Review of Economic Studies (2007) **74**, 733–762 0034-6527/07/00260733$02.00

Self-Correcting Information Cascades

JACOB K. GOEREE and THOMAS R. PALFREY
California Institute of Technology

BRIAN W. ROGERS
Northwestern University

and

RICHARD D. MCKELVEY
Deceased

First version received December 2004*; final version accepted August* 2006 (*Eds.*)

We report experimental results from long sequences of decisions in environments that are theoretically prone to severe information cascades. Observed behaviour is much different—information cascades are ephemeral. We study the implications of a theoretical model based on quantal response equilibrium, in which the observed cascade formation/collapse/formation cycles arise as equilibrium phenomena. Consecutive cascades may reverse states, and usually such a reversal is self-correcting: the cascade switches to the correct state. These implications are supported by the data. We extend the model to allow for base rate neglect and find strong evidence for overweighting of private information. The estimated belief trajectories indicate fast and efficient learning dynamics.

1. INTRODUCTION

An information cascade arises when a sequence of imperfectly informed decision-makers, each of whom observes all previous decisions, has reached a point after which all future decision-makers will rationally ignore their private information. Hence, learning ceases as subsequent decision-makers infer nothing new from observing any of the actions. Information cascades are predicted to occur, possibly after very few decisions, despite the wealth of information available and despite the common interest of all decision-makers (Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992). This result, if robust to variations in the basic model, has obvious and pernicious implications for economic welfare and raises problematic issues for various applications of mass information aggregation, such as bank runs, technology adoption, mass hysteria, and political campaigns.

We conducted laboratory experiments with very long sequences of decision-makers in canonical social learning environments. The data are examined and analysed through the lens of quantal response equilibrium (QRE), which makes systematic predictions about the long-run dynamics of choice behaviour, beliefs, and efficiency. Some of these predictions are essentially the opposite of Nash equilibrium. Because of the complicated dynamics implied by QRE, a careful test of many of these properties of QRE demands the observation of long sequences. In addition, we vary the informativeness of individuals' signals, which systematically affects the observable properties of QRE dynamics.

The QRE approach to the analysis of data enables two additional innovations. First, using a logit QRE error structure we are able to structurally estimate a parametric model of the

TABLE 1

Percentages of (broken) cascades in our data

	$q = 5/9$		$q = 6/9$	
	$T = 20$	$T = 40$	$T = 20$	$T = 40$
Number of sequences	116	56	90	60
Percentage with pure cascades (%)	4·3	0·0	13·3	13·3
Percentage without cascades (%)	0·0	0·0	0·0	0·0
Percentage with broken cascades (%)	95·7	100·0	86·7	86·7

base rate fallacy (QRE–BRF) and a cognitive hierarchy (CH) model of strategic sophistication. The existence and magnitude of judgement fallacies in these environments has important systematic implications about efficiency and dynamics. Second, this structural estimation approach yields estimates of the entire trajectory of public beliefs, for each sequence in the experiment. That is, the belief dynamics can be estimated indirectly without eliciting beliefs from the decision-makers.

We use the simplest possible social learning environment in our experiment because QRE makes especially crisp predictions in these environments, enabling relatively straightforward tests of the predictions while at the same time simplifying the structural estimation procedure. There are two equally likely states of nature, two signals, two actions, and T decision-makers. Nature moves first and chooses a state and then reveals to each decision-maker a private signal about the state. The probability a decision-maker receives a correct signal is $q > 1/2$ in both states of the world. Decision-makers choose sequentially, with each decision-maker observing all previous actions (and her private signal). A decision-maker receives a pay-off of 1 if she chooses the correct action and 0 otherwise. In this environment, learning never progresses very far in a Nash equilibrium. In fact, regardless of T, the equilibrium beliefs of all decision-makers are confined to an interval centred around $1/2$.

The need for an alternative theory of behaviour in these environments is obvious from looking at data from short decision sequences, such as those reported in Anderson and Holt (AH) (1997).[1] In that experiment, cascades are observed; however, some action choices are inconsistent with Nash equilibrium given the realized signals, and many subjects exhibit such behaviour. For example, Anderson and Holt (1997) observe that in their experiment with $q = 2/3$ and $T = 6$, more than 25% of the time subjects make a choice against the cascade after receiving a contradictory signal. And nearly 5% of subjects who receive a signal consistent with the cascade choose the opposite action. Such deviations become even more pronounced in the experiments reported below where we vary the signal precision, $q = 5/9$ and $q = 6/9$, and the number of decision-makers, $T = 20$ and $T = 40$. With this many decision-makers, we should observe cascades arising in 100% of the sequences according to the theoretical model of Bikhchandani *et al.* (1992). However, with $T = 40$, for instance, a cascade arises and persists in only eight out of 116 sequences ($< 7\%$).

