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Designing Environmental Markets: A Combinatorial Exchange for Trading Catch Shares

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Overfishing is a prime environmental concern. Catch share systems are an effective way to combat overfishing but they introduce economic inefficiencies when the allocation of shares does not align with industry needs. This paper describes a market-based approach to reallocating fishing shares in New South Wales (NSW), Australia. The design of the market needed to address several non-standard requirements, including the possibility of all-or-nothing offers, fair prices, and an endogenously determined subsidy. These features were crucial for the adoption of the reform but also led to computationally challenging allocation and pricing problems. The implemented exchange illustrates how computational optimization and market design can provide new policy tools that can be used to solve complex policy problems considered intractable only a few years ago. The exchange operated from May to July 2017 and effectively reallocated shares from inactive fishers to those who needed them most. It provides a template for the reallocation of catch shares elsewhere and of resource rights in other applications, e.g. water rights, pollution rights, and environmental offsets.

Key words: Overfishing, Fishery Management, Catch Shares, Market Design

Introduction

Advanced fishing technologies and increased demand for seafood have caused overfishing to become a prime environmental concern (Jackson et al. 2001). More than 30 percent of the world's fisheries are overexploited and require strict management to restore and maintain sustainability (Worm et al. 2006). Some reports warn that most of the world's commercial fisheries could collapse within decades (Costello et al. 2008).

To curb overfishing, policymakers worldwide have introduced catch share quotas, also called individual transferrable quotas (Branch 2009). First, a scientifically sound limit (a "cap") is set on the amount of fish that can sustainably be extracted from a fishery. Then a percentage of that limit is guaranteed to individual fishermen or groups of fishers. The use of catch share programs has grown considerably since their first implementations in the late 1970s (Costello et al. 2008). There are now over 150 catch-share programs worldwide, covering a wide variety of marine and freshwater species (Lynham 2014). Recent empirical evidence suggests that these programs successfully reduce overfishing (Melnychuk et al. 2012, Birkenbach et al. 2017).

However, catch share programs can introduce economic inefficiencies when the allocation of shares does not align with fishers' needs. The initial allocation of shares is typically based on historical catch numbers, a practice known as "grandfathering" (Lynham 2014). With time, this initial allocation becomes suboptimal when the industry undergoes structural change, the stock recovers, or quotas are tightened. To restore efficient outcomes, shares need to be reallocated, which raises several complex policy issues as we discuss next (Rosenberg 2017).

The Policy Problem in New South Wales

There are approximately 1,000 fishers catching around 15,000 tons of fish and prawns in New South Wales (NSW). Overfishing is a significant concern, which prompted the NSW government to introduce a catch share system, comprised of a 100 different share classes, more than a decade ago. Between 2002 and 2007, catch shares were distributed among fishers in an "equitable" manner, meaning that all fishers got an equal amount of shares within the share classes they were active. However, these shares were not effectuated until 2017 when the NSW government's *linkage program* required fishers to hold enough shares to justify their catch.

The uniform allocation of shares was likely not optimal when the shares were first allocated and it certainly did not match industry needs in 2017. The NSW fishing industry is characterized by the typical "20/80 rule," i.e. 20% of the fishers do 80% of the fishing. By making shares binding, the linkage program forced the most active fishers to either reduce their catch (which would adversely impact industry profitability) or to obtain additional shares. In fact, at the time the linkage program was enforced many of active fishers faced "share deficits" of up to 50%, i.e. they held only half of the shares required to justify their catch. To allow these fishers to reduce their share deficits the linkage program was supplemented with a "market for catch shares" where active fishers could buy shares from inactive ones.

One might naively assume that fishers can simply trade catch shares among themselves. One impediment to efficient bilateral trade is that fishers are geographically dispersed making it costly for buyers and sellers to match. Especially since fishers typically hold a portfolio of shares and, hence, sellers need to match with a variety of buyers. Another impediment is that even the most active and commercially-viable fishers are budget constrained and unable to finance the large number of shares needed to cover their share deficits. In recognition of the financial hardship the linkage program imposed on the most active fishers, the NSW government was willing to provide a subsidy of up to A$15 million to "grease the market." But it is unclear how this subsidy should be distributed to achieve efficient and fair outcomes in bilateral-trade settings, where prices for shares within the same share class may vary across different pairs of buyers and sellers. Finally, under bilateral trade, fishers who wish to exit the industry might end up selling only part of their portfolios, leaving them with a non-viable business as well as little proceeds – a possibility known as the "exposure problem."

To reduce transaction costs, ensure a fair and effective use of government subsidies, and avoid undesirable outcomes where selling fishers end up with fragmented portfolios, a centralized clearinghouse is needed. The importance of an efficient exchange can hardly be overstated. Without it, the introduction of the linkage program might have distorted catch levels and endangered the long-term viability of the NSW fishing industry. Indeed, if industry participants and other stakeholders did not believe the share reallocation would be done in a fair, transparent, and efficient manner, they would likely have rejected the introduction of the government's linkage program.

Solving the Reallocation Problem

One option is for the government to apply a fixed budget to buy back shares, which are then retired or redistributed among active fishers. The possibility of a government "buy-out" was considered in the context of the NSW reform, but it was deemed to have too many shortcomings. First, the buy-out provides no opportunity for "price discovery," i.e. there is no guidance as to what should be the (relative) prices for different share types, because there is no information about the demand of fishers who want to buy. As a result, it is unclear whether the correct shares are bought out, e.g. some shares may have little value to current owners but even less so to others. Second, acquired shares may go under-utilized for some time and it is unclear how the government should redistribute them. Allocating these shares "by formula" likely reintroduces inefficiencies as it would rest on imperfect assumptions about industry needs. Third, government buy-outs often face legal challenges (Ludicello and Lueders 2016) as the inefficiencies that arise from purchasing shares, as well as from redistributing them, are easily mistaken for favoritism toward specific market participants.

This paper proposes a radically different, market-based approach to the reallocation of fishery shares. By creating a market where inactive fishers ("sellers") and active fishers ("buyers") trade shares, transaction costs are reduced and delays minimized. Moreover, the market allows for price discovery, i.e. fishers can learn about the market value of their shares as reflected by the bids and asks submitted to the market. As a result, market-based exchanges hold the promise to yield more efficient allocations than government buyouts. Finally, government policy embedded in a market instrument typically faces less legal resistance as the outcome is determined endogenously by the buy/sell choices of participating fishers.

The desirability of market-based solutions has recently been flagged in the literature (Marszalec 2017), but no specific proposals have been put forward. One likely reason is that the market for fishery shares requires several non-standard features – standard market forms are inadequate. First, in standard markets (like those used in stock exchanges), different share types would be sold separately, which exposes sellers to the risk of selling some share types but not others, leaving them with insufficient proceeds and a commercially unviable share portfolio (i.e. the "exposure problem"). To avoid such fragmentation, sellers should be allowed to submit "all-or-nothing" asks. Second, all-or-nothing asks raise questions about pricing as a single seller may be matched with many buyers who each express their individual willingness to pay. Using a "discriminatory" price rule where each successful buyer pays their own bid seems natural but may cause envy and political stir. Participants feared paying too much or receiving too little for their shares compared to others. On the other hand, a uniform clearing price paid by all successful buyers can lead to paradoxically rejected bids and inefficiencies. Finally, a subsidy may be required to ensure industry acceptance of the "cap-and-trade" program and to stimulate participation in the market. In particular, the subsidy should aid those fishers for whom the introduction of the linkage program resulted in an immediate deficit of shares. These active and commercially-viable fishers were hit hardest by the linkage program, while inactive fishers were not impacted at all. The difficulty is that optimal subsidy levels should be determined *endogenously*, i.e. based on bids and asks received. Without this market information, it is impossible to know what are buyers' true preferences for shares, what are the correct market prices, and what are the costs to active fishers to effectively reduce their share deficits. For this reason, it is not possible to set correct subsidy levels upfront. And if the government simply promised to pay a rebate to the buyers after the exchange, it would expose itself to the possibility of having to pay much more than the planned subsidy. To summarize, even when data on individual share deficits is available, it was not possible to efficiently distribute the subsidy before or after the exchange takes place. Instead subsidies have to be distributed endogenously via discounts to market prices.

