ACE: A Combinatorial Market Mechanism

Leslie Fine, Jacob K. Goeree, Tak Ishikida, and John O. Ledyard

October 9, 2016

Abstract

In 1990 the South Coast Air Quality Management District created a tradable emissions program to more efficiently manage the extremely bad emissions in the Los Angeles basin. The program created 136 different assets that an environmental engineer could use to cover emissions in place of installing expensive abatement equipment. Standard markets could not deal with this complexity and little trading occurred. A new combinatorial market was created in response and operated successfully for many years. That market design, called ACE (approximate competitive equilibrium), is described in detail and its successful performance in practice is analyzed.

1 Intro

At the time of the Clean Air Act of 1990, the Los Angeles basin was the only region in the country classified as an extreme non-attainment area for exceeding the National Ambient Air Quality Standards for ozone. Under pressure at the national level from the Environmental Protection Agency (EPA), and at the state level from the California Air Resources Board (CARB) to significantly reduce emissions, the South Coast Air Quality Management District (SCAQMD) launched a program for trading permits in Nitrogen and Sulfur Oxides (NOx and SOx) in the Los Angeles basin. That program was strongly supported by environmental groups and large firms in the LA basin as the most cost effective way of attaining the desired reductions. This program, the REgional CLean Air Incentives Market (RECLAIM), was initialized in October of 1993 and has been operating since early 1994.[1]

The design of the tradable instruments was inevitably a compromise between regulatory interests and market efficiency. Regulators wanted to be able to control the timing and distribution of emissions as finely as possible with caps at many different locations and for many different time periods. To create liquidity, market designers wanted as few instruments as possible - a single aggregate cap would have been their preference. In the end, 136 different types of permits were created that could be used by a company to cover their emissions and, thereby, avoid the costs of abatement. This meant there were both substitutes and complements among the permits. Further, the markets for each of these permits would be illiquid. That created a big problem for the environmental engineers trying to choose between installing expensive abatement equipment or buying a portfolio of permits.

[1]Our description of RECLAIM in this subsection is taken liberally from Cason and Gangadharan (1998). See also Fine (2001), and Carlson, et al. (1993) for more information.

At the request of a number of firms, a new combinatoric market design was built and operated to help then and others deal with the complexities that the RECLAIM program created. In this paper, we provide a description and analysis of that market. There are three parts. In the rest of this section, we provide more of the details of the RECLAIM program and describe the problems that created for environmental engineers. In Section 2, we describe the ACE-RECLAIM market design with special emphasis in Section 3 on the algorithms that drove the market. Finally, in Section 4 we look at the performance of that market.

1.1 The RECLAIM program

RECLAIM is targeted at two major pollutants emitted from stationary sources: Nitrogen Oxides(NOx) and Sulfur Oxides(SOx). All facilities emitting 4 tons or more (per year) of NOx or SOx from permitted equipment were included in RECLAIM. Approximately 390 facilities were in the early NOx market, which collectively represented about 65 percent of the reported NOx emissions from all permitted stationary sources in the Basin. The SOx market consisted of 41 facilities, which represented about 85 percent of the reported SOx emissions from all permitted stationary sources.

Each facility in RECLAIM was allocated a certain number of RECLAIM Trading Credits (henceforth RTCs or simply *permits*) for equipment or processes that emit NOx or SOx. This allocation depended on the peak activity levels for each type of permitted equipment between 1989 and 1992. Each facility received an allocation for each year[2] between 1994 and 2000 based on a straight line rate of reduction calculated from the starting allocation to the allocation in the year 2000. For the years 2001 to 2003, the allocation levels were decreased further. Allocations for each year from 2004 to 2010 were

[2] A permit for 1998, for example, could only be used to cover emissions in 1998.

to be equal to the 2003 allocation unless the AQMD decided that further reductions would be required. Average annual percentage reduction rates for facilities ranged between 7.1 and 8.7 percent in the NOx market and between 4.1 and 9.2 percent in the SOx market (SCAQMD 1993). There is no banking allowed. At this point in the design there were 2x17 = 34 different types of permits to deal with. But more were yet to come.

Based on solid experimental evidence, the SCAQMD realized that this structure of permits would lead to extreme price volatility towards the end of each year. In a good business year, the need for permits by all firms would be high and at the end of the year prices could climb to as high as the fine for emitting too much. In a bad business year, the need for permits by all firms could be low and at the end of the year prices would drop, perhaps as low as zero. This price volatility would wreak havoc with rational planning efforts. To eliminate this problem, the SCAQMD adopted “cycles”. Permits were identified by year and by cycle. There were 2 cycles; one beginning Jan 1 and one beginning July 1. As an example, 1994 cycle 1 permits could only be used to cover pollution emitted from Jan 1 1994 to Dec 31 1994. 1994 cycle 2 permits could only be used to cover pollution emitted from July 1 1994 to June 30 1995. It was shown experimentally that such an overlapping structure of permits would serve to mitigate the extreme price volatility. In the end it was decided to accept a little more complexity in the number of types of permits for the reduced complexity due to excessive volatility. There were now 2x34 = 68 different types of permits. And yet more to come.

Due to regulatory worries about maintaining tight control over the distribution of pollution by the prevailing on-shore winds, the SCAQMD also identified permits by zones. The idea was that upwind zones could sell their permits to downwind zones but not vice versa. The SCAQMD originally wanted 37 zones. But, because the economists warned about serious complexities and thinness in trading, they settled for 2: an inland zone and a coastal zone. Firms located in a coastal zone could only use permits identified as coastal. Inland firms could use either coastal or inland permits. In the end, therefore, for perfectly good regulatory considerations, there were a total of 136 different assets (68 per pollutant) that could be used to cover a firm's pollution: 2 pollutants (NOx and SOx), 2 zones (inland and coastal), 2 cycles, and 17 years (1994-2010).

Put yourself in the position of an environmental engineer dealing with the complexities of the RECLAIM program. A typical exercise would have the engineer deciding between installing abatement equipment or buying permits. Suppose abatement equipment costs \$B to install and has a lifetime of T periods.[3] The equipment abates e_t units of emissions in period t. As an alternative, the engineer can cover the same emissions x by using some collection of the 136 permits. We let a_{kt} be the amount of permit of type k that can be used to cover one unit of emissions in t. For RECLAIM, $a_{kt} \in \{0, 1\}$. Suppose the engineer already has a portfolio of permits, $w \in \Re^K_+$. To avoid installing the abatement equipment she will have to buy a vector of permits $y \in \Re^K_+$ such that $y_k + w_k = \sum_t z_{kt}$ and $\sum_k a_{kt} z_{kt} \geq e_t$ where z_{kt} is the amount of permit k she will use to cover x_t.

If there is an active market for each of the 136 permits and a stable price

[3] A period can be a year, a quarter, a half-year, etc. For purposes of this example, it doesn't really matter.

of p_k for each, she has a fairly easy decision. Let

$$P = \min_{y,z} \sum_k p_k y_k$$

$$\begin{aligned} &\text{subject to} \\ y_k + w_k &= \sum_t z_{kt}, \forall k, \\ \sum_k a_{kt} z_{kt} &\geq x_t, \forall t. \\ y_k + w_k &\geq 0, \forall k \end{aligned} \tag{1}$$

The last constraint eliminates short sales. (1) is a straight-forward linear programming problem which is easy to solve. She installs the abatement equipment if and only if $\$B - pw \leq P$. If $P < \$B - pw$, she buys the vector of permits y^* that solves (1).

But such thick markets never existed. As noted by Cason and and Gangadharan (1998), the program placed minimal restrictions on how permits could be traded. Although the SCAQMD originally planned on putting out contracts for the design and management of a market system, brokers lobbied hard and succeeded in preventing that. Brokers suggested, without much real evidence, that traders could rely on brokers to help bring buyers and sellers together. RECLAIM brokers would thus bear the search costs for potential traders, of course for a fee.

Instead of leaving the market entirely to brokers, however, SCAQMD planners implemented an electronic bulletin board system (BBS) to help RECLAIM participants find trading partners and reduce search costs. The BBS was operated by SCAQMD, and anyone could obtain a password to access their computerized network. The BBS allowed firms to indicate trading interest by electronically posting offers to buy or sell RTCs. Other firms could then scroll through these offers and contact the offering firm to negotiate a transaction. Prices were not posted.

It turned out, however, this was not enough to enable firms to manage their environmental requirements.

1.2 The Environmental Engineer's Problem

She must identify and pursue a sequence of deals from the SCAQMD bulletin board: buying permits of type k without knowing for sure what price she will be able to negotiate for permits of type, say, k'. And she has to make the decision now. A simple toy example illustrates the complications and risks this type of thin market causes.

Example 1 *(Permits as complements: the need for AND bids)*

There are 2 markets: one for A and one for B. Bidder 1 is our engineer who needs to buy both A and B. Bidder 2 is only in market A. Bidders 3 and 4 are only in market B. The amounts they want to buy or sell and their true values for that transaction are in the following table.

Bidder	*$*	*Amount of A*	*Amount of B*
1	*400*	*50*	*50*
2	*- 50*	*- 50*	
3	*- 180*		*- 30*
4	*- 120*		*- 20*

It is important to remember, for this and other examples, that <u>each bidder knows only her own $ value</u> and does not know those of the others.

If Bidder 1 has to participate separately in market A and market B, she has to decide what her separate bids in A and B will be. Suppose she assumes *prices will be similar in the 2 markets and decides to bid 200 for 50 units in each (or 4 per unit). Having done that, suppose she buys her 50 units of A for 3 per unit or 150 total. So far so good. Now she must go into the market for B. She has 2 options. If she is unable to complete a trade for B, resell the A she bought and pay 400 to abate. Suppose she takes a loss from that of L. The cost of this outcome is* $5 + \frac{L}{50}$ *per unit. That is how much she is now willing to bid for B. But 3 and 4 are only willing to sell at a price greater than 6. If* $L > 50$ *she will pay something more than 6 for B. She will not have to abate (the socially desirable outcome) but will have paid more than* 400 *for the permits. This is not a good outcome. If* $L < 50$ *she will sell the 50 A and abate (the socially undesired outcome). She will also take a loss of L and therefore will have paid more than* 400*. This is not a good outcome. So, no matter what she does at this point she loses. Whatever the ultimate price of B, she paid too much for A.*

She could have tried to negotiate harder when she bought A but if seller 2's willingness-to-accept had been 5 and seller 4 and 5 had a willingness-to-accept of 2, this would lead her to not buying A initially. After buying B she could go back to seller 2, but he might have completed another deal with someone else. No matter how she plays it she may end up not buying permits, which is the undesired outcome.