Table 1 gives an illustration of a few ways that the standard theory fares badly. At a minimum, a plausible theory should explain two systematic features of the data. First, off-the-Nash-equilibrium-path actions occur with significant probability. The theory as it stands does not place adequate restrictions off the equilibrium path. Second, deviations from equilibrium are systematic, indicating that such behaviour is informative! Why? Because going off the equilibrium path (*i.e.* choosing an action opposite to the cascade) happens much more frequently

1. Indeed AH use a recursive version of logit equilibrium to describe their data.

TABLE 2

Frequency of confirmatory/contrary signals when cascades are (not) broken

	$T = 20$			$T = 40$		
$q = 5/9$	Decision\Signal	Confirming	Contrary	Decision\Signal	Confirming	Contrary
	Don't Break	48·5%	26·7%	Don't Break	45·0%	23·5%
	Break	3·1%	21·7%	Break	7·0%	24·5%
	#Obs = 1305			#Obs = 1234		
$q = 6/9$	Decision\Signal	Confirming	Contrary	Decision\Signal	Confirming	Contrary
	Don't Break	53·2%	28·2%	Don't Break	59·8%	31·4%
	Break	2·4%	16·2%	Break	1·6%	7·2%
	#Obs = 1080			#Obs = 1923		

Notes: Obs, observation.

if the player received a signal contradicting the cascade choices, see Table 2. Indeed, when a break occurs, the observed frequency with which the received signal was contradictory is 83%.[2] This should come as no surprise as a deviation following a confirmatory signal is a worse deviation (*e.g.* in terms of expected pay-offs and also intuitively) than a deviation following a contradictory signal.

The introduction of a random component in QRE ensures that all paths can be reached with positive probability, so Bayes' rule places restrictions on future rational inferences and behaviour when a deviation from a cascade occurs. Deviations from optimal play occur according to a statistical process, and players take these deviations into account when making inferences and decisions. Moreover, deviations or mistakes are pay-off dependent in the sense that the likelihood of a mistake is inversely related to its cost.[3]

In this paper, we demonstrate that QRE predicts the temporary and self-correcting nature of cascades and also predicts several features of the long-run dynamics, as a function of signal informativeness. QRE predicts that with an infinite horizon, the true state will be revealed with probability 1, that is, learning is complete. While no finite experiment can formally test this prediction, our ability to structurally estimate public beliefs with QRE allows us to draw inferences about the rate at which beliefs are converging to full revelation.[4]

Following the pioneering paper of Anderson and Holt (1997), there have been a number of studies exploring different questions related to information cascades. Hung and Plott (2001) replicate the original findings and also explore information aggregation in a voting mechanism. Çelen and Kariv (2004) differentiate between information cascades and herds. Huck and Oechssler (2000), Nöth and Weber (2003), Oberhammer and Stiehler (2003), and Dominitz and Hung (2004) explore whether decision-makers are following Bayes' rule in their updating process and find evidence that Bayes' rule is systematically violated. Some of the other extensions involve cascades in networks (Choi, Gale and Kariv, 2005), the effect of advice (Çelen, Kariv and Schotter, 2005), costly signals (Kübler and Weizsäcker, 2004), and herd behaviour in stock markets (Cipriani and Guarino, 2005; Drehmann *et al.*, 2005). The negative relationship between the duration of a cascade and the probability of collapse is demonstrated in Kübler and Weizsäcker

2. When averaged over the four treatments. In the ($q = \frac{5}{9}, T = 20$), ($q = \frac{5}{9}, T = 40$), ($q = \frac{6}{9}, T = 20$), and ($q = \frac{6}{9}, T = 40$) treatments, the numbers are 87%, 78%, 87%, and 82% respectively.

3. We only consider *monotone* QRE, where choice probabilities are monotone in expected utilities, see McKelvey and Palfrey (1995, 1998) and Goeree, Holt and Palfrey (2005).

4. Longer sequences of decisions could possibly be obtained from an Internet experiment, where agents are successively invited to participate (see Drehmann, Oechssler and Roider, 2005).

(2005) across several different studies and is consistent with our own findings and with the predictions of the QRE model.