Contributions

The recent subsidized share trading market in New South Wales (NSW) is a first-of-a-kind market design for the reallocation of catch shares and the largest combinatorial exchange for fishery access rights run to date. The requirements for this market were challenging and likely play a role in other environmental applications (e.g. trading pollution permits, water rights, environmental offsets):

- *Computational challenges in large-scale combinatorial exchanges:*

  Allowing for all-or-nothing offers avoids exposure problems but also introduces several design complexities. First, the allocation or winner-determination problem is an NP-hard combinatorial optimization problem. While combinatorial auctions have been analyzed in operations research for more than a decade, the size of this market was remarkable. With around 600 participating fishers, 100 share classes, and 1300 bids in each round it was all but clear that the allocation problem could be solved to optimality or even near-optimality.

- *Development of new payment rules for combinatorial exchanges:*

  Second, stakeholders required "fair" prices in the sense of anonymous and linear prices for each share class. This led to fundamentally new economic insights about pricing in combinatorial exchanges (Bichler et al. 2018). Our theoretical analysis of these prices is of interest beyond the fishery market and relevant also to other applications of combinatorial exchanges as they can be found in energy markets or in logistics.

- *Endogenous distribution of a subsidy to incentivize participation:*

  The government decided to provide a subsidy to participating fishers in need. This was implemented via a lexicographically-ranked objective function, where the subsidy was first used to help active fishers with large share deficits, then active fishers interested in buying but without such a deficit, and finally fishers that exit the industry. The endogenous distribution of subsidies in the exchange was a way to achieve sufficient participation from fishers and reallocate the shares effectively to those in need, but led to new design problems not covered by existing theory.

The market was conducted between May 1, 2017 and June 30, 2017 in three rounds. In each of these rounds buyers and sellers could revise their bids via a Web-based bid submission system. To solve allocation and prices, a series of large-scale mathematical programs had to be solved after each round. The branch-and-cut algorithm was executed on a compute cluster. Various modeling approaches and cuts helped solving the problems to optimality, which cannot generally be expected for a problem of this size.

Impact

The market solved a long-standing policy problem in New South Wales. For over a decade there had been a political struggle between fishers and the government about the correct way to reallocate catch shares. The proposed market design addressed all key concerns and was decisive to finally get all stakeholders on board.

The market completely reshaped the fishery industry in NSW. Close to 600 fishing businesses registered for the market and placed 740 buy bids, 432 sell offers, and 107 package offers. Importantly, previously underutilized shares were transferred to active fishers, which was the primary policy goal. The market saw 86% of the buy bids of active fishers with a share deficit get matched, reducing 75% of the share deficit and 95% of the share deficit in the high priority minimum shareholding share classes. In total 35,954 shares were traded in the market with the expense of \$12.86 million of the availabe \$15 million subsidy funds. These cost savings were informed by our evaluation of alternative scenarios, which demonstrated the decreasing returns of higher subsidy expenditures for the government's main objectives.

The Honourable Minister Niall Blair acknowledged the success of the approach as follows:

> *"I feel very confident in stating that we have been able to achieve a result for the taxpayer of NSW and the shareholders in the commercial fishing industry that is far beyond what any traditional approach to industry reform would have achieved."(Blair 2017)*

Lynham (2014) reports that 154 fisheries worldwide have introduced catch share systems, and that the number is growing. The market design we implemented in New South Wales can serve as a template to effectively reallocate fishing shares elsewhere. First, the need for package trading has been motivated in a number of academic articles (Marszalec 2017, Iftekhar and Tisdell 2012, Teytelboym 2018), and it is likely to be a concern in other environmental markets as well. Second, linear and anonymous prices satisfy fairness considerations such as the equal treatment of equals (Bichler et al. 2018), a desirable feature not only for the market in New South Wales. Finally, the possibility to distribute a subsidy endogenously within a market can set important incentives for participation, in markets where this is an issue otherwise.

The need for package trading to accommodate synergistic preferences is by no means unique to the fishing industry. Package trading plays a role in the assignment of airport time slots (Pellegrini et al. 2012), railway slots (Borndörfer et al. 2006), the assignment of time slots at loading docs in retail logistics (Karaenke et al. 2018), pollution rights (Fine et al. 2017), rights on native vegetation offsets (Nemes et al. 2008), water rights (McAdams 2017), etc. We provide a practical market design with important features for such types of exchanges and as such, the impact of our work extends well beyond the fisheries application discussed in this paper.

Bid Language and User Interface

Let us first describe how bidders submitted bids and the types of bids allowed, which will be useful in the discussion of the mathematical model to solve the allocation problem next.

Buyers and sellers in the market for catch shares had different requirements with respect to the expressivity of bids submitted. Some sellers are unprofitable fishers who intend to retire, while others may want to get rid of unprofitable (unused) shares only, still keeping others. Those fishers who want to quit the business need to be able to specify all-or-nothing bids. Such a fisher has only one bid to submit, which includes his endowment in various share classes. The bid has a single ask price for the whole set of shares, and represents the least amount the seller wants to get for his endowment. Selling part of his shares would not be an option as it might render his operations even less profitable. Also, accepted exit bids lead to an extra payment that is paid out of the subsidy and intended to cover fishing license cost. Figure 1 shows a screen for the submission of a package bid. If a fisher wants to keep his licenses and sell only a part of his endowment, he can submit a set of independent sell-side bids in individual share classes independently. For example, he might sell 125 shares of share class A for \$10 and/or 80 shares of share class B for \$15.

Buyers want to win shares in one or more share classes. They can submit several bids on multiple shares in a share class. Synergies across share classes were of less concern to buyers, which is why they were not allowed to submit exclusive-or (XOR) package bids across multiple share classes. Buyers could bid a unit price for a quantity interval. For example, a fisher may want to buy shares