This is certainly a contrived example but imagine trying to string together trades for 15 different assets in 15 different markets. The example illustrates *the exposure problem*[4] that the environmental engineer faces when she must deal in <u>separate</u> markets for each type of permit or engage in a sequence of <u>separate</u>, complex multi-lateral negotiations. A permits and B permits are complements, like left and right shoes. Getting one without the other is no

[4]The exposure problem was identified and discussed at some length in by Bykowsky, Cull and Ledyard in 1995. The published version is Bykowsky et. al. (2000).

help. A buyer who has to deal in separate markets for A and B is *exposed* to the risk of incomplete transactions. To protect for this she may either bid lower than she really would be willing to or she may not participate at all. This destroys potential gains from trade and reduces trade below that which would be optimal.

Example 2 *(Swaps and Endowments: the need for OR bids)*

Continuing Example 1, suppose Bidder 1's company had an initial allocation of permits from the SCAQMD and is willing to commit some of them to this project. Suppose she has 60 of A and 40 of B.[5] *If she is going to use permits instead of abatement equipment, she will have to buy the following stream of permits:* $(-10, 10)$. *As in Example 1, she has a problem in deciding how much to offer in market A and how much to bid in market B. This is a swap she is willing to enter into if and only if* $-10p_A + 10p_b \leq 400 - 60p_A - 40p_b$ *or* $50p_A + 50p_B \leq 400$. *What she would like to do is to enter a deal to "sell 10 A and buy 10 B if* $50p_A + 50p_B \leq 400$*" OR "sell 60 A and 40 B if* $50p_A + 50p_B > 400$*" but not both. But the thin market won't allow such deals. Instead she must negotiate separately for A and B. Suppose when she goes to the A market, the price for A is \$4.50 and, believing that the price of B will be equal or greater than that, she sells her 60 permits. Then suppose that when she gets to the B market she learns the price for B is \$1.50. She will have to complete her sale of the 40 B at that price and install abatement equipment, paying out* $400 - 270 - 60 = 70$. *If, instead, she had only sold 10 A at the price of \$5 and bought 10 B at the price of \$1.50, she would have made \$30. The difference is \$100. And she may be the least cost abater, which makes this bad not only for her but also for society.*

When markets are thick and prices are stable, buyers and sellers with

[5] This is as if A are permits for 1995 and B are permits for 1996 and the firm is given a declining initial allocation.

complex needs are no more at risk than buyers and sellers with simple needs. But when markets are thin and prices are unpredictable, buyers and sellers with complex needs are at risk. Gangadharan (2000) has estimated that these problems were particularly bad in the early years of the RECLAIM program. The program consists of firms with very different industrial structures, which use different technologies to produce very heterogeneous products which means that matching a single buyer with a single seller with coincident wants is extremely unlikely. She found that these problems reduced the probability of trading by about 32%.

Individuals faced with this risk of incomplete trading often turn to brokers who generally claim to be able to find the collection of trades that will satisfy the bidder's price point. That is what happened with RECLAIM. The brokers were more than willing to help, but they charged 40% on each side of a trade, did not publish their prices, and did not let firms know what the alternatives in the market really were. They operated dark markets.[6]

After their experience with brokers, a collection of the firms that had been deeply involved in the design of RECLAIM came to us and asked that we consider creating a marketplace that would help them. Without knowing the language or even the exact concept, what they described as desirable was a *combinatorial exchange*. They wanted a transparent market place into which

[6] A quote from Duffie (2012) shows how pervasive the problem is. "Over-the-counter (OTC) markets for derivatives, collateralized debt obligations, and repurchase agreements played a significant role in the global financial crisis. Rather than being traded through a centralized institution such as a stock exchange, OTC trades are negotiated privately between market participants who may be unaware of prices that are currently available elsewhere in the market. In these relatively opaque markets, investors can be in the dark about the most attractive available terms and who might be offering them. This opaqueness exacerbated the financial crisis, as regulators and market participants were unable to quickly assess the risks and pricing of these instruments." This also happened in the early years of RECLAIM.

they could submit bids that expressed their complex needs and willingness-to-pay and from which they would get fair trades at understandable prices. The rest of the paper is about the market we built for them.

2 ACE-RECLAIM

The market, designed for RECLAIM, was initially operated under the name Automated Credit Exchange (ACE). The design was based on research conducted at Caltech in 1993.[7] The actual internet-based implementation was built by a team from Net Exchange that included Takashi Ishikida, Charles Polk, and Lance Clifner. It utilized an iterated combined-value call market to trade in quarterly trading sessions. We believe that it was the first internet-based commercial exchange. ACE-RECLAIM opened in April of 1995 and, by 1999, it had accounted for approximately 90% of all priced trades in the RECLAIM market.[8]

A functioning market has two main pieces: a market mechanism (the inside piece) and a market process (the outside piece). The inside piece includes the bid forms, the winner determination algorithm and the pricing algorithms. The outside piece includes the participation rules, bidding rules, what is displayed to bidders and when, stopping rules, enforcement rules, etc. We will first describe the outside piece of the ACE-RECLAIM market.

The market flowed as follows:

- Qualification and escrow

  The first step in any market is the qualification of the market participants. Since the SCAQMD rules allowed anyone to hold permits

[7] For a detailed description of the design and experimental test-bedding of the ACE-RECLAIM mechanism, see Ishikida et al (2001).

[8] For more on participation in ACE-RECLAIM, see section 4.1.

and therefore to trade them, anyone was allowed to participate in the market. But combinatorial markets create a problem that separated markets do not. Trades are likely to be multi-lateral and complex. This means that if anyone defaults by not delivering cash or permits at the end of the process, then many winners may find their trades invalidated. To avoid this unfortunate outcome, ACE-RECLAIM required any participant to provide in escrow, before the market opened, the cash they would be willing to spend and the permits they were willing to sell. ACE-RECLAIM then used these data to provide constraints on the bids the bidders were allowed to make. In particular, no bidder could bid in a way that would violate the cash and permit bounds, no matter what bids were accepted. This removed the possibility of default.

- Bid submission

  Bids were submitted in two ways: either online, through a custom Windows bidding interface on a computer connected via a modem to the ACE-RECLAIM computer, or via fax to the market manager who would then input the bids herself.

- Provisional winners, prices, and payments.

  After bids are submitted, the ACE-RECLAIM computer calculated provisional winners, prices, and payments using the market mechanism described in section 3.

- Stopping Rule, Resubmissions, and the Improvement Rule

  ACE-RECLAIM was run as an iterative process. It proceeded in rounds. At the end of each round (once provisional winners, prices, and payments were announced) a stopping rule is calculated. The stopping rule for ACE-RECLAIM was very simple: (a) there must be at least

3 rounds, (b) after round 2, if surplus and volume do not increase by at least 5% from the previous round, the market will end, and (c) the market will end after round 5 if (b) has not taken place. If the rule is satisfied, the process goes to Clearance and Settlement to finish up.

If the rule is not satisfied, the process returns to allow more bids. There are two features of ACE-RECLAIM that come into play at this point. All winning bids must be re-submitted as they are. They can then be improved on. Improvement can happen in several ways. Without going into too much detail, improvement usually means (a) for a bid to buy, a higher willingness to pay is entered or (b) for a bid to sell, a lower willingness to accept is entered.

Remark 1 *ACE can also be run as a one-shot sealed bid or a continuous mechanism. Each such version can be used with a variety of stopping rules, e.g. based on eligibility and activity similar to the FCC's SMR rules or based on the increase in surplus over the last round (e.g. stop if less than 5%).*

- Sunshine

  An important aspect of combinatorial trading is that often bids of several individuals have to fit together, like pieces of a jigsaw puzzle, in order for them to trade. Thus it is often important to know what others are willing to trade or to show others what you are willing to trade. ACE-RECLAIM allowed individual bidders to choose how much of their own bids would be made public. They could choose (a) nothing, (b) quantities of permits only, or (c) quantities of permits and dollar amounts bid. This was called, respectively, (a) no sunshine, (b) partial sunshine, and (c) sunshine. ACE-RECLAIM is an iterative process and many bidders chose (a) in early rounds and (b) or (c) in later rounds as they became more desperate to find a match of some kind.

- Clearance and Settlement

  Once the stopping rule comes into play, ACE-RECLAIM takes the results of the last round and implements these: sending orders to the RECLAIM database for the appropriate reallocation of permits and sending orders to the escrow holder for the appropriate reallocation of cash between accounts.

3 ACE

ACE is the market mechanism (the inside piece) for ACE-RECLAIM. ACE is an acronym for Approximate Competitive Equilibrium, the philosophy behind the market mechanism design. It is a generalization of the well-know Uniform Price Double Auction (UPDA) which produces high efficiencies in simple markets.[9] UPDA is usually run like a sealed bid auction. Subjects submit bids (P, Q). If $Q > 0$, it is a buy order to be read as "I will buy up to Q units of the good for any price less than or equal to P". If $Q < 0$ then it is a sell order to be read as "I am willing to sell up to Q units of the good for any price greater than or equal to P". After all bids are submitted, UPDA picks winners by choosing that set of trades that would maximize the reported surplus. The price at which all transactions take place is the midpoint between the marginal units (both accepted and rejected). All winning buyers pay less than (sellers receive more than) or equal to their bid. It is the same price for all, there is no subsidy or tax for the market, and there is a strong incentive to reveal one's true willingness-to-pay (or accept) except at margin. Since the probability that any bid is marginal can be very low, this gives ACE a serious shot at being virtually incentive compatible.

[9]See Smith et.al. (1982), Friedman (1993), and McCabe et. al. (1993).

Since the ACE market mechanism is to operate in a world that is more complex than UPDA was designed for, the bid structure, winners determination, and price determination pieces all had to be modified. The mechanism allows bidders to submit contingent orders. The winners determination algorithm then maximizes reported surplus, as in UPDA. The pricing algorithm approximates the UPDA price rule but still leaves bidders as well off after as if they had not participated.