The remainder of the paper is organized as follows. Section 2 describes the basic model and presents the main theoretical properties of QRE dynamics, which deliver hypotheses that are directly testable with data from our experiment. Section 3 explains the experimental design. Section 4 contains a descriptive analysis of the data, focusing on cascade dynamics and choice behaviour. Section 5 presents an econometric analysis of the basic model and extensions that better explain the data. Section 6 discusses the belief dynamics implied by the structural estimation and the resulting efficiency properties of the data. The Appendix contains proofs.

2. THE BASIC MODEL

There is a finite set $\mathcal{T} = \{1, 2, \ldots, T\}$ of agents who sequentially choose between one of the two alternatives, A and B. Agent t chooses at time t, and let $c_t \in \{A, B\}$ denote agent t's choice. One of the alternatives is selected by nature as "correct", and an agent receives a pay-off of 1 only when she selects this alternative, otherwise she gets 0. The correct alternative (or state of the world), denoted by $\omega \in \{A, B\}$, is unknown to the agents who have common prior beliefs that $\omega = A$ or $\omega = B$ with probability $\frac{1}{2}$. Further, they receive conditionally independent private signals s_t regarding the better alternative. If $\omega = A$ then $s_t = a$ with probability $q \in (\frac{1}{2}, 1)$ and $s_t = b$ with probability $1 - q$. Likewise, when $\omega = B$, $s_t = b$ with probability q and $s_t = a$ with probability $1 - q$.

We will be concerned with the evolution of agents' beliefs and how these beliefs co-evolve with actions. Agent t observes the actions of all her predecessors, but not their signals. Thus a history H_t for agent t is simply a sequence $\{c_1, \ldots, c_{t-1}\}$ of choices by agents $1, \ldots, t-1$, with $H_1 = \emptyset$. Agents care about the history only to the extent that it is informative about which alternative is correct. So, let $p_t \equiv P(\omega = A \mid H_t)$ denote the (common knowledge) posterior belief that A is correct given the choice history H_t, with $p_1 = \frac{1}{2}$, the initial prior. We first determine agent t's private posterior beliefs given the public beliefs p_t and given her signal s_t. Applying Bayes' rule shows that if $s_t = a$, agent t believes that alternative A is correct with probability

$$\pi_t^a(p_t) \equiv P(\omega = A \mid H_t, s_t = a) = \frac{q\, p_t}{q\, p_t + (1-q)(1-p_t)}. \tag{2.1}$$

Likewise,

$$\pi_t^b(p_t) \equiv P(\omega = A \mid H_t, s_t = b) = \frac{(1-q)\, p_t}{(1-q)\, p_t + q(1-p_t)} \tag{2.2}$$

is the probability with which agent t believes that A is correct if her private signal is $s_t = b$. A direct computation verifies that $\pi_t^a(p_t) > p_t > \pi_t^b(p_t)$ for all $0 < p_t < 1$. In other words, for any interior public belief, an agent believes more strongly that $\omega = A$ after observing an a signal than after observing a b signal.

2.1. *Nash equilibrium*

We first discuss the dynamics of beliefs and choice behaviour in a Bayesian Nash equilibrium. The unique trembling hand perfect equilibrium of the game, identified by Bikhchandani *et al.* (1992), involves rapid convergence to an information cascade.

This pure cascade Nash equilibrium works as follows.[5] The first agent chooses A if $s_1 = a$ and chooses B if $s_1 = b$, so that her choice perfectly reveals her signal. If the second agent's signal agrees with the first agent's choice, the second agent chooses the same alternative, which is strictly optimal. On the other hand, if the second agent's signal disagrees with the first agent's choice, the second agent is indifferent, as she effectively has a sample of one a and one b. For comparison with the QRE discussed next, we assume that the second agent randomizes uniformly when indifferent.[6] The third agent faces two possible situations: (i) the choices of the first two agents coincide, and (ii) the first two choices differ. In case (i), it is strictly optimal for the third agent to make the same choice as her predecessors, even if her signal is contrary. Thus her choice imparts no information to her successors, resulting in the onset of a cascade. The fourth agent is then in the same situation as the third and so also makes the same choice, a process that continues indefinitely. In case (ii), however, the choices of the first two agents reveal that they have received one a signal and one b signal, leaving the third agent in effectively the same position as the first. Her posterior (before considering her private information) is $p_3 = \frac{1}{2}$, so that her signal completely determines her choice. The fourth agent would then be in the same situation as the second agent described above and so forth. Thus a cascade begins after some even number of agents have chosen and $|\#A - \#B| = 2$, where $\#A$ is the number of decision-makers who have chosen A and $\#B$ is the number of decision-makers who have chosen B.