Figure 1 **The Screenshot Illustrates the Submission of a Package Bid**

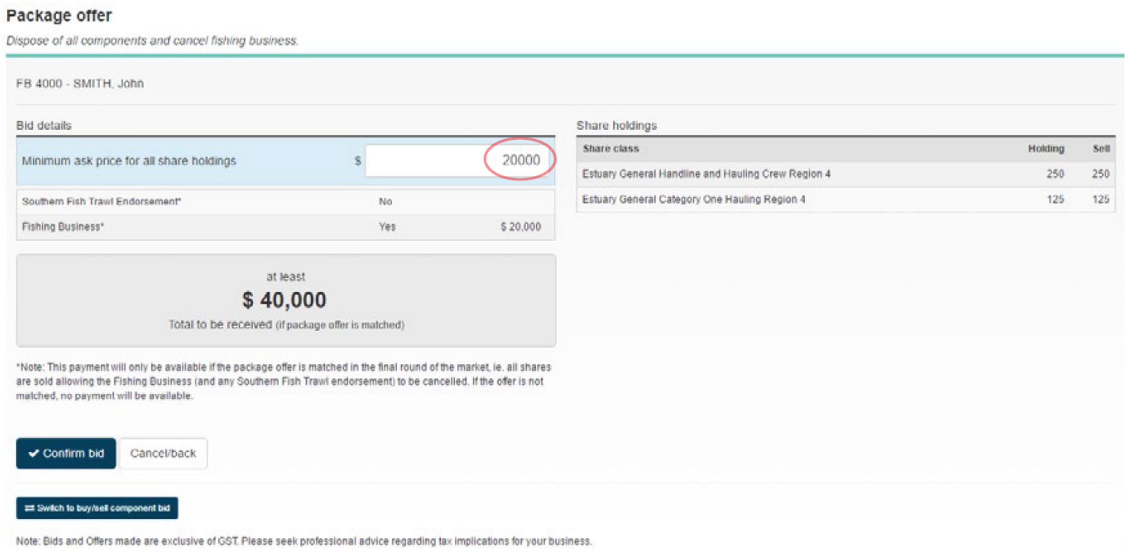
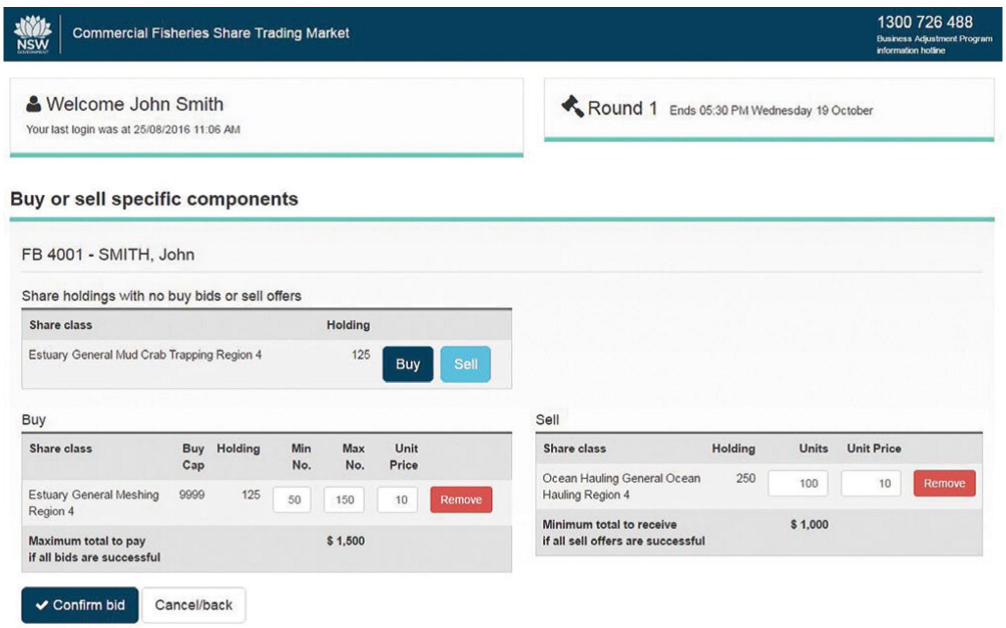


Figure 2 **The Screenshot Illustrates the Submission of Bids to Buy or Sell Shares in Specific Share Classes**



from share class A. He needs at least 125 units and at most 250 units and is willing to pay up to 3$ for each unit in this quantity interval. Figure 2 shows an example of the bid submission page for a buyer who can either buy or sell shares.

Modeling and Solution

The reallocation of catch shares based on bids and asks received in the market was facilitated via a sequence of mathematical programs implementing a sequence of lexicographically-ordered policy goals. This sequence of mathematical programs enforced constraints that determined linear and anonymous prices, respected “all-or-nothing” bids for packages, and ensured that neither the government nor any of the participants made a loss. We next introduce the objectives and constraints in an informal way (the precise mathematical model can be found in the Appendix).

Objective Functions

The main objective of the centralized market was to ensure that shares could be reallocated such that existing catch levels remained possible after the introduction of the linkage program. This reallocation had to be based on voluntary trades as the government could not just take shares from inactive fishers and reallocate them to active ones. And even if it could, the government would not be privy to detailed information about fishers’ preferences, making it impossible to reallocate efficiently. In contrast, the market can reallocate efficiently as the bids and asks received reflect fishers’ preferences.

To alleviate any financial hardship imposed by the linkage program the government was willing to inject a subsidy of up to A$15 million in the market. As discussed above, this subsidy had to be distributed endogenously based on the bids and asks received. Furthermore, different types of fishers had to be prioritized with regard to how much they should benefit from the subsidy. Active fishers with a share deficit should benefit as much as possible, and any residual budget should be used to help active fishers without a deficit or fishers that wanted to exit the market. This led to four different priorities for the government, which were turned into mathematical programs that were solved in a sequence.

The first priority (P1) was to ensure that the demands of active fishers with a deficit were satisfied as much as possible given the available subsidy. The second priority (P2) focused on active fishers without a share deficit. The third priority (P3) was to use the residual subsidy to support the retiring fishers who wanted to exit the market. The final priority (P4) was to use any left-over subsidy to aid inactive fishers who did not report any catch in the year before.

The mathematical programs implemented the different priorities by maximizing trades for the subset of fishers who were considered by the priority. The programs were run sequentially. After each optimization the objective function was added as a constraint in the subsequent optimization to make sure that the objective function value did not deteriorate.

Constraints

A number of constraints are enforced to make sure that the allocation was feasible and that the prices satisfy the payment rules laid out by the government. *Demand-supply constraints* guarantee that in every share class the total number of units bought does not exceed the total number of units sold.

Individual rationality constraints make sure that no participant will incur a loss. This means that no winning seller should receive less than his quoted ask price and no winning buyer is paying more than he was willing to pay at most. Also the government must not spend more than the pre-defined subsidy in the market, which we guarantee via a total subsidy or *budget constraint.*

The *prices* computed by the optimization formulation are such that each winner pays the same price for a share class (aka. linear and anonymous prices) and receives the same subsidy if he is eligible for it. An important argument for linear and anonymous prices in the discussion during the design was the perceived fairness of such prices. If two sellers on the exchange sell the same package, but they get very different payments, this would be seen as unfair and not equitable by participants. Proportional fairness and equitability of payments was a particularly important design goal for those sellers, who exit the market. The subsidy was used to lower the payment by active fishers. In other words, all active fishers received the same discount on the market price in a share class, while inactive fishers paid the full market price determined by the mathematical program.

Linear and anonymous prices are also employed on day-ahead energy markets for similar reasons (Van Vyve 2011). Unfortunately, linear and anonymous prices that constitute a competitive equilibrium in a combinatorial exchange are typically not feasible or only with restrictive assumptions on the value functions (Kelso and Crawford 1982, Gul and Stacchetti 1999, Baldwin and Klemperer 2018). Similar to day-ahead energy markets, we allowed paradoxically rejected losing bids

(Meeus et al. 2009). Such buy (sell) bids are losing, even though they were higher (lower) than the market price. However, prices were set such that they could not exceed the bid price plus the relative subsidy set for a share class for a winning bidder, i.e. individual rationality constraints were satisfied.

The Multi-Round Auction Process

There was significant value uncertainty in the market and the government was concerned that the results of a single sealed-bid auction might lead to regret among the participants. This was the first time that such an exchange took place and most fishers were inexperienced in any form of electronic trading. Therefore, the auction was organized in multiple rounds. If the initial round did not meet a predefined internal goal and objective function values in terms of the priorities, the government could organize up to two more rounds. Bidders did not know a priori, which round would be the last, in order to mitigate gaming behavior.

