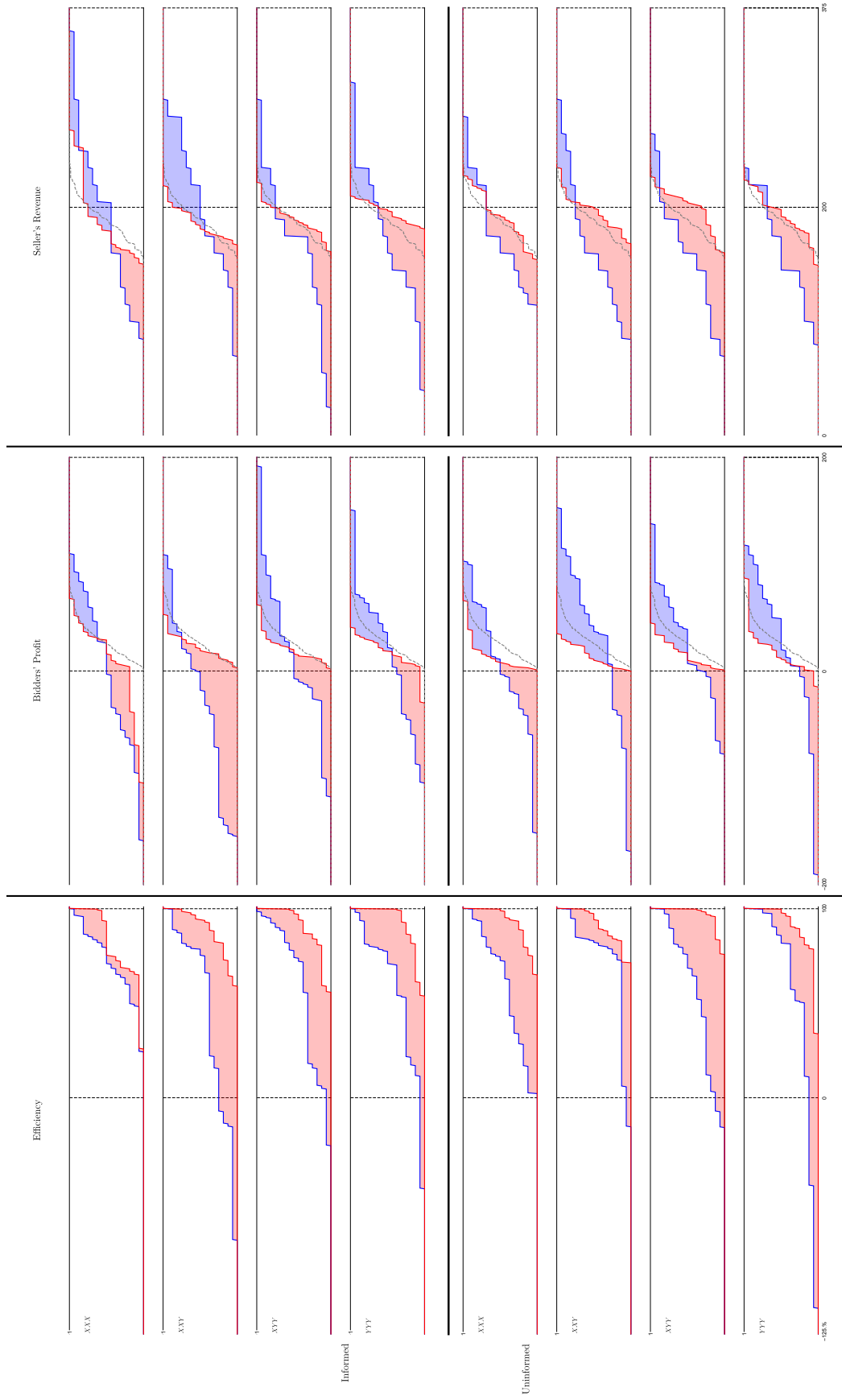


Figure 10: Distributions by group.

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