One quantity of interest is the probability that "correct" and "incorrect" cascades will form in equilibrium. First, the probability of being in neither cascade vanishes rapidly as t grows. The probability of eventually reaching a correct cascade is $\frac{q(1+q)}{2-2q(1-q)}$, and the complementary probability of eventually reaching an incorrect cascade is $\frac{(q-2)(q-1)}{2-2q(1-q)}$.[7,8] Once a cascade has formed, all choices occur independently of private information, and hence public beliefs remain unchanged. The points at which public beliefs settle are the posteriors that obtain after two consecutive choices for the same alternative, beginning with the uninformative prior.

2.2. *Quantal response equilibrium*

We now describe the logit QRE of the model described above. In the logit QRE, each individual t privately observes a pay-off disturbance for each choice, denoted ε_t^A and ε_t^B. The pay-off-relevant information for agent t is summarized by the difference $\varepsilon_t \equiv \varepsilon_t^A - \varepsilon_t^B$. Denote agent t's type by $\theta_t = (s_t, \varepsilon_t)$. The logit specification assumes that ε_t are independent and obey a logistic distribution with parameter λ.[9,10] The disturbance, ε_t, can be interpreted in several different ways. For example, it could represent a stochastic part of decision-making due to bounded rationality, or it could be an individual-specific preference shock that occurs for other reasons. Irrespective of the interpretation of the noise, the resulting logit choice model implies that the stronger the belief that A is correct, the more likely action A is chosen. The logit QRE model assumes that

5. As we will see, almost all choice sequences in our laboratory data are inconsistent with the behaviour implied by this Nash equilibrium.

6. This randomization holds in any logit QRE. There are other Nash equilibria where players randomize with different probabilities when indifferent, all resulting in herds and information cascades.

7. After the first two choices, the probabilities of the three regimes, correct cascade, no cascade yet, or incorrect cascade, are $\frac{1}{2}q(1+q)$, $q(1-q)$, and $\frac{1}{2}(q-2)(q-1)$, respectively. More generally, after $2t$ choices, these probabilities are $\frac{1}{2}q(1+q)\left(\frac{1-(q(1-q))^t}{1-q(1-q)}\right)$, $(q(1-q))^t$, $\frac{1}{2}(q-2)(q-1)\left(\frac{1-(q(1-q))^t}{1-q(1-q)}\right)$. Taking limits as t approaches infinity yields the long-run probabilities of the three regimes.

8. Thus as q increases from $\frac{1}{2}$ to 1, the probability of landing in a good cascade grows from $\frac{1}{2}$ to 1.

9. This arises when ε_t^A and ε_t^B are i.i.d. extreme-value distributed.

10. The properties derived in this section hold for all atomless error distributions that have full support over the interval $[-1, 1]$. The logit specification is convenient because its behaviour is determined by a single parameter with a natural "rationality" interpretation.

the distribution of the pay-off disturbances is common knowledge.[11] The logit QRE is calculated as the sequential equilibrium of the resulting game of incomplete information, where each player observes only her own type θ_t.

It is straightforward to characterize the optimal decision of agent t given her type θ_t and the history H_t (which determines public beliefs p_t). The expected pay-off of choosing A is $\pi_t^{s_t}(p_t)+\varepsilon_t$ and that of selecting alternative B is $1-\pi_t^{s_t}(p_t)$. Thus given agent t's signal, the probability of choosing A is given by[12]

$$P(c_t = A \mid H_t, s_t) = P(\varepsilon_t > 1-2\pi_t^{s_t}(p_t)) = \frac{1}{1+\exp(\lambda(1-2\pi_t^{s_t}(p_t)))}, \tag{2.3}$$

and B is chosen with complementary probability $P(c_t = B \mid H_t, s_t) = 1-P(c_t = A \mid H_t, s_t)$. When $\lambda \to \infty$, choices are fully rational in the sense that they do not depend on the private realizations ε_t and are determined solely by beliefs about the correct alternative. It is easy to show that the logit QRE converges to the pure cascade Nash equilibrium in which indifferent subjects randomize uniformly.[13] On the other hand, as λ approaches 0, choices are independent of beliefs and become purely random.