This uncertainty should counteract gaming where some bidders just try to win with a very low bid and learn about the market. We did see a few bids in the first round that were either far higher or far lower than others which could either be an attempt to game the market or a result of value uncertainty. However, the bids were relatively stable across all three rounds as the data analysis in the last section shows. Overall, the multiple rounds helped to reduce anxiety, set the expectation of fishers and let them revise their valuation, which was also important for the acceptance of the market.

There were only minimal activity rules across rounds. Bidders needed to have a confirmed bid in a round in order to participate in any subsequent round (if held). Bidders could freely modify their bids within a round (prior to the closing time) or from one round to the next. This could include changing from an exit package offer to individual buy/sell bids, from buy to sell or vice versa, adding bids for new share classes or deleting previous individual bids, or changing quantity or price. Each new bid superseded the previous bid from that shareholder. Bids that were not modified remained valid from one round to the next until the conclusion of the market, and could become successful in a later round even if they were not successful in an earlier round. Stricter activity rules were considered too complex for the large number of inexperienced fishers participating.

After a round closed, allocation, prices, and subsidy were computed for a number of different scenarios based on predetermined variations in parameters such as total subsidy, subsidy reserved for exit bids and bounds on the subsidy per share class. An evaluation panel consisting of government and independent experts, with oversight from the projects independent probity advisor, analyzed the effect of these scenarios on achievement of the market objective in each scenario. The evaluation panel then chose one of the allocations as final allocation or devised another round. If there was another round, one of the allocations and the associated prices were chosen as a basis for the feedback to the bidders.

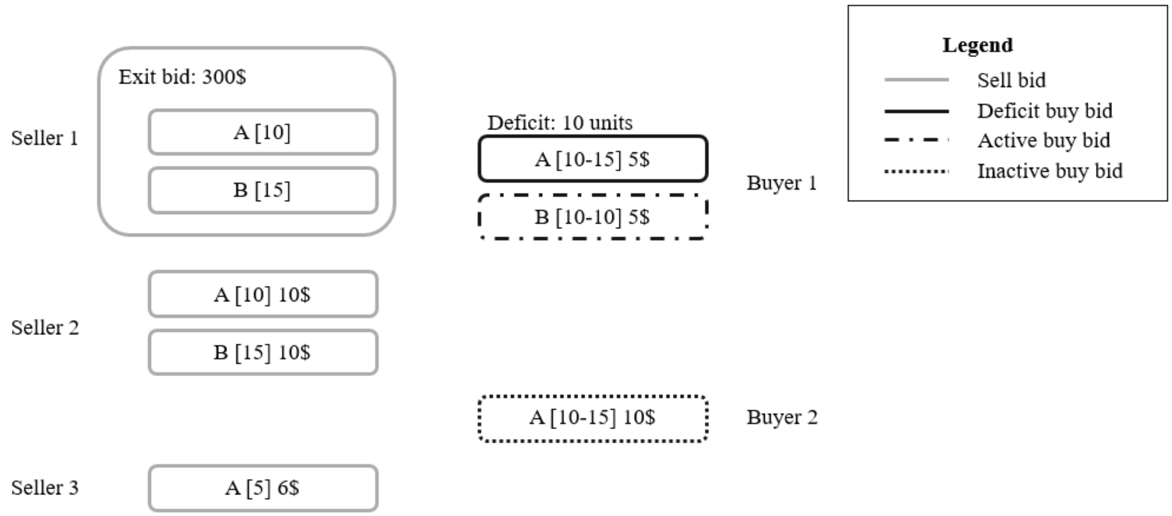
After a round the government provided individual results to each bidder (i.e. whether bids were successful or unsuccessful and market prices for each share class plus the subsidy they would have received if it was the last round), via a combination of email and a secure website. If there was another round, the government also informed bidders about the next round (e.g. the opening and closing times, the fact that bids carried over unless modified and the fact that both share allocations and prices could change in a new round). Once a round was determined to be the final round, the government advised bidders of the final outcomes and next steps for payment.

Illustrative Example

Let us illustrate the combinatorial exchange with a toy example shown in Figure 3. We have 3 sellers: Seller 1 is providing a package bid with two blocks of shares with a total ask of 300$, two other sellers submit individual sell bids. On the buy side we have two participants: Buyers 1 has a deficit of 10 units in share class A and is willing to buy from 10 to 15 units and wants to pay 5$ at maximum; he is also active in share class B, in which he wants to buy 10 units firm and quotes the same price. Buyer 2 is also willing to buy shares in A, but he was not active in this share class yet, and thus is not eligible for a discount.

If we take a look at the bids from the example in Figure 3, we see that if there is no subsidy, then no trade will happen: buy and sell prices simply do not match. The only match can be observed

Figure 3 An Example Bid Instance



between Seller 3 and Buyer 2, yet Buyer 2 needs twice as many shares as Seller 3 can provide, thus they will not trade as well.

In priority **P1** the program maximizes volume (price times number of units bought) of deficit shares acquired by buyers. In the example we have only Buyer 1 with a deficit of 10 units in share class A. A solution will be to accept the bid of Buyer 1 in class A and that of Seller 2 in the same class. Then, the price for share class A is 10$ and the government provides a discount of 5$ per unit to make this trade happen. In total, the government spends 50$ of the predefined subsidy. The result of priority P1 is shown in Figure 4. For further priorities, Buyer 1 should not get less than 10 shares in class A.

Priority **P2** aims to maximize the volume of trades for bids of active bidders, who don't have a share deficit. We have two active bids in our example, both from Buyer 1: In share class A and in share class B. Note, even though Buyer 1 has a deficit of 10 units only, he is willing to buy up to 15 units, which should be take into account in P2. The best option is again to match Buyer 1 with Seller 2, this time in both share classes and with a single bid of Seller 3 in order to satisfy his demand of 15 units in share class A (see Figure 5).

Note that Seller 2 wants to sell 15 units in share class B, while Buyer 1 demands only 10 units. This means that the government needs to buy out the remaining 5 units for which the market price