3.1 Bids

The mechanism allows bidders to submit contingent orders. These can be ANDs (I want A if and only if I can also secure B) or ORs (I want either A or B).

Participants submit bids that are numbered $i = 1, ..., N$.

3.1.1 AND Bids

The basic bid of ACE is an AND bid. It allows a bidder to express interest in a collection of complements, all of which are needed. An AND bid is intended to protect the bidder from exposure in situations like Example 1.

Definition 1 *A simple AND bid numbered i is (b^i, x^i, F^i) where $b^i \in \Re, x^i \in \Re^K$, and $F^i \in [0, 1]$. K is the number of commodities.*

The bid is read as " I will pay up to $b^i f^i$ for the vector $x^i f^i$ as long as $F^i \leq f^i \leq 1$".

The vector $x^i \in \Re^K$ contains the quantities offered or demanded, where K is the number of commodities. If $x^i_k > 0$, then x^i_k units are demanded. If $x^i_k < 0$, then x^i_k units are offered for sale. It is not required that all x^i_k

be positive or negative, i.e. swaps are allowed. The number $b^i \in \Re$ is the maximum amount the bidder is willing to pay for x^i. ($|b^i|$ is the minimum amount a seller is willing to sell for if $b^i < 0$.) The number $F^i \in [0, 1]$ is the minimum fill number.

When $F^i = 0$, the bid is similar to a (P, Q) bid in a uniform price auction where bidders are willing to accept any amount $q \leq Q$ at a payment $t \leq Pq$. $F^i > 0$, indicates a minimum fill requirement which introduces non-convexities and discontinuities into the design problem. This is the most challenging part of the ACE design. When $F^i > 0$, the simple UPDA approach won't work, particularly the pricing.

Example 3 *(Minimum Fill required #1)*

Consider a market with a single commodity labeled A, with 1 buyer and 3 sellers. Buyer 1 is willing to pay 20 for 3 units but does not want to pay anything if he gets less than 3 units. That is, he requires his order be filled at the 100% level or not at all.

Bidder	*$*	*Amount of A*	*% fill required*
1	*24*	*3*	*100*
2	*- 2*	*- 1*	*0*
3	*- 4*	*-1*	*0*
4	*- 10*	*-1*	*0*

Gains from trade are maximized if all 4 participate in the deal. Even though bidder 4 requires $10 per unit and bidder 1 is only willing to pay $8 per unit, bidder 4 makes the deal possible for bidders 2 and 3 and should be included.

The minimum fill requirement creates several problems here. First, if 1 bids $8 per unit, 4 will never trade and 1 will not get the 3 units he wants. Second, if 1 agrees to pay 4 any amount greater than $10, both 2 and 3 would want the same deal. But then 1 cannot afford to pay every one at the rate of

\$10/unit. Often this configuration of values and desires will lead to no trade. The \$8 in potential gains from trade is lost.

A standard thin market has difficulty dealing with minimum fill requirements. In the above example, the standard market makes it difficult to include Bid 4 in the final allocation even though it should be. In our next example, the standard market makes it difficult to exclude someone from the final allocation even though they should be.

Example 4 *(Minimum Fill required #2)*

In this example, there is an additional buyer and Seller 4 has a lower reservation price.

Bidder	*\$*	*Amount of A*	*% fill required*
1	*24*	*3*	*100*
2	*- 2*	*- 1*	*0*
3	*- 4*	*-1*	*0*
4	*- 6*	*-1*	*0*
5	*10*	*1*	*0*

Here the allocation that maximizes the gains from trade would include 1,2,3, and 4 and exclude 5. The price would probably be somewhere between 6 and 8. But 5 is more than willing to pay upto \$10 and so might work hard to be included in the allocation. In fact, at that proposed price, 5 could offer 2 a deal. 5 can offer to buy 2's unit for \$9. There is no price at which 5 is not willing to buy and 1 is willing to buy. Often this configuration of values and desires will lead to a single trade between 1 and 5 yielding a surplus of \$8. Whereas the maximum surplus possible is \$12. Potential gains from trade are lost.

3.1.2 OR bids

An OR bid allows a bidder to express interest in a collection of substitutable possibilities. An OR bid is intended to protect a bidder from exposure in situations like Example 2.

Definition 2 *An OR bid numbered i is* $\{(b^{ij}, x^{ij}, F^{ij})\}_{j=1}^{J_i}$.

The bid is read as "I will accept one and only one $j \in J_i$ in which case I will pay up to $b^{ij} f^{ij}$ as long as $f^{ij} \in [0, 1]$." If $J_i = 1$, then this is exactly the same as the AND in section 3.1.1.

Each (b^{ij}, x^{ij}, F^{ij}) is a simple AND bid. An OR bid requires that, at most, only one of these be accepted. That means there are $\delta^{ij}, \forall j \in J_i$ such that $f^{ij} \in [\delta^{ij} F^{ij}, \delta^{ij}], \sum_j \delta^{ij} \leq 1$, and $\delta^{ij} \in \{0, 1\}$.

3.1.3 Characteristic Bids

A special OR was designed for ACE-RECLAIM to give environmental engineers a user-friendly way of expressing interest and value over a wide range of substitutable possibilities. Here we provide a slightly more general version. As we described in Section 1.2, the environmental engineer has a vector of permits $w \in \Re^K_+$ and wants to cover a stream of pollution $e \in \Re^T_+$. The engineer needs to buy a vector of assets x such that $x_k + w_k = \sum_t z_{kt}, \forall k$, and $\sum_k a_{kt} z_{kt} \geq e_t, \forall t$. And, if they want to avoid short sales, they would add the constraint that $x^i + w^i \geq 0$.

The characteristic bid generalizes the OR bid to allow an infinite variety of possible satisfactory trades to be considered without exposing the bidder to the risk of having more than one of those options active at a time.

Definition 3 *A characteristic bid numbered i is* $(b^i, A^i, e^i, w^i, F^i)$.

A characteristic bid is to be read as "I will accept one trade x^i as long as $A^i_t z_t \geq f^i e^i_t, x^i + w^i = \sum_t z_t, x^i + w^i \geq 0$, and $f^i \in [F^i, 1]$ in which case I will pay up to f^ib^i for it.

If $F^i = 1$, then the characteristic bid lets the market know that the engineer will buy any combination of assets x as long (i) as she can add them to her endowment w in a way that covers e and (ii) it costs her no more than her abatement costs, \$B. She leaves it to the market to decide what works.

If $F^i = 0$, then the engineer is also allowing the market to consider buying her endowment w as long as she is paid at least \$0. If there are prices p then I would want to agree to that trade if and only if $(0, -w^i)$ solved $\max_{(f^i,x^i)} f^ib^i - px^i$ subject to $A^i_t z_t \geq f^i e^i_t, x^i + w^i = \sum_t z_t, x^i + w^i \geq 0$, and $f^i \in [0, 1]$. In particular, letting $f^i = 1$, this means that she would agree to the trade if and only if $pw \geq \$B - P$ where P solves (1). As we will see below, the ACE market mechanism algorithms are designed with exactly this in mind.

Remark 2 *A characteristic bid is a generalization of a simple bid. Suppose* (b, x, F) *is a simple bid. Consider the characteristic bid* $(B, A, e, w, G) := (b, I, x, 0, F)$*. This bid agrees to pay up to* fb *for any* y *such that* $y \geq fx$*. Since* $y = fx$ *is the dominant possibility this is just another form of a simple bid.*

3.2 Winners Determination

Once all bids are submitted, a winners determination algorithm determines what trades will be matched and implemented. For ACE, winners are determined by maximizing the "reported surplus" of the trades subject to the restrictions imposed by the bidders and subject to no excess demand.

Definition 4 *(Winners Determination)*

Let $\mathcal{S} = \{$ *AND bids*$\}$*. Let* $\mathcal{O} = \{$ *OR bids*$\}$*. Let* $\mathcal{Z} = \{$ *characteristic bids*$\}$*. The* winner determination problem *is:*

$$
\begin{aligned}
S &= \max_{(f,y)} \sum_{i \in \mathcal{S} \cup \mathcal{Z}} b^i f^i + \sum_{i \in \mathcal{O}} \sum_{j \in J_i} b^{ij} f^{ij} && (2)\\
&\quad \textit{subject to} \\
&\quad f^i \in \{0\} \cup [F^i, 1] \ \textit{for all}\ i \in \mathcal{S} \cup \mathcal{Z} && (3)\\
&\quad A_t^i z_t^i \geq f^i e_t^i, x^i + w^i = \sum_t z_t^i, x^i + w^i \geq 0,\ \textit{for all}\ i \in \mathcal{Z} && (4)\\
&\quad f^{ij} \in [\delta^{ij} F^{ij}, \delta^{ij}], \delta^{ij} \in \{0,1\}, \sum_{j \in J_i} \delta^{ij} \leq 1\ \textit{all}\ j \in J_i,\ \textit{all}\ i \in \mathcal{O} && (5)\\
&\quad \sum_{i \in \mathcal{S}} x_k^i f^i + \sum_{i \in \mathcal{Z}} x_k^i + \sum_{i \in \mathcal{O}} \sum_{j \in J_i} x_k^{ij} f^{ij} \leq 0, k \in K. && (6)
\end{aligned}
$$

(3) is the minimum fill requirement for all bids except ORs. (4) is the restriction placed by characteristic bids. (5) is the restriction placed by OR bids. (6) is the requirement that there be no excess demand. In the RECLAIM ACE market, if $\sum x^i f^i < 0$ at the optimum, the transactions were made and the un-transacted credits were retired. That is, free disposal was possible for the market. We retain that feature here.

Remark 3 *For ease of computation, the algorithm actually split the market into disjoint segments (no overlapping bids). This also aided the price computations which we pick up in the next section.*

Remark 4 *Maximizing surplus is not the only rule one might use. For example, if one is interested in extracting revenue from the mechanism, then maximizing surplus is rarely the best strategy. But for now we stay with surplus maximization.*

We will need to identify winners and losers as we proceed.