The belief dynamics also depend on λ. To derive the evolution of the public belief that A is correct, note that given p_t, there are exactly two values that $p_{t+1} = P(\omega = A \mid H_t, c_t)$ can take depending on whether c_t is A or B. These are denoted p_t^+ and p_t^- respectively. The computation of the posterior probabilities p_t^+ and p_t^- given p_t is carried out by agents who do not know the true state and so cannot condition their beliefs on that event. In contrast, the transition probabilities of going from p_t to p_t^+ or p_t^- (*i.e.* of a choice for A or B) depend on the objective probabilities of a and b signals as dictated by the true state. Thus when computing these transition probabilities, it is necessary to condition on the true state. Conditional on $\omega = A$, the transition probabilities are

$$\begin{aligned}
T_t^{\omega=A} &\equiv P(c_t = A \mid H_t, \omega = A)\\
&= P(c_t = A \mid H_t, s_t = a)P(s_t = a \mid \omega = A) + P(c_t = A \mid H_t, s_t = b)P(s_t = b \mid \omega = A)\\
&= \frac{q}{1+\exp(\lambda(1-2\pi_t^a(p_t)))} + \frac{1-q}{1+\exp(\lambda(1-2\pi_t^b(p_t)))},
\end{aligned}$$

with the probability of a B choice given by $1-T_t^{\omega=A}$. Similarly, conditional on $\omega = B$, the probability agent t chooses A is

$$T_t^{\omega=B} = \frac{1-q}{1+\exp(\lambda(1-2\pi_t^a(p_t)))} + \frac{q}{1+\exp(\lambda(1-2\pi_t^b(p_t)))}.$$

Using Bayes' rule, we now obtain the two values that p_{t+1} may take as

$$p_t^+ \equiv P(\omega = A \mid H_t, c_t = A) = \frac{p_t T_t^{\omega=A}}{p_t T_t^{\omega=A} + (1-p_t)T_t^{\omega=B}} \tag{2.4}$$

and

$$p_t^- \equiv P(\omega = A \mid H_t, c_t = B) = \frac{p_t(1-T_t^{\omega=A})}{p_t(1-T_t^{\omega=A}) + (1-p_t)(1-T_t^{\omega=B})}. \tag{2.5}$$

11. In general, the distributions of pay-off disturbances in a logit QRE need not be the same for every decision-maker, but these distributional differences would be assumed to be common knowledge.

12. Note that indifference occurs with probability 0 under the logit specification and hence plays no role.

13. This is because for any $\lambda \in (0,\infty)$, an agent chooses in an equiprobable way when indifferent.

These expressions can be used to derive the following properties of the belief dynamics (see Appendix for proofs), where without loss of generality we assume the true state is $\omega = A$.

Proposition 1. *For all $\lambda > 0$ there is a unique logit QRE with the following properties:*

(i) *Beliefs are interior:* $p_t \in (0,1)$ *for all* $t \in \mathcal{T}$.
(ii) *Actions are informative:* $p_t^- < p_t < p_t^+$ *for all* $t \in \mathcal{T}$.
(iii) *Beliefs about the true state rise on average:* $E(p_{t+1} \mid p_t, \omega = A) > p_t$ *for all* $t, t+1 \in \mathcal{T}$.
(iv) *Beliefs converge to the truth: conditional on* $\omega = A$, $\lim_{t\to\infty} p_t = 1$ *almost surely.*

The intuition for the full information aggregation result (iv) is as follows. First, for all possible histories and signal realizations, both options are chosen with strictly positive probability. This implies there are no herds, and learning never stops. Second, choice probabilities are increasing in expected pay-offs, so actions reveal some (private) information. Hence, for any belief, a decision-maker is more likely to choose A with an *a* signal than with a *b* signal. As a result, more and more information gets revealed over time leading to full information aggregation in the limit.

2.3. *Classification of cascades observed in the laboratory*

We distinguish several kinds of cascade-like behaviour.[14] A *pure A (B) cascade* is said to form at time $t \leq T$ if after period $t-1$ the number of $A(B)$ choices exceeds the number of $B(A)$ choices by 2 for the first time, and *all* choices from t to T are $A(B)$ choices. Thus, for example, if $T = 6$ and the sequence of choices is $\{A, B, A, A, A, A\}$, then we say a pure A cascade forms at $t = 5$. In periods 5 and 6, we say the decision-makers are in a pure A cascade. Note that any pure cascade beginning at time t will have length $T - t + 1$.

A *temporary A (B) cascade* or *A (B) craze*[15] is said to form at time $t \leq T$ if after period $t-1$ (but not after period $t-2$) the number of theoretically informative A (B) choices[16] exceed the number of theoretically informative B (A) choices by 2 and some decision-maker τ, with $t \leq \tau \leq T$, makes a contrary choice. The number of periods decision-makers follow the cascade, $\tau - t$, defines its *length*. Thus in the sequence of decisions $\{A, A, B\}$ we say that an A cascade of length 0 occurs at $t = 3$.