Figure 4 **Computation of Priority 1**

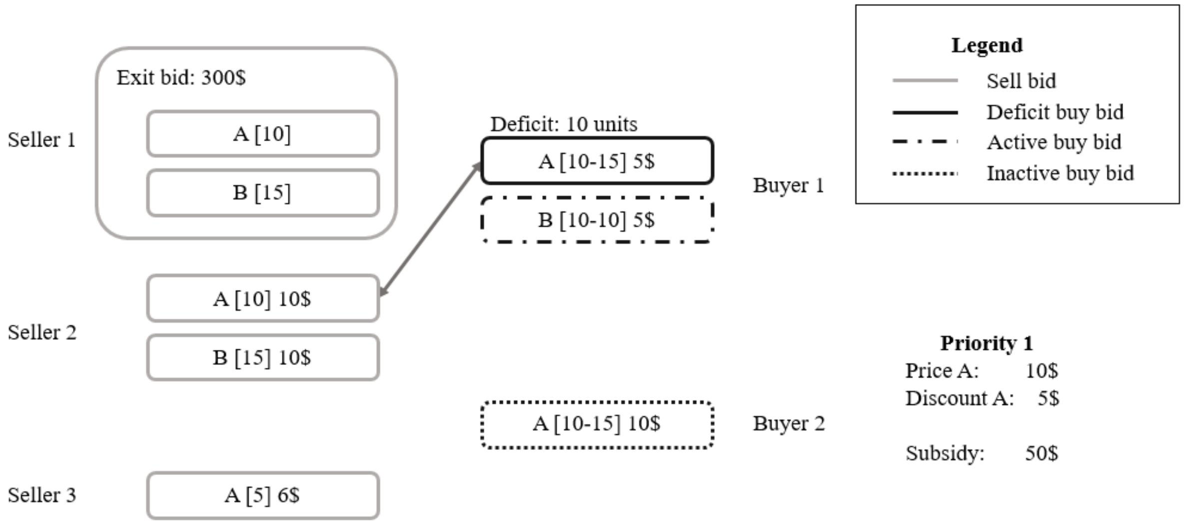
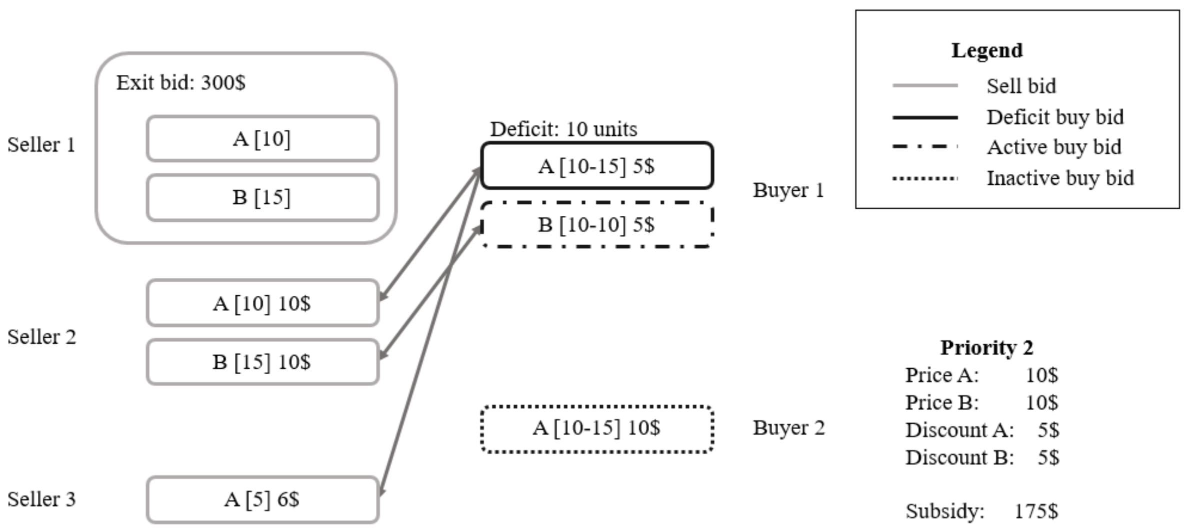


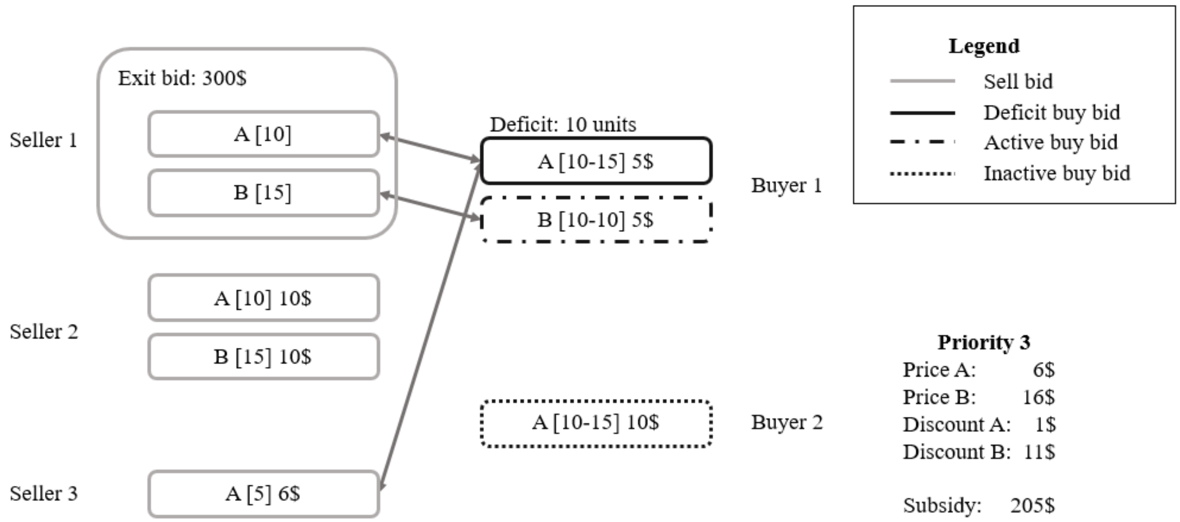
Figure 5 **Computation of Priority 2**



needs to be paid. The resulting prices are 10$ for A and B, discounts of 5$ for both share classes. The government buys out 5 units in share class B for the full market price in order to keep the exchange budget-balanced. The subsidy needed for such an allocation is thus 175$ (25 times 5$ for discounts and 50$ for the purchase of the remaining 5 units in share class B).

The reform is also meant to consolidate the market by allowing fishers to quit the business and sell all their possessions and fishing licenses. This is the focus of priority **P3**. In our example this makes a difference for the sell side: even though it is cheaper to accept bids from Seller 2, the exit

Figure 6 Computation of Priority 3



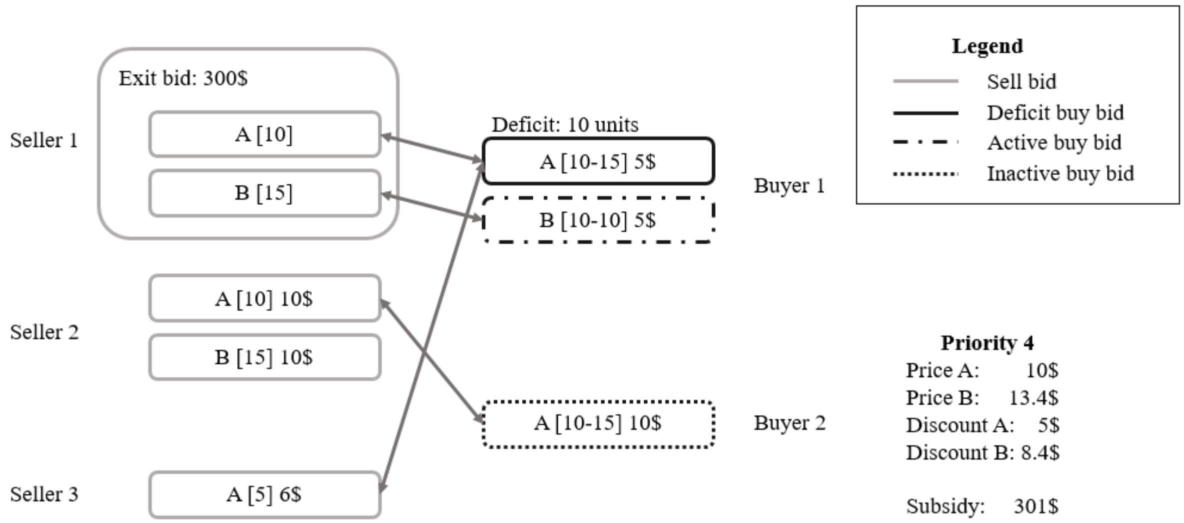
bid is preferred in P3 as a policy goal (see Figure 6). Buyer 1 trades with Seller 1 and Seller 3. In order to use the subsidy efficiently we charge 6$ for A and 16$ per unit of B. The Seller 1 gets exactly the 300$ he asked for, and the government again needs to buy 5 shares in share class B. The total subsidy spent is thus 205$ (15$ for discounts in A, 110$ for discounts in B and 80$ for buying out 5 shares in class B).