Definition 5 *Let* (f^*, y^*) *be the values that solve (2) and let* $f^{*i} := \sum_{j \in J_i} f^{*ij}$ *for each* $i \in \mathcal{O}$*. The set of* winners *is denoted by* $W = \{i | f^{*i} > 0\}$*. The set of* losers *is denoted by* $L = \{i | f^{*i} = 0\}$. *Those winners for whom* $0 < f^{*i} < 1$ *will be referred to as* marginal winners.

3.3 Pricing

ACE pricing is about (a) providing useful signals to the market that reflect aggregate demand and supply, (b) not extracting any revenue, (c) achieving some measure of incentive compatibility, and (d) not rewarding inflexible bidders (i.e. those for whom allowing $f^i = 0$ in (2) would increase the surplus). Ideally, all of this can be accomplished if we can find competitive equilibrium prices that support the allocation found by the winners determination problem.

3.3.1 Competitive Equilibrium Prices

Competitive equilibrium allocations and prices satisfy two conditions: (i) each individual's allocation is just what they would want to buy at those prices and (ii) there is no excess demand. For ACE, the bids are the individuals. We want prices that support the winners determination allocation as a competitive equilibrium. They satisfy *ex-post self-selection*, or incentive compatibility, for the bidders.[10]

For simple bids, competitive equilibrium prices π would satisfy $b^i - \pi x^i \geq 0$ if $i \in W$ and $b^i - \pi x^i \leq 0$ if $i \in L$. For other bids, things are a little more complex.

For OR bids, let $f^{*ij} > 0$ in the winners determination solution; that is, j is the winning part of the OR. $f^{*ik} = 0$, for all $k \neq j$. Prices π would be regret-free if (a) $b^{ij} - \pi x^{ij} \geq 0$ and $b^{ij} - \pi x^{ij} = 0$ if $0 < f^{*ij} < 1$ and (b) $b^{ik} - \pi x^{ik} \leq b^{ij} f^{*ij} - \pi x^{ij} f^{*ij}$ for all $k \neq j$. Note that if ij is a winner with

[10] These are also sometimes called *no arbitrage* or *no re-contracting* constraints.

$f^{*ij} = 1$, this does not require ik to be a loser (i.e. $b^{ik} - \pi x^{ik} < 0$) only that it not be as good as ij at those prices. It does imply that if $\sum_{j \in J_i} f^{*ij} = 0$, then all the J_i bids should be losers at the prices π.

For characteristic bids, let the solution to the winners determination problem (2) be (f^{*i}, x^{*i}). Prices would be regret-free if $b^i f^{*i} - \pi x^{*i} \geq b^i f^i - \pi x^i$ for all x^i such that $A_t^i z_t \geq f^i e_t^i, x^i + w^i = \sum_t z_t, x^i + w^i \geq 0, f^i \in [F^i, 1]$.

Remark 5 *For RECLAIM Zone free bids, the restriction to regret-free prices was implemented with the constraint that the inland price be less than or equal to the coastal price. Because inland credits could not be used to cover coastal emissions, it imposed a simple requirement on the prices of the two zones no matter what year-cycle is involved. Only the inland firms care about the relative prices. If the coastal price is less than the inland price, inland firms would want to only buy coastal credits, thus driving the price up. If the inland price is less than the coastal price, the coastal firms could not drive the inland price up. No-regret prices are also arbitrage-free prices.*

The no-arbitrage conditions are:

- *If there are* k, k', l, i *such that* $a_{kl}^i a_{k'l}^i > 0, \overline{X}_k^i > 0,$ *and* $\overline{X}_{k'}^i > 0,$ *then* $p_k^b = p_{k'}^b$.
- *If there are* k, k', l, i *such that* $a_{kl}^i a_{k'l}^i > 0, \overline{X}_k^i > 0,$ *and* $\overline{X}_{k'}^i = 0,$ *then* $p_k^b \leq p_{k'}^b$.
- *If there are* k, k', l, i *such that* $a_{kl}^i a_{k'l}^i > 0, \overline{X}_k^i < 0,$ *and* $\overline{X}_{k'}^i < 0,$ *then* $p_k^s = p_{k'}^s$.
- *If there are* k, k', l, i *such that* $a_{kl}^i a_{k'l}^i > 0, \overline{X}_k^i < 0,$ *and* $\overline{X}_{k'}^i = 0,$ *then* $p_k^s \geq p_{k'}^s$.

If the p^b and p^s did not satisfy these then $\overline{X}^i$ would not be a cost-minimizing coverage of $\overline{e}^i$ at those prices.

Definition 6 *(competitive equilibrium prices)*

Let (f^*, x^*, δ^*) be the values of f that solve the winner determination problem (2).[11] *Competitive equilibrium prices, π, satisfy the following:*

$$f^{*i}(b^i - \pi \cdot x^i) \geq f^i(b^i - \pi \cdot x^i) \text{ for all } f^i \in [F^i, 1], i \in \mathcal{S} \tag{7}$$

$$\sum_{j \in J_i} f^{*ij}(b^{ij} - \pi x^{ij}) \geq \sum_{j \in J_i} f^{ij}(b^{ij} - \pi x^{ij}) \text{ for all } f^i, \delta^i \text{ such that} \tag{8}$$
$$\sum_{j \in J_i} \delta^{ij} \leq 1, f^{ij} \in [\delta^{ij} F^{ij}, \delta^{ij}], \delta^{ij} \in \{0,1\},\ i \in \mathcal{O}$$

$$f^{*i} b^i - \pi x^{*i} \geq f^i b^i - \pi x^i \text{ for all } (f^i, x^i) \text{ such that} \tag{9}$$
$$f^i \in [F^i, 1], A^i_t z^i_t \geq f^i e^i_t, x^i + w^i = \sum_t z^i_t, x^i + w^i \geq 0, \text{ for all } i, \quad i \in \mathcal{Z},$$

$$\pi \cdot [\sum_{i \in \mathcal{S}} x^i f^{*i} + \sum_{i \in \mathcal{Z}} x^{*i} + \sum_{i \in \mathcal{O}} \sum_{j \in J_i} x^{ij} f^{*ij}] = 0. \tag{10}$$

$$\pi_k \geq 0 \text{ for all } k. \tag{11}$$

(7) - (9) are the regret-free conditions on bidders. They require that *ex post* each bid is allocated in a way that maximizes the bid's surplus at those prices subject to the bid's restrictions. (10) is Walras Law which requires that $\pi_k = 0$ when there is an excess supply of k. These conditions, along with *no excess demand* (6) from the winners determination problem, imply that the winners determination allocation and the prices π are a competitive equilibrium for the economy described by the bids.

There are two possible problems at this point. Competitive equilibrium prices may not exist. And, even if they do exist, they may not be unique.

[11] x^* is relevant for characteristic bids. δ^* is relevant for OR bids. f^* is relevant for all types of bids.

Non-uniqueness: If (2) is convex, prices satisfying (7) -(11) exist.[12] The dual variables of the linear-programming problem will serve as these prices. But they may not be unique. There are 2 reasons.

One, in a thin market when a package matches another package, the range of prices satisfying no-regret can be wide. Consider the following example.

Example 5 *(Opposing swaps)*

There are 2 AND bids.

Bid	*$*	*Amount of A*	*Amount of B*
1	*3*	*1*	*-1*
2	*-3*	*-1*	*1*

Bid 1 indicates, for example, that some one is willing to pay 3 to swap 1 unit of B for 1 unit of A. In this case, p_A and p_B are not separately identified and all we can conclude is that $p_A - p_B = 3$. This is not a problem, since no bidder cares which particular price vector is selected from those.

Two, the bounds in definition 6 may not be tight. This occurs for example if there is no marginal winner.

Example 6 *(Single asset - No marginal winner)*

Bid	*$*	*Amounts*	*$/unit*
1	*500*	*500*	*1*
2	*-400*	*-500*	*0.8*

For this example, any price between $0.80 and $1 will be a competitive equilibrium price. This is a problem, because the price selected determines the distribution of the surplus among buyers and sellers.

[12]Since (2) maximizes surplus, and "preferences" are quasi-linear, the winners allocation is Pareto-optimal. We can therefore apply the second welfare theorem to establish the existence of prices supporting that allocation.

To deal with non-uniqueness in prices, ACE chooses the prices that maximize the winning bidders' minimum per-unit surplus. This is the intuitive equivalent to Myerson-Satterthwaite's (1983) k-double auction when $k = 1/2$. It equalizes the per-unit surplus between buyers and sellers on the margin. In example 6, ACE will set the price to \$0.9.

Non-existence: Non-existence is way more serious than non-uniqueness. The prime cause of the non-existence of equilibrium prices is inflexibility due to a minimum fill requirement. In this section, we show how the inflexibility causes the non-existence through examples and discuss how it is resolved. For simplicity, the examples are set in a single asset market.

Example 7 *(Excess supply at winners determination)*

In this example, it is possible to find a regret-free price at the winners determination allocation but payments will not balance.

Bid	*\$*	*Amount*	*Min. scale(%)*	*\$/unit*
1	*2500*	*2000*	*0*	*1.25*
2	*500*	*500*	*0*	*1.00*
3	*-1500*	*-3000*	*100*	*0.50*

If all orders were flexible, a competitive equilibrium would exist at the price of \$0.5 per unit and 500 units of order 3 would not trade. Because of the sell order's inflexibility, the market must absorbs 500 units of excess supply. But, because of this imbalance between the amounts bought and the amounts sold, no single price will allow balance of all payments and charges. With inflexibility someone must pay for this extra 500 units, or no trade will occur.

In Example 7, if the extra 500 units were not taken by the market maker, no trade would occur. So it is not unreasonable to have Bids 1 and 2 pay for it. A natural way to do that, while preserving some measure of incentive compatibility, is to charge buyers a higher price than is paid to sellers. To implement that, we would charge each buy unit at \$0.8181 and pay each sell unit at \$0.68182. But this rewards the inflexibility of Bid 3. If they were perfectly flexible, they would gain a surplus of 0 and get only 2500 units. If they are inflexible and we used this pricing scheme, they would get $3000 \cdot \$0.18182 = \$545.476$. An alternative would be to pay B3 at what she bid, \$0.50/unit, and charge the buyers \$1.20. But this punishes not only the inflexible part of Bid 1 but also the flexible part. A better alternative is to pay the inflexible part (500 units) at \$0.50 and then split the prices in a way that equalizes the per-unit surplus at the margin. In Example 7, this would mean each buyer would be charged \$0.80 and the seller would be paid \$0.70 for the 2500 flexible units and \$0.50 for the 500 inflexible units. ACE does something similar to this.