Temporary cascades are particularly interesting because subsequent play of the game is off the Nash equilibrium path. Moreover, if the sequence is long enough it is possible for a new cascade to form after a temporary cascade has broken. Following AH, we define a simple counting procedure to classify sequences of decisions and determine whether a new cascade has formed. This *ad hoc* counting rule roughly corresponds to Bayesian updating when the probability that indifferent subjects follow their signals equals the probability that subjects who break cascades hold contrary signals.[17] Under the counting rule, every A decision when *not* in a cascade increases the count by 1 and every B decision when *not* in a cascade decreases the count by 1. Recall that we enter the first cascade of a sequence when the count reaches 2 or -2. Then the decisions during the cascade do not change the count, until there is an action that goes against the cascade, which decreases the count to 1 if it was an A cascade or increases the count to -1

14. One might argue for using the term "herd" instead of cascade, since cascade refers to belief dynamics, while "herds" refer to choice dynamics. In the context of QRE, this distinction is artificial. All choices occur with positive probability at every point in time, and learning never ceases.

15. According to the *Oxford English Dictionary* (1980), a craze is defined as a "great but often short-lived enthusiasm for something".

16. Choices made during a (temporary) cascade are called theoretically uninformative.

17. These conditions are approximated in our data, where we find 85% of indifferent subjects go with their signals, and 84% of cascade breakers received contrary signals.

if it was a B cascade. The count continues to change in this way, until the count reaches either 2 or -2 again, and then we are in a new cascade, which we call a *secondary* cascade.

We distinguish three different kinds of secondary cascades. One possibility is that actions cascade on the same state as the previous cascade: a *repeat cascade*. The other possibility is that the actions cascade on a different state: a *reverse cascade*. A *self-correcting cascade* is a cascade that reverses from the incorrect state to the correct state.

2.4. *Hypotheses*

A wide range of observable implications follow from the theoretical results about the logit equilibrium in these dynamic games of incomplete information. We distinguish four categories of hypotheses depending on their object: cascade length and frequency, self-correction of cascades, efficiency of decisions, and belief dynamics. Most of these hypotheses are in the form of the comparative statics with respect to the two main treatment parameters, q and T.

Testable implications of the logit QRE. *For all $\lambda > 0$ observed behaviour in the unique logit QRE will have the following properties:*

1. **Cascades: frequency and length**[18]
   - (C1) For any q and sufficiently large T, the probability of observing a pure cascade is decreasing in T, converging to 0 in the limit. For any q and $T > 2$, the probability of observing a temporary cascade is increasing in T, converging to 1 in the limit.
   - (C2) For any T, the probability of a pure cascade is increasing in q.
   - (C3) For any q, the expected number of cascades is increasing in T.
   - (C4) For sufficiently large T, the expected number of cascades is decreasing in q.
   - (C5) The probability that a cascade, which has already lasted k periods, will break in the next period is decreasing in k.
   - (C6) For any q, the average length of cascades is increasing in T.
   - (C7) For any T, the average length of cascades is increasing q.
2. **Self-correction**
   - (SC1) Incorrect cascades are shorter on average than correct cascades.
   - (SC2) Incorrect cascades are more likely to reverse than correct cascades (self-correction).
   - (SC3) Correct cascades are more likely to repeat than incorrect cascades.
   - (SC4) Later cascades are more likely to be correct than earlier ones.
   - (SC5) A decision-maker with a contradictory signal is more likely to break a cascade than a decision-maker with a confirmatory signal.
3. **Efficiency: the probability of correct decisions**
   - (E1) The *ex ante* (*i.e.* before decision-maker t has drawn a private signal) probability of a correct decision is increasing in t. An interim version of this statement is true, but only conditional on receiving an incorrect signal.[19]
   - (E2) The probability of a correct decision is higher for a correct than for an incorrect signal.
   - (E3) The probability of a correct choice is increasing in q.

18. Several of these hypotheses are only sensible if T is sufficiently large. At least two periods are required for any cascade to form, and at least six periods are required to observe a cascade and its reversal. For example, $\{A, A, B, B, B, B\}$ is the shortest possible sequence for a reverse from an A cascade to a B cascade, and $\{A, A, B, A\}$ is the shortest possible sequence for a repeated A cascade.

19. It is *not* true conditional on receiving a correct signal. To see this, note that the interim probability of a correct decision at time $t = 1$ with a correct signal approaches 1 as λ diverges as it is optimal to follow one's signal. In later periods, it is bounded away from 1 because of the probability of a cascade on the wrong state.