Priority 4 focuses on inactive bids, and in our example there is a room for improvement in this regard: Buyer 2 can be matched with the bid of Seller 2 in class A. This will increase the subsidy paid to Buyer 1 in class A, and thus the overall subsidy used (see Figure 7). All buy bids are satisfied (Buyer 2 gets only 10 units), and the price for A is 10$. For B the price increased to approximately 13.4$ in order to satisfy the exit bid of 300$. This means, the discount for class A is 5$ and for class B 8.4$. The total subsidy used is now 301$ (150$ discounts for A, 84$ discounts for B and 67$to buy out 5 units in class B), and the priorities of the government have been satisfied as far as possible in a lexicographic manner.

Results

We first analyze the bidding activity of participants in the auction, before we interpret the results and analyze the impact on the market.

Figure 7 **Computation of Priority 4**



Progress of the Auction

Let us briefly discuss the progress of the auction, the bidding activity and the changes across rounds in the auction in 2017. Table 1 shows the number of bids submitted in each round. Interestingly, this number is almost the same in each round, although there were no activity rules employed across rounds. Since bidders did not know, which round would be the last, they participated actively from the start. The number of winners was increasing largely because the government decided to increase the total subsidy after each round from 6 to 11 million dollars in total based on the bids received.

Most losing buyers increased their bids across rounds and losing sellers typically decreased their bids, while most winners did not change their bid. From 589 losing buyers in round 1, 322 increased their bid. The median increase was more than 52%. However, only a small number of buyers with a very large relative increase are responsible for this large relative number.

From 417 losing bids in round 2 153 were increased by 30% (median), while 212 losing bids remained untouched. The distributions of relative changes and median values have to be taken with a grain of salt as some buy bids in the initial rounds were just very low or even close to zero. Once the bid was increased to levels of the market price in the first round this could lead to a very large relative increase. Note that bids and resulting market prices varied a lot across different

Table 1 Number of Bids per Round

bid type	round 1	round 2	round 3
buy	747	739	740
-winning	158	322	446
-losing	589	417	294
sell	421	439	432
-winning	35	90	131
-losing	386	349	301
exit	95	101	107
-winning	34	51	62
-losing	61	50	45

Table 2 Number of Bid Price Changes of Buyers

bid type	round 1 to round 2			round 2 to round 3		
Buy bid	↑357	↓47	∼287	↑223	↓60	∼413
-winners	↑35	↓19	∼103	↑70	↓42	∼201
-losers	↑322	↓28	∼184	↑153	↓18	∼212

share classes. A majority of the share classes traded at prices less than $200, while a few valuable share classes had share prices of several thousand Australian dollars.

Similarly, from 386 losing sell-side bids or asks 188 lowered their bids by 32% in the median while 117 did not change between round 1 and 2. Between round 2 and 3 142 of the 349 losing sell-side bids were decreased by 30% (median). 166 losing asks remained untouched after round 2.

Outcomes

The main objective of the market was to reallocate shares so that existing catch levels would remain possible after the introduction of the linkage program. Or, stated differently, the main goal of the market was to provide an opportunity to undo the share deficits created by the linkage program. The exchange can be considered great success in light of this. Close to 600 fishers participated and

Table 3 Number of Ask Price Changes of Sellers

bid type	round 1 to round 2	round 2 to round 3
Sell bid	$\uparrow$52 $\downarrow$191 $\sim$141	$\uparrow$21 $\downarrow$153 $\sim$227
-winners	$\uparrow$6 $\downarrow$3 $\sim$24	$\uparrow$9 $\downarrow$11 $\sim$61
-losers	$\uparrow$46 $\downarrow$188 $\sim$117	$\uparrow$12 $\downarrow$142 $\sim$166

Table 4 Number of Ask Price Changes of Sellers with an Exit Bid

bid type	round 1 to round 2	round 2 to round 3
Exit bid	$\uparrow$6 $\downarrow$39 $\sim$45	$\uparrow$10 $\downarrow$34 $\sim$52
-winners	$\uparrow$4 $\downarrow$1 $\sim$29	$\uparrow$6 $\downarrow$12 $\sim$33
-losers	$\uparrow$2 $\downarrow$38 $\sim$16	$\uparrow$4 $\downarrow$22 $\sim$19

86% of the buy bids from active fishers with a deficit were matched. Their overall share deficit was reduced by 95% in the high-priority share classes.

In total, A$14.8 million was paid to the sellers: the buyers paid A$3.2 million and the remaining A$11.2 million was subsidized by the government. An additional A$0.4 million was spent by the government to buy out shares of sell-side package bids for which there was no buyer. Also, 60% of the package offers were matched and 62 businesses successfully sold all their shares. These exiting fishers received A$10.1 million for their shares. Around A$5.9 million went to sellers who just sold part of their endowment of catch shares.

The government had set aside A$15 million to subsidize the market but ended up spending only A$11.6 million. This decision was informed by our evaluation of alternative scenarios, including the effects of higher subsidy levels. This showed that the higher government's main objectives were largely unaffected by higher expenditure levels. This aspect of our approach saved taxpayers several millions.

Because of the linear and anonymous prices, as well as the subsidy, buyers typically paid less than their submitted bid. In total, winning buyers bid A$6.2 million, but only had to pay A$3.2 million. Similarly, the winning sellers asked for A$12.7 million in total, but received payments of

A$14.8. The subsidy of A$11.2 millions enabled these trades and set enough incentives for fishers to participate. These results would not have been possible with a government buy-out.

Lessons Learned

Market-based solutions are increasingly being considered for the allocation or reallocation of resource shares (Marszalec 2017). A recent fisheries reform bill of the Faroe Islands, for instance, proposes to use auctions to allocate 25% of the country's fishing shares (N.N. 2017). Initial allocations decided by auction tend to be more efficient than those resulting from grandfathering, but changes in the industry, fish stock, or overall quota will necessitate revisions to the initial share allocation.

This paper describes the implementation of a combinatorial exchange to reallocate fishing rights in New South Wales (NSW), Australia. The design of the exchange addressed several non-standard requirements, including all-or-nothing offers, fair prices, and an endogenously determined subsidy. The exchange was conducted in May and June 2017 and successfully reformed the NSW fishing industry. The exchange may serve as a template for the reallocation of fishing rights in other jurisdictions. Lynham (2014) reports that there are 154 catch share systems world-wide and that the number is growing. Moreover, similar exchanges can be used to reallocate resource rights in other domains, e.g. to trade water rights, pollution rights, or environmental offsets.

An important takeaway from this paper is that operations research and market design have significant potential to address challenging policy problems. And this potential is not limited to the fisheries domain. A very different application is the recent incentive auction in the US, which reallocated spectrum from TV stations to telecoms (Leyton-Brown et al. 2017). Also in this case, new and innovative designs were required to address the non-standard requirements imposed by the market. While there is no "one size fits all" in market design, these examples highlight that template designs that are portable across jurisdictions and domains can successfully be developed.

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Appendix. Mathematical Model

Mathematical Model

Let us now introduce the optimization models formally. These models were solved after each round determining allocation and prices.

Notation

We denote the set of all buy bids as $\mathcal{B}$; this set can be naturally divided into set of active buy bids $\mathcal{B}^{\mathcal{A}}$ and set of inactive buy bids $\mathcal{B}^{\mathcal{I}}$, which have no common elements, i.e. $\mathcal{B}^{\mathcal{A}} \cap \mathcal{B}^{\mathcal{I}} = \emptyset$. Each buy-side bid $j \in \mathcal{B}$ is related to only one share class $l \in \mathcal{L}$ and is a tuple $< l, {b_j}^l, \underline{\beta}^l_j, \overline{\beta}^l_j >$, where ${b_j}^l$ is unit price, which a fisher can pay for a single share, and $\underline{\beta}^l_j$ and $\overline{\beta}^l_j$ are the lower and the upper bounds on the number of units fisher wants to acquire. In addition, with each active buy-side bid $j \in \mathcal{B}^{\mathcal{A}}$ a deficit value d^l_j is associated. This deficit value is the shortage of shares for this particular fisher in a share class l, which is computed by the government prior to the auction.