Example 8 *(Excess supply from AND bid)*

In this example, we illustrate another way in which a single price can be found to satisfy no-regret but will not balance payments. There are no minimum fill requirements here.

Bid	*\$*	*Amount of A*	*Amount of B*
1	*- 100*	*50*	*- 30*
2	*300*	*25*	*10*
3	*-100*	*- 75*	
4	*100*		*10*

All these bids will win in the winners determination problem. But there will be an excess supply of 10 units of of B. It is easy to find prices that satisfy no-regret for all bids. For example, $p_a = 2, p_b = 9$ will do the job. But because there is an excess supply, the market will absorb the 10 units and, as *in example 7, someone will have to pay the* $\$9 \cdot 10 = \$90$ *to Bid 1.*

ACE will deal with this by splitting the buy and sell prices and then choosing them so as to equalize the minimum per unit surplus across orders.

Example 9 *(Incompatible surplus requirements)*

In this example it is not even possible to find a regret-free price at the winners determination allocation.

Order	*$*	*Amount*	*Min. scale(%)*	*$/unit*
1	*2000*	*2000*	*100*	*1.00*
2	*-425*	*-500*	*0*	*0.85*
3	*-980*	*-1000*	*0*	*0.98*
4	*-525*	*-500*	*0*	*1.05*

Sell order 4 is allocated because buy order 1 is an all-or-none order and there is enough surplus from the part of trade made among orders 1,2, and 3 to compensate for the surplus loss resulting from matching a part of order 1's demand and sell order 4. Without order 4 no trades will occur so 4 is included in the winners determination allocation.

Since sell order 4 asks more per unit than buy order 1 bids per unit, no single price can satisfy no-regret for both 4 and 1. For allocations in these examples, individual prices will have to be charged to some bids in order to satisfy no-regret. One option is to pay 4 what they bid, $525, and then split the buy and sell prices for the others to pay that. Since Order 1 is inflexible, they would pay $1 per unit (as in example 7 we don't want to reward inflexibility) while Orders 2 and 3 would be paid $0.9833 per unit.

3.3.2 The ACE pricing algorithm

As we found out in the previous section, allowing bids that are able to express bidders' preferences opens up the possibility that competitive equilibrium prices will not exist. So one must settle for something less. In this section, we describe how ACE handles this.

We want prices that (a) provide useful signals to the market that reflect aggregate demand and supply, (b) do not extract or inject any revenue into the process, (c) achieve some measure of incentive compatibility, and (d) do not reward inflexible bidders. ACE accomplishes (a) by finding prices that are "as close as possible" to competitive equilibrium prices, accomplishes (b) by requiring payments and receipts to add up to zero, accomplishes (c) by ignoring losing bids[13] and choosing prices between all marginal winning bids (a double auction approach), and accomplishes (d) by paying inflexible bids at exactly what they bid.

ACE first identifies those inflexible bids or parts of those bids that prevent regret free prices from existing, as for example Bid 4 in Example 9. Those inflexible bids are set aside and will be paid or pay at what they bid. There is now a price that is regret-free for the remaining bids. Unfortunately, payments may not balance at that price, as happens in Example 7. So ACE then splits the prices into a price vector for buyers, p^b, and a price vector for sellers, p^s, and searches for a pair that (i) satisfy no-regret for the remaining bids (or parts of bids), (ii) maximize the minimum per unit surplus for all

[13]In a thick market, losing bids can be used to drive incentive compatibility by leading winning bids to reveal their true values. In thin markets, losing bids can be used to manipulate prices and detract from incentive compatibility. Since ACE is designed for thin markets (it is not needed in thick markets), ACE ignores the losing bids from the winners determination problem (2) when determining prices. ACE ignores both losing simple bids (those with $f^i = 0$) and the losing parts of OR bids (those with $f^{ij} = 0$).

of the remaining bids, and (iii) balance payments and receipts of all bids (including those set aside).

Step 1: *Ignore losing bids and the losing parts of OR bids.*

Begin with the allocation (f^*, y^*) from the winners determination problem (2). Let $\overline{I} \subset I$ be the set of allocated orders, (those $i \in \mathcal{S} \cup \mathcal{Z}$ with $f^{*i} > 0$ and those $i \in \mathcal{O}$ with $f^{*ij} > 0$). Let $\overline{\mathcal{S}} = \mathcal{S} \bigcap \overline{I}$, $\overline{\mathcal{Z}} = \mathcal{Z} \bigcap \overline{I}$, let $\overline{\mathcal{O}} = \mathcal{O} \bigcap \overline{I}$. For $i \in \mathcal{S} \cup \mathcal{Z}, \overline{B}^i = f^{*i} b^i$. For $i \in \mathcal{O}, \overline{B}^i = b^i \sum_{j \in J_i} f^{*ij} b^{ij}$. For $i \in \mathcal{S}, \overline{X}^i = f^{*i} x^i$. For $i \in \mathcal{O}, \overline{X}^i = \sum_{j \in J_i} f^{*ij} x^{ij}$. For $i \in \mathcal{Z}, \overline{X}^i = y^{*i}$, and $\overline{e}^i = f^{*i} e^i$.

Step 2: *Determine which units are extra-marginal.*

To determine those parts of the winning bids that are inflexible, ACE solves the following *fully flexible winners determination* problem.

$$\begin{array}{cc}
\max\limits_{(g,x)} & \sum_{i \in \overline{I}} \overline{B}^i g^i \\
\text{subject to} & |x^i| \le |\overline{X}^i|, i \in \overline{\mathcal{Z}} \\
& A_t^i z_t^i \ge g^i \overline{e}^i, x^i + w^i = \sum_t z_t^i, x^i + w^i \ge 0, i \in \overline{\mathcal{O}} \\
& \sum_{i \in \overline{\mathcal{S}} \cup \overline{\mathcal{O}}} g^i \overline{X}_k^i + \sum_{i \in \overline{\mathcal{Z}}} x_k^i \le 0, k \in K \\
& 0 \le g^i \le 1, i \in \overline{I}
\end{array} \tag{12}$$

Let $(\overline{g}, \overline{y})$ be the solution of (12). The inflexible part of bid i is $(1 - \overline{g}^i)\overline{X}^i$ for $i \in \overline{\mathcal{S}} \cup \overline{\mathcal{O}}$, and is $\overline{X}^i - \overline{y}^i$ for $i \in \overline{\mathcal{Z}}$.[14] ACE sets this part of these orders aside. They will be filled and they will pay $(1 - \overline{g}^i)\overline{B}^i$. Prices on

[14]A characteristic bid may not be itself inflexible but a part of it may be matching an inflexible part of another bid.

the rest of the orders, called the fully flexible orders, will have to "pay" for $D = \sum_{i \in \overline{I}} (1 - \overline{g}^i)\overline{B}^i$. This usually requires that prices be split into buy and sell prices.

Step 3: *Find a vector of buy prices and a vector of sell prices that is regret-free for as many of the fully flexible orders as possible, that balances all payments, and maximizes the minimum surplus per order.*

Let $\overline{G} := \{i \in \overline{I} | \overline{g}^i > 0\}$ and $D := \sum_{i \in \overline{I}} (1 - \overline{g}^i)\overline{B}^i$. $\overline{G}$ is the set of bids with a fully-flexible component. D is the contribution to the total payment from the units paying at their bid/ask prices.

For any vector y, let $y^+ := (\max\{0, y_1\}, \max\{0, y_2\}, ..., \max\{0, y_K\})$ and $y^- := (\min\{0, y_1\}, \min\{0, y_2\}, ..., \min\{0, y_K\})$. $\overline{X}_k^{i+}$ is the amount of asset k potentially bought at the market (buy) price, and $|\overline{X}_k^{i-}|$ is the amount of asset k potentially sold at the market (sell) price.

Solve the following pricing problem.

$$\max_{(m, p^b, p^s)} m \tag{13}$$

$$\text{subject to}$$

$$p^b \overline{X}^{i+} + p^s \overline{X}^{i-} + m^i \sum_{k \in K} |\overline{X}_k^i| = \overline{B}^i, i \in \overline{G} \tag{14}$$

$$\begin{aligned} p^b \overline{g}^i \overline{X}^{i+} + p^s \overline{g}^i \overline{X}^{i-} &\leq p^b y^{i+} + p^s y^{i-}, \text{ for all } y^i \text{ such that} \\ A_t^i z_t^i &\geq g^i \overline{e}_t^i, y^i + w^i = \sum_t z_t^i \geq 0, i \in \overline{G} \cap Z \end{aligned} \tag{15}$$

$$\sum_{i \in \overline{G} \setminus (\overline{G} \cap Z)} (p^b \overline{g}^i \overline{X}^{i+} + p^s \overline{g}^i \overline{X}^{i-}) + \sum_{i \in \overline{G} \cap Z} (p^b x^{i+} + p^s x^{i-}) + D = 0, \tag{16}$$

$$m^i \geq m, i \in \overline{G}$$

$$p_k^b \geq p_k^s \geq 0, k \in K$$

In (14), m^i is the per-unit surplus that this bid will receive if the prices are (p^b, p^s). To be precise, each of the terms in (14) should be multiplied by $\overline{g}^i$, but, since they all just cancel, we are leaving them out. The regret-free condition for all bids is contained in (14) when $m^i \geq 0$. (15) is the regret-free condition for $i \in Z$. It should be noted that the complete statement is $\overline{g}^i\overline{B}^i - \overline{g}^i(p^b, p^s) \cdot (\overline{X}^{i+}, \overline{X}^{i-}) \geq \overline{g}^i\overline{B}^i - (p^b, p^s) \cdot (y^{i+}, y^{i-})$. But the $\overline{g}^i\overline{B}^i$ cancels out. Finally, (16) is the requirement that all payments and receipts balance.

Remark 6 *Note that (15), as well as (19) below, really involves an infinite number of constraints that define a convex set. As pointed out in Remark 5, for the RECLAIM implementation we were able to convert (15) to a finite set of constraints on the prices. Each application requires its own conversion. We do not have a general approach to accomplishing that.*

If there is no inflexibility, then $D = 0$, and $p^b = p^s$ in the solution to (13). That is, a competitive equilibrium price will be found.