4. Beliefs: informational efficiency

(B1) For each q, on average the public belief on the true state is closer to 1 in the final period of the $T = 40$ treatments than in the $T = 20$ treatments.

(B2) For each t, on average the public belief on the true state is closer to 1 in the $q = 6/9$ treatments than in the $q = 5/9$ treatments.

(B3) For all treatments, on average the public belief on the true state is increasing in t.

These hypotheses follow from a few basic properties implied either by QRE or by the informative signal process itself. We list them below,[20] and refer to them in the ensuing discussion that explains the intuition of the hypotheses. For any positive value of λ:

1. There is a (positive) lower bound on the probability a decision-maker chooses either decision because pay-offs are bounded. This lower bound is independent of beliefs.
2. The higher the public belief on a state, the greater the probability the decision-maker will choose the optimal action for that state.
3. If a decision-maker breaks a cascade, he is more likely to have a contradictory signal than a confirmatory signal.
4. The higher is q, the more likely it is that any given cascade be correct.
5. In a correct cascade, confirmatory signals are more likely than contradictory signals.
6. In an incorrect cascade, confirmatory signals are less likely than contradictory signals.
7. When an action is taken at time t, the public belief on the corresponding state increases. That change in public belief is an increasing function of q.
8. The higher the public belief on the true state, the higher the probability the decision-maker receives a signal favouring that state.
9. The expected change in beliefs in the true state from t to $t+1$ is always positive.

Hypothesis (C1) (which applies to $T > 2$) follows from 1, which implies that the probability a cascade breaks in any round is strictly positive. Hypothesis (C2) follows from 3, 4, and 5. Hypothesis (C3) follows because the probability a first cascade has formed is increasing in T, the probability a cascade has formed and broken is increasing in T, the probability a cascade has formed and broken, and then another one has formed is increasing in T, and so forth.

Hypothesis (C4) is more complicated and can only be proved for T sufficiently large. For example, if $T = 2$, then the expected number of cascades is simply the probability that exactly one cascade occurs, which is the probability of two either correct or incorrect signals, which is $q^2 + (1-q)^2$. This expression is increasing in q. The difficulty is that there are two opposing effects of increasing q. The probability of a cascade forming is increasing in q, but the probability of a cascade breaking is decreasing in q. For sufficiently large T, the latter effect dominates because decisions are more frequently in a cascade than not in a cascade. The higher is λ, the greater must be T for this to be true.

Hypothesis (C5) follows from 2 and 6. Hypotheses (C6) and (C7) follow from 2, 6, and 8 and the fact that the probability of a cascade breaking once you are in a cascade is decreasing in q. Hypothesis (SC1) follows from 3, 4, and 5. The logic behind the next two hypotheses about self-correction, (SC2) and (SC3), is fairly obvious. They follow from 7 and the fact that decision-makers are more likely to receive correct than incorrect signals. Hypothesis (SC4) is a consequence of the self-correction process and follows from 2 and 8. Hypothesis (SC5) is equivalent to 3.

The efficiency hypotheses address the frequency of correct decisions. First, on average, efficiency will increase over time because the expected public belief converges monotonically to the

20. The proofs are straightforward and are omitted.

true state (the *ex ante* part of E1). Second, decision-makers who receive a correct signal are obviously more likely to make the correct decision than decision-makers with incorrect signals (E2), but this difference will decline over time because the public belief on the true state converges to 1 (interim part of E1). Third, efficiency should be positively affected by signal informativeness in three ways. There is the direct effect that more good signals are received with a higher q, but there are two indirect effects as well: with more informative signals, social learning is faster because actions are more informative, and conditional on being in a cascade, the cascade is more likely to be correct.[21] Because all these three effects go in the same direction, there should be a difference in efficiency in the different q treatments.

All the C, SC, and E hypotheses are tested with simple direct tests on sample means. However, (C3), (C4), (C6), and (C7) can be strengthened because of the comparative statics results on length and frequency of cascades for *the entire distribution* of lengths and frequencies, not just the means.

Because beliefs, unlike actions, are not directly observable, we test the B hypotheses by estimating beliefs using our QRE structural estimation approach. Because the analysis of beliefs in our data is quite different and depends on the estimation procedure, we discuss the results about beliefs later, after presenting the QRE estimates of the underlying parameters of the model.

While some of these properties are also true for the initial few decisions in the pure cascade Nash equilibrium, the effects vanish quickly with longer sequences. An exception is (E3). In the perfect Nash equilibrium, the probability of a correct decision is approximately equal to the probability of ending up in a correct cascade, which quickly approaches $q^2/(q^2+(1-q)^2)$ and rises with q.