Further, we denote by $\mathcal{S}$ the set of individual sell side bids and by $\mathcal{S}^E$ the set of exit bids (package bids). An individual bid $i \in \mathcal{S}$ in a share class $l \in \mathcal{L}$ consists of a quantity q^l_i that a fisher wants to sell and a unit price s^l_i she wants to get for a single share at a minimum.

An exit bid $i \in \mathcal{S}^E$ consists of a package vector $q_i = \{q^l_i\}_{l \in \mathcal{L}}$, which contains the number of units owned by a retiring fisher in each share class (some elements of the vector are zeros) and the total price s_i which a fisher wants to get for his package at a minimum. In addition, if an exit bid is accepted, a compensation for a fishing license p^E is payed from the subsidy that the government provides to the market.

With each bid we associate an allocation variable, which describes whether a particular bid is accepted, and, in case of buy bids, how many units are allocated. A buy bid is represented by an integer variable β^l_j. To have a feasible allocation, the value of β^l_j needs to be within the quantity bounds specified by the buyer, $[\underline{\beta}^l_j, \overline{\beta}^l_j]$ or zero. For this, we introduce an additional binary variable ζ^l_j which is 1 if and only if the corresponding β^l_j is positive:

$$\beta^l_j \leq \overline{\beta}^l_j \zeta^l_j \qquad \forall j \in \mathcal{B}, l \in \mathcal{L} \qquad \text{(BQty1)}$$

$$\underline{\beta}_j^l \zeta_j^l \leq \beta_j^l \qquad \forall j \in \mathcal{B}, l \in \mathcal{L} \qquad \text{(BQty2)}$$

Acceptance of an individual sell or exit bid is determined by binary variables σ_i^l where $i \in \mathcal{S}$ and $\sigma_i \in \{0,1\}$ where $i \in \mathcal{S}^E$ respectively.

Objective Function

The lexicographic policy goals are described as objective functions in a series of mathematical programs. This means that priority $k+1$ is taken into account after priority k. Let's describe each priority as an objective function for a mixed-integer linear program. In what follows "main constraints" refers to the constraints of the problem described in the subsequent sections.

Priority P1 focuses on buy bids with a deficit, and can be written as:

$$\max \sum_{j \in \mathcal{B}^{\mathcal{A}}} \sum_{l \in \mathcal{L}} (\kappa_j^l b_j^l) \qquad \text{(Priority 1)}$$

$$\text{s.t. } \kappa_j^l \leq \beta_j^l, \kappa_j^l \leq d_j^l \qquad \forall j \in \mathcal{B}^{\mathcal{A}}, l \in \mathcal{L} \qquad \text{(Deficit)}$$

(Main constraints)

So the goal is to maximize the volume of units bought up to a deficit level for active fishers. The new variable κ_j^l is introduced in order to consider the buy bid of an active fisher only up to her deficit level d_j^l. As a result of the optimization, we get an allocation $(\{\beta_j^{l,P1}\}_{j \in \mathcal{B}^{\mathcal{A}}})$, which becomes a new constraint for priority 2. This new constraint ensures that the buyers with a deficit are satisfied as far as possible and also later priorities and optimization runs do not negatively impact this policy goal.

Priority P2 considers all active buyers, and thus we formulate the objective function as

$$\max \sum_{j \in \mathcal{B}^{\mathcal{A}}} \sum_{l \in \mathcal{L}} (\beta_j^l b_j^l), \qquad \text{(Priority 2)}$$

$$\beta_j^l \geq \beta_j^{l,P1} \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : d_j^l > 0 \qquad \text{(Transition 1)}$$

(Main constraints)

Note that Transition 2 constraint implies Transition 1. Priority P3 aims to maximize number of accepted exit bids, i.e. package bids of bidders who want to leave the market:

$$\max \sum_{i \in \mathcal{S}^E} \sigma_i, \qquad \text{(Priority 3)}$$

$$\beta_j^l \geq \beta_j^{l,P2} \qquad \forall j \in \mathcal{B}^{\mathcal{A}} \qquad \text{(Transition 2)}$$

(Main constraints)

Priority P4 just maximizes the total volume of shares bought, such that even inactive fishers are taken into account:

$$\max \sum_{j \in \mathcal{B}} \sum_{l \in \mathcal{L}} (\beta_j^l b_j^l), \qquad \text{(Priority 4)}$$

$$\beta_j^l \geq \beta_j^{l,P2} \qquad \forall j \in \mathcal{B}^{\mathcal{A}} \qquad \text{(Transition 2)}$$

$$\sigma_i \geq \sigma_i^{P3} \qquad \forall i \in \mathcal{S}^E \qquad \text{(Transition 3)}$$

(Main constraints)

All priorities P1-P4 were solved in a sequence. Overall, due to constraints on allocation, the solution to each priority should satisfy objective values of the previous priority solutions. To speed up the overall process, we allowed to "warm start" the solution process of later priorities. This means, we provide the solution of the last priority to the solver. The last solution is feasible also for the new priority, but may be improved with respect to the new objective.

Let us now discuss the main constraints that we need to consider in all models for all priorities.

Demand-Supply Constraint

We need to guarantee that in every share class the total number of units bought does not exceed the total number of units sold. This is implemented via the following demand-supply constraint:

$$\theta^l + \sum_{j \in \mathcal{B}} \beta_j^l = \sum_{i \in \mathcal{S}^E} q_i^l \sigma_i + \sum_{i \in \mathcal{S}} q_i^l \sigma_i^l, \qquad \forall l \in \mathcal{L} \qquad \text{(Demand-Supply)}$$

where θ^l is an integer-valued variable describing the artificial demand by the government, which facilitates trades in cases where sell-side bids do not have matching demand on the buy side. This variable can only be positive when it is beneficial for the current objective function (one of priorities 1-4).

For all units bought, the government has to pay the full market price, because we have anonymous prices and also require the budget to be balanced. Such shares bought by the government were sunset after the market and the money paid by the government to sellers had to come from the total subsidy.

Individual Rationality Constraints

An important requirement for the market is to guarantee that no participant will incur a loss. This means that no winning seller should receive less than his quoted ask price and no winning buyer is paying more than his bid. Thus for winning buyers, the following constraint should be satisfied

$$\rho^l \leq b_j^l, \qquad \forall j \in \mathcal{B} : \beta_j^l > 0, l \in \mathcal{L} \tag{IRB´}$$

where ρ^l is continuous variable representing linear price in share class l. Active fishers may pay a subsidized price, i.e. the government pays a discount on the market price δ^l in share class l:

$$\rho^l \leq b_j^l + \delta^l. \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : \beta_j^l > 0, l \in \mathcal{L} \tag{IRBA´}$$

Note that these constraints should be satisfied only for winning buyers. We rewrite them as valid linear constraints using big-M constraints:

$$\rho^l \leq b_j^l + \delta^l + (1 - \zeta_j^l)M \qquad \forall j \in \mathcal{B}^{\mathcal{A}}, l \in \mathcal{L} \tag{IRBA}$$

$$\rho^l \leq b_j^l + (1 - \zeta_j^l)M \qquad \forall j \in \mathcal{B}^{\mathcal{I}}, l \in \mathcal{L} \tag{IRBI}$$