Step 4: *Check to see if appropriate prices have been found. If not, set aside more bids and repeat.*

A) If the value of the objective function of (13) is nonnegative, ACE is done with this phase. ACE then enters into a clearance and settlement phase, determining what is allocated to each bid and how much they pay or are paid. We describe that process in Section 3.3.3.

B) If the value of the objective function of (13) is negative, then an appropriate market price system does not exist because there is not enough surplus at the margin to pay for the inflexible parts of the winning bids.[15]

[15] This would be the case, for example, if Bid 2 in Example 7 were \$275 for 500 units or \$0.55 per unit.

In this case, ACE chooses some more units/orders to set aside and to pay at their bid/ask prices and then recomputes. This iterates until ACE finds an appropriate market price system for those bids that have not been set aside.

After the t-th iteration, there are two sets of orders. G_t is the set of the orders which have been set aside and which will pay at their bid. An order in G_t is (B^i, Y^i). H_t is the set of orders, or parts of orders, which have not yet been set aside. There are two types of orders in H_t: simple orders and cycle-zone free orders (B^i, Y^i) and characteristic orders (B^i, A^i, E^i, w^i). H_0 is set to be $\overline{G}$.

At the $t+1$-st iteration, we solve

$$\max_{(m,p^b,p^s)} \; m$$

$$\text{subject to}$$

$$p^b Y^{i+} + p^s Y^{i-} + m^i \sum_{k \in K} |Y^i_k| = B^i, i \in H_t \tag{17}$$

$$p^b Y^{i+} + p^s Y^{i-} = B^i, i \in G_t \tag{18}$$

$$p^b Y^{i+} + p^s Y^{i-} \geq p^b y^{i+} + p^s y^{i-}, \text{ for all } y^i \text{ such that}$$

$$A^i_t z^i_t \geq g^i \overline{e}^i_t, y^i + w^i = \sum_t z^i_t \geq 0, i \in H_t \cap Z \tag{19}$$

$$\sum_{i \in H_t \setminus (H_t \cap Z)} (p^b Y^{i+} + p^s Y^{i-}) + \sum_{i \in H_t \cap Z} (p^b y^{i+} + p^s y^{i-}) + \sum_{i \in G_t} B^i = 0, \tag{20}$$

$$m^i \geq m, i \in H_t$$

$$p^b_k \geq p^s_k \geq 0, k \in K$$

If the value of the objective function is nonnegative, then ACE is done with the pricing computation and goes to clearance and settlement (see Section 3.3.3). If the value of the objective function is negative, then the orders that attain the minimum per unit surplus among the orders in H_t are now removed from H_t and added to G_t to get H_{t+1} and G_{t+1}.[16]

If H_{t+1} is empty, the surplus distribution is determined and ACE goes to clearance and settlement.[17] Otherwise ACE repeats Step 4.

3.3.3 Payments

When the ACE pricing algorithm is finished, there are two sets of orders, G_t and H_t, and two market price vectors, p^b and p^s. The orders in G_t will pay what they bid. The orders in H_t will pay at the market prices.

[16] We perform an extra step to determine the orders that are *truly* attaining the minimum per unit surplus. The step is necessary because of the non-uniqueness of prices; at some prices more orders can attain the minimum surplus than at other prices.

Let W_t be the value of the objective function of LP (17). Let $H_t^{\min}$ be the set of orders whose per unit surplus w^js are found equal to W_t. Then solve the following LP for each j in $H_t^{\min}$.

$$\begin{array}{rc} \max & w^j \\ \text{sub. to} & w^i \geq W_t, i \in H_t^{\min}, \\ & \text{all the constraints in (17)} \end{array}$$

If the value of the objective function of the above is W_t, order j is considered attaining the minimum per unit surplus at the t-th iteration.

[17] There still is a chance that the price is not uniquely determined after the surplus distribution is determined because bids are as in Example 5. In such a case we choose prices that support the surplus distribution and are averaged. Suppose a buy order of \$1000 for 500 units each of items A and B matches a sell order of \$900 for 500 units each of items A and B. The per unit surplus of each order is \$0.5. This surplus distribution can be achieved, for example, ($p_A = \$19, p_B = \$0$). We choose ($p_A = p_B = \$9.5$). This 'averaging' is often consistent to the reference price information made available to traders in early round when there was no trade because asking prices by sellers are higher than bid prices by buyers. Suppose the buyer's bid is \$900 and the seller's ask is \$1000 in the example in this footnote. If there are no other orders involving assets A and B in the market, \$0.9 is returned as the highest average bid price for both assets A and B and \$1.0 is returned as the lowest average ask price for A and B.

3.3.4 A computational note

The pricing algorithm took the split of the markets[18] further after removing losing bids (losing bids can overlap with winning ones and the removal can allow further division). This occurred between Steps 1 and 2. This was done to limit the number of bids that have to pay the deficit from the 'pay-as-you-bid' portion of allocation and as a consequence of the decision not to impose arbitrage-free conditions on losing bids.

4 The Performance of ACE in RECLAIM

Although there have been many papers assessing the performance of RECLAIM,[19] ACE has been mostly invisible[20] because those assessments were aimed at the performance of RECLAIM and not at the performance of the markets themselves.

In this section, we will examine the performance of the ACE market in the RECLAIM program for the period from 1996 to 2000.[21] The ACE data explored in this section cover the markets from April 1996 to January of 2000.[22] There are two main reasons for choosing this time period. First, this is the period covered in Fine (2001). Second, it is the historically relevant period. RECLAIM began in late 1993. In the beginning, there were several avenues

[18]See Remark 3.

[19]See, for example, Fowlie, Holland, and Mansur (2012), Fromm and Hansjurgens (1996), McCann (1996), and Thompson (2000).

[20]The exceptions are Klier et.al. (1997), written before the first ACE market, and Fine (2001).

[21]This section draws heavily on Chapter 4 of Fine (2001) who had access to the ACE data from Net Exchange.

[22]Markets were held at-least quarterly, with some years having 5 or 6 markets. Specifically, the data are from the following auctions: April 1996, July 1996, August 1996, October 1996, February 1997, April 1997, July 1997, October 1997, January 1998, April 1998, July 1998, October 1998, January 1999, April 1999, July 1999, August 1999, October 1999, and January 2000.

for trading but the main ones were a bulletin board run by the SCAQMD and semi-annual auctions run by the brokerage firm Cantor-Fitzgerald. ACE began in April 1996. In 2001, the California electricity market crisis caused serious disruptions in the operations of RECLAIM and by 2000 ACE had been sold by Net Exchange to Aeon. At that point the ACE data become suspect.[23]

To assess the performance of ACE in RECLAIM, we will examine the participation, bidding and pricing behavior exhibited by the participants in ACE market.

4.1 Participation

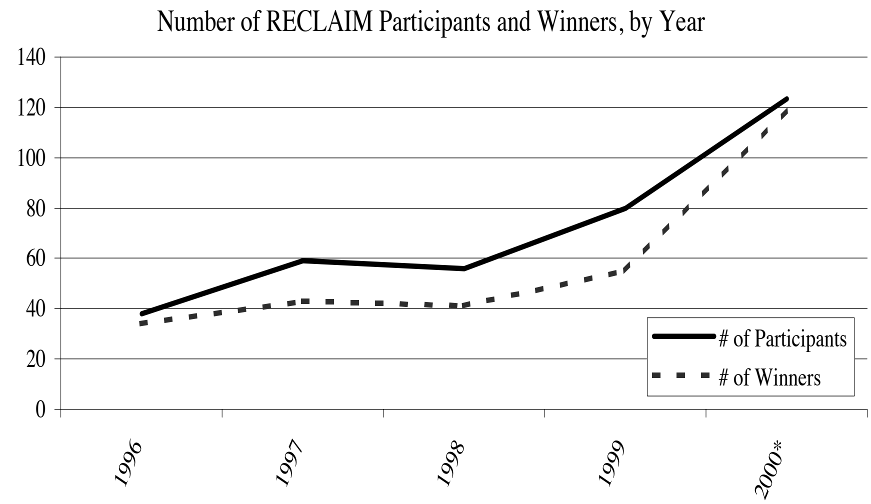
Over time more and more of the members of RECLAIM universe used ACE markets. In Figure 1, we show the number of bidders and winners participating in ACE markets from 1996 to 2000. Even though the number of RECLAIM facilities did not dramatically changed over this time period, the number of bidders and winners in the ACE markets increased yearly. In 1996 only 10% of RECLAIM members conducted trades through ACE. By 1999, 20% did. By 2000, the number increased to about 30%.

Even more dramatically, by 2000 the ACE market had accounted for approximately 90% of all trades-for-a-price[24] in the RECLAIM market. In 1999, RECLAIM recorded 239 trades-for-a-price (219 in the NOx market and 20 in the SOx market). ACE conducted 213 of these trades, 208 of which were for NOx permits. That means 95% of all serious NOx trades went through the ACE markets.

As the RECLAIM program progressed, environmental engineers learned

[23] For more on this, see section 4.4 below.

[24] We are only interested in trades for a price, since zero price trades are either inter-facility within a firm or transfers to or from a permit broker, and are therefore not truly part of the competitive market.



Note: 2000 values are based on January 2000 data, divided by the proportion of participants and winners represented relative to the entire year in 1997, 1998, and 1999.

Figure 1: Participation in the ACE market, by session and year

that the non-ACE trading mechanisms involved extreme transactions costs relative to the ease and efficiency of the ACE market. They were choosing the more efficient option.

4.2 Biding Patterns

In each ACE market session, there are over 100 assets available for purchase. There is a significant stream of futures available to a RECLAIM trader. The ACE market is a powerful institution, allowing for bids that describe very complex preferences. Are the bidders using these options, or are the complex bidding features of the ACE mechanism simply window-trimming? In Figure 2 we display the percentages of package bids submitted and transacted. 18% of the bids submitted and 14% of the transactions are for packages. This seems to be a relatively small use of the combinatorial capability of the ACE algorithm. But taking a slightly different look at the data gives a deeper insight.

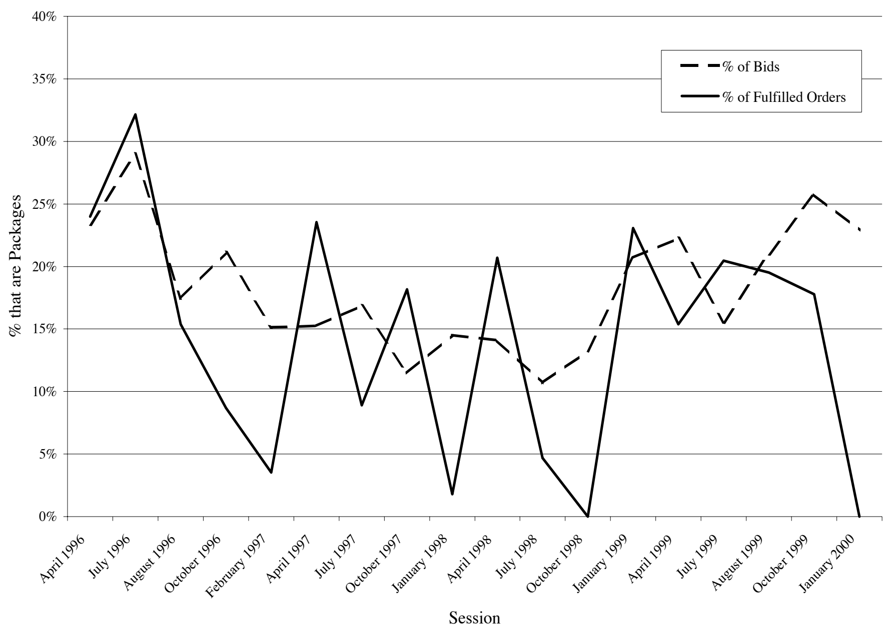


Figure 2: Percentage of bids and fulfilled orders that are for packages

If we consider the RTCs as different assets because the date of effectiveness is different, we have a very thin data set from which it is difficult to perceive bidding and pricing patterns. So instead, we consider the RTCs not as dated objects (in terms of their year of validity) but as a stream of future contracts dated relative to the current market. In Figure 3, we show the number of bids and fulfilled orders in the market as a function of the vintage of the earliest item in the bid.[25]

[25]We only consider the spot and futures markets up to 17 permits in the future (for a total of 9 years of forward contracts in any given trading session).

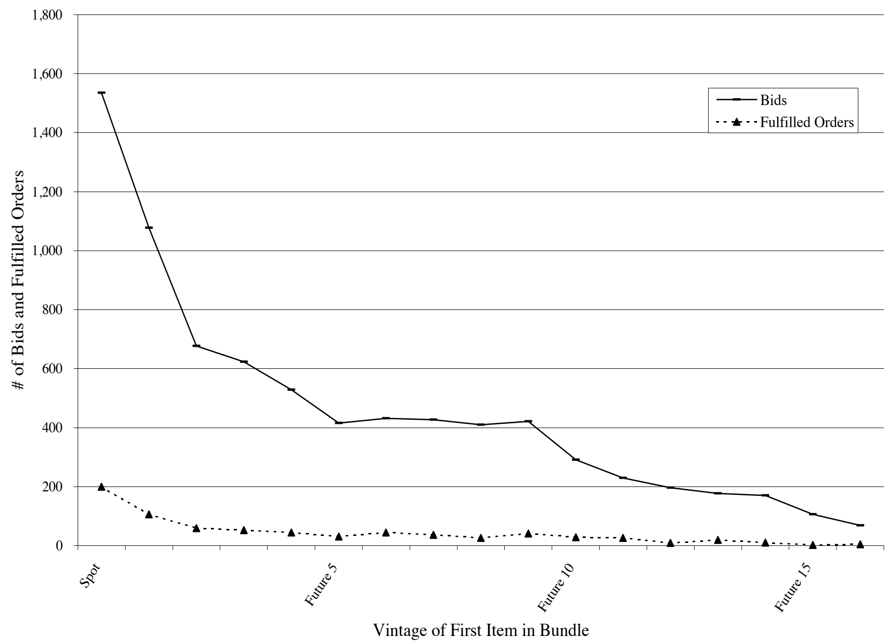


Figure 3: Relative vintage of assets in bids and fulfilled orders

Once we make this adjustment and consider the permits in terms of their relative vintage, a pattern emerges. The number of bids entered is inversely proportional to the time horizon. Further, looking at Figure 4, it can be seen that there are really two markets: a spot or short-term and a planning or long-term market. The bidding patterns in the two markets are in stark contrast to each other. In the spot market (the earliest vintage currently available), approximately 91% of the bids are for a single asset. Additionally, there are no swappers at all in the spot market. That is, of those 9% of bids that are for more than one asset, the bids are either pure buys or pure sells. This is not unexpected. The spot market is a time for rebalancing planned with actual emissions. This can be easily done with single asset bids.

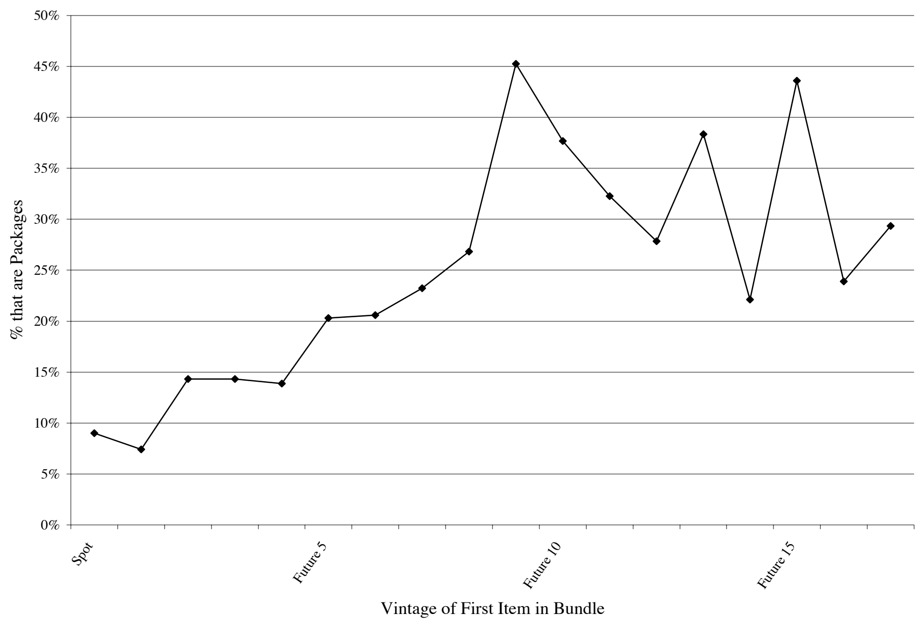


Figure 4: Percent of bids that are packages, by relative vintage

In the longer-term market the bidding is far more frequently for packages. 32% of bids are combined value (for the 72 permits more than 3 years in the future). This is also not unexpected. The futures market is a place to evaluate trade-offs between investment in abatement equipment and RTC acquisition. As we showed in section 1.2, this involves swaps and packages. The frequency of package bidding in the long-term market is quite similar to that observed in the experiments discussed in Fine, Bossaerts, Ledyard (2002). There it was shown that 20-30% package bidding was more than enough to create the liquidity necessary to allow efficient trading when there are strong complementarities as there are in RECLAIM.

4.3 Pricing Patterns

It is important to look separately at two different time trends: how prices move across ACE markets and how prices move across permit vintages.

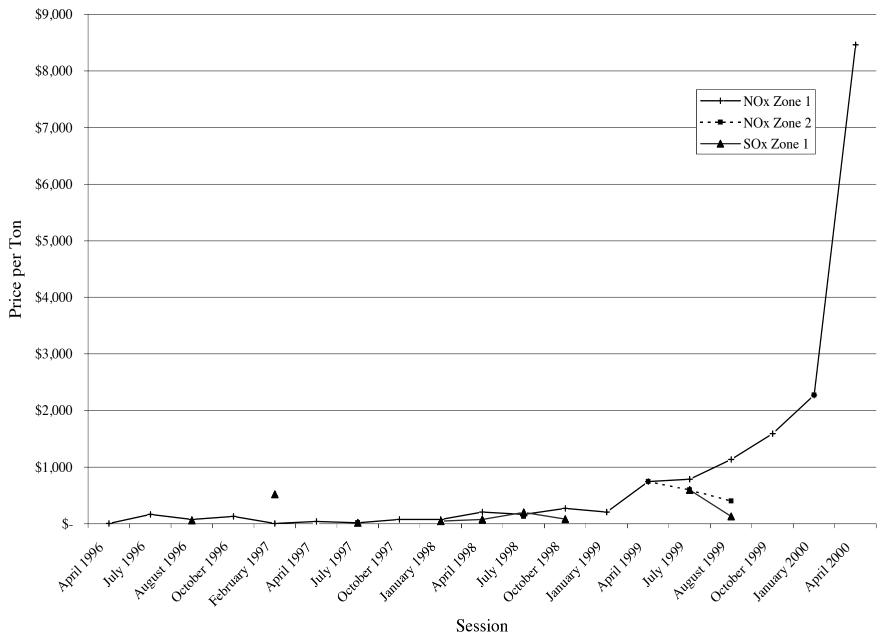


Figure 5: Spot market pricing in ACE markets, April 1996 to April 2000

In Figure 5, we detail the evolution of the NOx and SOx spot ACE price across ACE markets from April 1996 to January 2000. There is almost no trade in any of the markets except for NOx Zone 1, so henceforth we will use the data from this asset to demonstrate the patterns that emerge. The initial supply of RTCs was in excess of the reported emissions at the market's inception in 1994. The supply of RTCs was then reduced each year and in 1997-98, dipped below the total 1994 emissions level. It is at this point, that prices began an uphill climb.[26] From July 1997, the spot market price strictly increased over time. This is because facilities looked down the road, recognized the impending shortage of RTCs, and began rolling permits forward. In late 1999, the price of RTCs dramatically increases shortly after the expected cross of RTC availability and reported emissions. However, even this highest price falls quite short of the AQMD's anticipated price of $11,257 per ton.[27]

In fact, prices have stayed quite low. Assuming these prices accurately reflect the marginal abatement costs, it appears that the planning flexibility offered by the RECLAIM program resulted in lower than expected marginal control costs, perhaps from the shift to a facility-wide performance standard[28] Additionally, prior to the start of RECLAIM, facilities had incentive to misrepresent their emissions and true costs of abatement. Indeed the incentive historically has been to overstate the control costs during public hearings in order to deflect proposed command-and-control type regulations. This strategic reporting further accounts for the gap between predicted and actual RTC prices.