Similarly, we can formulate individual rationality constraints for the sell-side:

$$s_i \sigma_i \leq \sum_{l \in \mathcal{L}} q_i^l \rho_i^l \qquad \forall l \in \mathcal{L}, \forall i \in \mathcal{S}^E : \tag{IRP}$$

$$s_i^l \sigma_i^l \leq \rho^l \qquad \forall i \in \mathcal{S} \tag{IRS}$$

Budget Constraint

To encourage active participation, the government provides a subsidy Δ, which is distributed endogenously. Some amount Δ^E of this subsidy was determined for buying out fishing licenses of those fishers who exit the market:

$$\sum_{i \in \mathcal{S}^E} p^E \sigma_i \leq \Delta^E \tag{Exit subsidy}$$

The residual subsidy Δ^R (i.e. $\Delta = \Delta^R + \Delta^E$) covered discounts for active buyers δ^l and the money necessary for buying shares through the government which helped facilitate sales of package bids where there was no matching buy-side quantity in some of the share classes (θ):

$$\sum_{j\in\mathcal{B}^A} \beta_j^l \delta^l + \sum_{l\in\mathcal{L}} (\rho^l - \delta^l)\theta^l \leq \Delta^R \tag{Subsidy}$$

Unfortunately, the previous constraint contains the product of continuous and integer variables, which can only be linearized through introduction of a big number of additional binary variables and constraints.Therefore, we rewrote the constraint in such a way that only inactive bids are linearized (the number of inactive participants was expected to be lower than the number of active fishers). We multiplied the amount sold to sellers σ_i with the discount δ^l and subtracted the amount bought by inactive buyers $\sum_{j\in\mathcal{B}^I} \beta_j^l \delta^l$, which also yields the discounts for the active buyers:

$$\begin{aligned}&\sum_{i\in\mathcal{S}^E} \sum_{l\in\mathcal{L}} q_i^l \delta^l \sigma_i + \sum_{i\in\mathcal{S}} \sum_{l\in\mathcal{L}} q_i^l \delta^l \sigma_i + \\ &\sum_{l\in\mathcal{L}} (\rho^l - \delta^l)\theta^l - \sum_{j\in\mathcal{B}^I} \beta_j^l \delta^l \leq \Delta^R \end{aligned} \tag{Subsidy}$$

This constraint is still non-linear, and all 4 terms contain products of two variables. However, the total number of variables after linearization should be lower than in the first version of the constraint because of smaller number of inactive bids. The product of binary and integer variables in terms 1 and 2 ($\delta^l\sigma_i$) are straightforward to linearize. The terms 3 and 4 are more challenging as they are products of integer and continuous variables. We first convert the integer variables θ^l and β_j^l to a sum of binary variables:

$$\theta^l = \sum_{n=0}^{logN/2} 2^n \theta_n^l \tag{Government Bids}$$

$$\beta_j^l = \underline{\beta}_j^l + \sum_{n=0}^{log(\overline{\beta}_j^l - \underline{\beta}_j^l)/2} 2^n \mu_{j,n}^l \tag{Inactive Bids}$$

Then we can linearize these terms as products of binary and continuous variables. The number of shares the government can buy was restricted by a parameter N in constraint (Government bids). This parameter N was set to 50% of the supply in a share class.

Payment Computation

The formulation in the previous subsections determined an allocation that satisfies the lexicographic objective function of the government. The resulting prices are not necessarily unique. The government decided to minimize the variance of prices in different share classes as a secondary objective. For this, the variables $\sigma_i^{l*}, \sigma_i^*, \beta_j^{l*}$ become fixed parameters, but prices ρ^l remain variables. A quadratic optimization minimizes the differences in prices across share classes l.

$$\min \sum_{l\in\mathcal{L}} (\rho^l)^2 \tag{Prices}$$

$$\text{s.t. } s_i \le \sum_{l\in\mathcal{L}} q_i^l \rho^l \qquad \forall l \in \mathcal{L}, \forall i \in \mathcal{S}^E : \sigma_i^* = 1 \tag{IRP}$$

$$s_i^l \le \rho^l \qquad \forall i \in \mathcal{S} : \sigma_i^{l*} = 1 \tag{IRS}$$

$$\rho^l \le b_j^l + \delta^l \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : \beta_j^{l*} > 0, l \in \mathcal{L} \tag{IRBA}$$

$$\rho^l \le b_j^l \qquad \forall j \in \mathcal{B}^{\mathcal{I}} : \beta_j^{l*} > 0, l \in \mathcal{L} \tag{IRBI}$$

$$\begin{aligned}&\sum_{i\in\mathcal{S}^E}\sum_{l\in\mathcal{L}} q_i^l \sigma_i^* \rho^l + \sum_{i\in\mathcal{S}}\sum_{l\in\mathcal{L}} q_i^l \sigma_i^{l*} \rho^l - \\ &\sum_{j\in\mathcal{B}^I} \beta_j^{l*} \rho^l - \sum_{j\in\mathcal{B}^A} \beta_j^{l*} (\rho^l - \delta^l) + \\ &\sum_{i\in\mathcal{S}^E} p^E \sigma_i^* \le \Delta\end{aligned} \tag{Subsidy}$$

$$\underline{\delta}^l \rho^l \le \delta^l \le \overline{\delta}^l \rho^l \qquad l \in \mathcal{L} \tag{Subsidy bounds}$$

$$\rho^l, \delta^l \in \mathbb{Z}_{\ge 0} \qquad \forall j \in \mathcal{B}, l \in \mathcal{L} \tag{Int}$$

Finally, after unique prices were determined, the subsidy by the government was minimized. For this, the variables ρ^{l*}, the resulting prices for each share class from the previous model, were fixed, and the following last optimization problem was solved.

$$\min \sum_{j\in\mathcal{B}^A} \beta_j^{l*} \delta^l \tag{Subsidy}$$

$$\text{s.t. } \rho^{l*} \le b_j^l + \delta^l \qquad \forall j \in \mathcal{B}^{\mathcal{A}} : \beta_j^{l*} > 0, l \in \mathcal{L} \tag{IRBA}$$

$$\underline{\delta}^l \rho^{l*} \le \delta^l \le \overline{\delta}^l \rho^{l*} \qquad l \in \mathcal{L} \tag{Subsidy bounds}$$

$$\delta^l \in \mathbb{Z}_{\ge 0} \qquad \forall l \in \mathcal{L} \tag{Int}$$

Implementation

Different model formulations and solver implementations with various problem sizes were tested during the development phase, before we decided on the model described earlier. The final model was tested with up to 2000 bids on artificially generated bid data, which was significantly more than what we experienced in the field. These problems could typically be solved to near-optimality and many also to optimality. Near-optimality describes integrality gaps of of less than 6%. Only very rarely did we experience larger integrality gaps after half an hour for very large problem sizes. The differences in the allocation of a near-optimal solution and the optimal solution were typically very small or did not exist. However, proving the optimal solution could sometimes take a disproportionally long time. Depending on the subsidy provided problems became harder to solve such that the computations could take several hours. However, on average we used less than 30 minutes for each optimization run. Parameter tuning in the branch-and-cut solver helped to solve the problem sizes. For example, for priority P3 we modified the branching priorities for package bid variables, in order to branch first on low ask bids. The specific model formulation presented in this section played a significant role to keep the problems tractable.

The final round of the market included 1,280 bids leading to a mathematical program of around 6,000 variables and 10,230 constraints. We were able to solve the problem to optimality with standard branch-and-cut solvers on an Intel i7-7600U CPU with 2.80 GHz, two cores and 16 GB memory. We do not report a detailed analysis of the runtimes as they depend very much on the specific parameters of the problem. However, with nowadays solver technology problems of this size can be solved to optimality or near-optimality.