The April 2000 data indicate that prices were now approaching the predicted levels. The marginal cost of Best Available Control Technology (or BACT) for NOx was believed to be in the range of $3.50 to $4.50 per pound. The spot market price for permits in the April 2000 market was $4.23. In

[26]Why do we observe non-zero prices in the spot market? While the number of RTCs available across both cycles has historically been in excess of reported emissions, this does not mean that in any given market there is an excess supply. Firms were either assigned to Cycle 1 or Cycle 2, and then allocated RTCs. Therefore, it is quite possible that there may not be enough permits of a given cycle to cover that entire year's emissions without benefit of permits from another cycle.

[27]Johnson and Pekelney (1996) built the Emissions Trading Model (ETM) to assess the potential economics and environmental impacts of RECLAIM's emissions trading program. It estimated trades that were likely to occur under the program and linked them to a general equilibrium model of the regional economy.

[28]See Bohi and Burtraw (1997) for an excellent discussion of the impacts on control costs.

2000, the RECLAIM program was just reaching the market transition point that Johnson and Pekelney (1996) believed would occur.

Now let us turn to how prices change across vintages. Again we can see differences between the short and long-term markets. As Figure 6 shows, from the spot market to about 7 permits or 3.5 years into the future, the price of permits increases. Once we look to 4 years in the future and beyond, the pricing is remarkably stable. The combined-value mechanism, including its pricing algorithm, provides the liquidity necessary for stable, meaningful prices in the long-term market. This allows the long-term market to exhibit the stable pricing many economists feared impossible.[29]

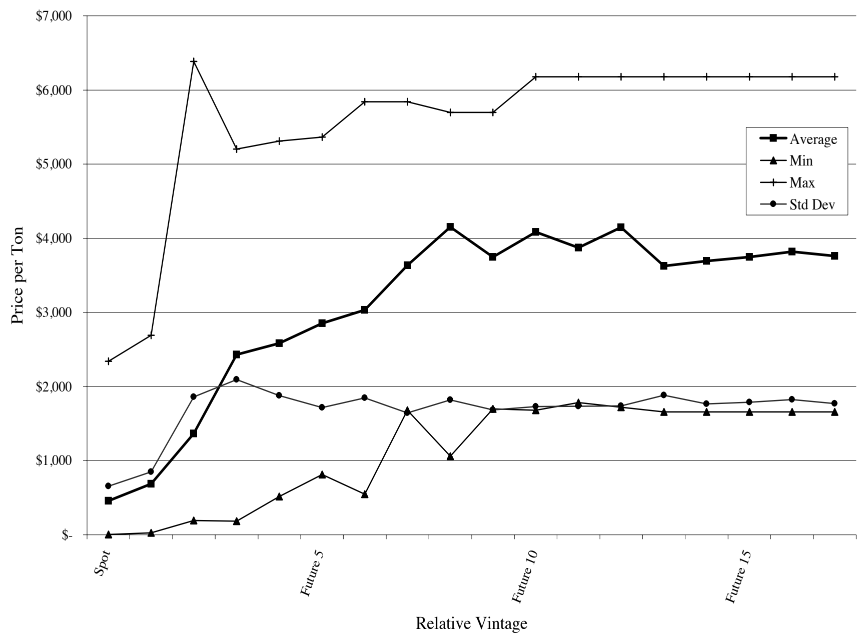


Figure 6: Price of NOx Zone 1 RTCs, as a function of relative vintage

[29] See Hausker (1992).

4.4 But, Who Will Guard the Guardians?

We cannot leave off our evaluation of ACE-RECLAIM without acknowledging a serious, practical problem that market designers generally just wave their hands at.[30] How can we keep the operators of the algorithms honest? The post-2000 history of ACE-RECLAIM illustrates the problem. In 1999/2000, Net Exchange, who had developed and run the ACE algorithms, sold ACE-RECLAIM to Aeon, the company that had handled most of the back-end details of the auctions, such as marketing, escrowing, etc. Between 2000 and 2004, something went very wrong. In June of 2004, Anne Sholtz, the president of Aeon was arrested for being in a "scheme to defraud" numerous companies.[31] This included trading in front of the bidders and selling RTCs that had no counter side. She was sentenced in 2008.

The algorithms were fine, trustworthy and reliable; the operator was not. How to design to protect against such misbehavior remains an open research question.

4.5 Conclusion

Karl Hausker (1992) eloquently voiced the concerns of many economists of the time about tradable permit programs such as RECLAIM. He was concerned that the long-term market would suffer from extreme thinness due to uncertainty, transactions costs, and other sources of market inefficiency. Although the long-term market does seem to have been thinly traded, the combined-value market mechanism used in the ACE market overcame this illiquidity, as predicted in Bossaerts, Fine, and Ledyard (2002). The ACE market mechanism became, over the first four years of the RECLAIM emis-

[30]One exception can be found in the Nobel Lecture of Leonid Hurwicz, "But, Who Will Guard the Guardians?" See Hurwicz (2007). We have adopted his title for this section.

[31]See LA Times (2004) and Pasadena Weekly (2010).

sions credit trading program, the market venue of choice for polluting facilities.

The short-term and long-term markets exhibited dramatically different bidding and pricing patterns. The short-term trader places single-asset bids, making one-time adjustments to her predicted emissions levels. For the long-term planner, the combined value market provided clear, stable pricing and the ability to plan a pollution stream without the risk of attaining only part of that stream.

Since a single-asset bid is simply a degenerate form of a combined-value bid, imposing the combined-value structure on the short-term market certainly did no harm. Indeed, the additional liquidity provided by those few traders who traded in bundles including both short-term and long-term RTCs provided an important bridge between the two markets, improving liquidity in both.

References

Bohi, D. and D. Butraw (1997) "SO_2 Allowance Trading: How do Expectations and Experience Measure Up," *The Electricity Journal* 10 (7), 67-75.

Bossaerts, P., L. Fine, and J. Ledyard (2002) "Inducing Liquidity In Thin Financial Markets Through Combined-Value Trading Mechanisms," *European Economics Review*, 46(9): 1671-1695.

M. Bykowsky, R. Cull, and J. Ledyard (2000) "Mutually Destructive Bidding: The FCC Auction Design Problem," *Journal of Regulatory Economics*, 17(3): pp. 205-228.

Carlson, D., C. Forman, J. Ledyard, N. Olmstead, C., Plott, D. Porter, and A. Sholtz (1993) *An Analysis and Recommendation for the Terms of the RECLAIM Trading Credit*, Pacific Stock Exchange Technical Report, April 27, 1993.

Cason T. and L. Gangadharan (1998) "An Experimental Study of Electronic Bulletin Board Trading for Emission Permits", *Journal of Regulatory Economics*, 14: 55-73.

Duffie, D. (2012) *Dark Markets: Asset Pricing and Information Transmission in Over-the-Counter Markets*, Princeton Lectures in Finance.

Fine, L. (2001) "Cooperative and Market-based Solutions to Pollution Abatement Problems", PhD Dissertation, California Institute of Technology, Pasadena, CA, USA

Fowlie, M., S. Holland and E.T. Mansur (2012) "What do Emissions Markets Deliver and to Whom? Evidence from Southern California's Nox Trading Program," *American Economic Review*, 102 (2), 965-993.

Friedman, D. (1993) "How trading institutions affect financial market performance: some laboratory evidence", *Economic Inquiry*, 31(3), 410-435.

Fromm, O. and B. Hansjurgens (1996) "Emission trading in theory and practice: an analysis of RECLAIM in Southern California," *Environment and Planning*, 14, 367-384.

Gangadharan, L. (2000) "Transactions Costs in Pollution Markets: An Empirical Study", *Land Economics*, 76:601-614.

Hausker, K. (1992) "The Politics and Economics of Auction Design in the Market for Sulfur Dioxide Pollution," *Journal of Public Analysis and Management*, 11, 553-72.

Hurwicz, L. (2007), $http://www.nobelprize.org/nobel\_prizes/economic-sciences/laureates/2007/hurwicz-lecture.html$

Ishikida, T., J. Ledyard, M. Olson, and D. Porter (2001) "Experimental Testbedding of a Pollution Trading System: Southern California's *RECLAIM* Market", *Research in Experimental Economics* 8:185-220.

Johnson, Scott L. and D. M. Pekelney (1996) "Economics Assessment of the Regional Clean Air Incentives Market: A New Emissions Trading Program for Los Angeles," *Land Economics*, 72 (3), 277-97.

Klier, T. H., R. H. Mattoon, and M. A. Prager (1997) "A Mixed Bag: Assessment of Market Performance and Firm Trading Behavior in the NOx RECLAIM Programme," *Journal of Environmental Planning and Management* 40 (6), 741-74.

Los Angeles Times (2004), $http://articles.latimes.com/2004/jun/17/local/me-smog17$

McCabe, K., S. Rassenti, and V. Smith (1993) "Designing a uniform-price double-auction; an experimental evaluation", In: D. Friedman and J. Rust (Eds.), *the Double Auction Market, SFI Studies in the Sciences of Complexity, Proc.* vol XIV, 307-332, Addison-Wesley.

McCann, R. (1996) “Environmental Commodities Markets: 'Messy' versus 'Ideal' Worlds,” *Contemporary Economic Policy*, 14, 85-97.
Myerson, R. and M. Satterthwaite (1983) “Efficient Mechanism for Bilateral Trading” *Journal of Economic Theory* 29, 265-281

Pasadena Weekly (2009), $http://www.pasadenaweekly.com/cms/story/detail/an\_air\_of\_deceit/7616/$

SCAQMD (1993) *Annual Report*

Smith, V. L., A. W. Williams, W. K. Bratton, and M. G. Vannoni (1982) “Competitive Market Institutions: Double Auctions vs. Sealed bid-offer auctions”, *American Economic Review*, 72, 58-77.

Thompson, D. (2000) “Political Obstacles to the Implementation of Emissions Markets: Lessons from RECLAIM,” *Natural Resources Journal*,” 40, 645-697.