3. EXPERIMENTAL DESIGN

The two innovations of our experimental design are the use of much longer choice sequences and the use of different signal precisions. These innovations allow us to assess the predictions of the logit QRE model in ways that are not possible with past designs and to gain insights into how the basic models might be improved.

The experiments reported here were conducted at the Social Science Experimental Laboratory at Caltech and the California Social Science Experimental Laboratory at UCLA between September 2002 and May 2003. The subjects included students from these two institutions who had not previously participated in a cascade experiment.[22]

The experiments employ a 2×2 design, where we use two values of both the signal quality q and the number of individuals T. Specifically, q takes values 5/9 and 6/9, and T takes values 20 and 40. The number of sequences in each experimental session is denoted M. Table 3 summarizes the design.

In each session, a randomly chosen subject was selected to be the "monitor", and the remaining subjects were randomly assigned to computer terminals in the laboratory. All interaction among subjects took place through the computers; no other communication was permitted. Instructions were given with a voiced-over Powerpoint presentation in order to minimize variations across sessions.[23] After logging in, the subjects were taken slowly through a practice match (for which they were not paid) in order to illustrate how the software worked and to give them a chance to become familiar with the process before the paid portion of the experiment commenced.

21. Another minor effect going in the same direction is that with a higher q the posterior beliefs are, on average, further from $\frac{1}{2}$, so the expected pay-off difference between a correct and incorrect action is generally increasing in q.

22. There was one subject who had previously participated in a related pilot experiment.

23. The instructions are available as supplementary material at http://www.restud.com.

TABLE 3

Experimental sessions

Session	T	q	M	Subject pool
03/14/03A	20	5/9	30	Caltech
09/26/02B	20	5/9	30	Caltech
09/19/02A	20	5/9	26	Caltech
04/03/03AB	20	5/9	30	UCLA
04/14/03A	20	6/9	30	UCLA
04/14/03C	20	6/9	30	UCLA
04/14/03E	20	6/9	30	UCLA
05/05/03D	40	5/9	17	UCLA
05/05/03F	40	5/9	19	UCLA
05/05/03G	40	5/9	20	UCLA
04/16/03B	40	6/9	20	UCLA
04/21/03C	40	6/9	20	UCLA
04/21/03E	40	6/9	20	UCLA

Before each match, the computer screen displayed two urns. For the $q = 5/9$ treatment, one urn contained five blue balls and four red balls and the other contained four blue balls and five red balls. For the $q = 6/9$ treatment, one urn contained six blue balls and three red balls and the other contained three blue balls and six red balls. The monitor was responsible for rolling a die at the beginning of each game to randomly choose one of the urns with equal probabilities. This process, and the instructions to the monitor (but not the outcome of the roll) were done publicly. At this point, the subjects saw only one urn on the computer screen, with all nine balls coloured grey, so that they could not tell which urn had been selected. Each subject then independently selected one ball from the urn on their screen to have its colour revealed. Then, in a random sequence, subjects sequentially guessed an urn. During this process, each guess was displayed on all subjects' screens in real time as it was made, so each subject knew the exact sequence of guesses of all previous subjects. After all subjects had made a choice, the correct urn was revealed, and subjects recorded their pay-offs accordingly. Subjects were paid \$1·00 for each correct choice and \$0·10 for each incorrect choice.[24] Subjects were required to record all this information on a record sheet, as it appeared on their screen. Due to time constraints, the number of matches (sequences of T decisions) was $M = 30$ in each $T = 20$ session and $M = 20$ in each $T = 40$ session.[25] After the final game, pay-offs from all games were summed and added to a show-up payment, and subjects were then paid privately in cash before leaving the laboratory.

4. RESULTS I: CASCADES, SELF-CORRECTION, AND EFFICIENCY

In this section, we examine the aggregate properties of our data. The analysis is focused by the hypotheses in the previous section about cascade frequency and length, self-correction of cascades, and efficiency of decisions.

4.1. *Infrequency of pure cascades; frequency of temporary cascades*

In AH's experiment with only $T = 6$ decision-makers, all cascades were necessarily very short. In contrast, our experiments investigated sequences of $T = 20$ and $T = 40$ decision-makers,

24. Previous studies have used different incentive structures. Anderson (2001) varied the magnitude of incentive payments and found no systematic behavioural effects, except when incentive payments were removed entirely.

25. A few sessions contained fewer sequences due to technical problems, see Table 3.

TABLE 4

Percentages of pure cascades by treatment

	Our data				AH data			QRE–BRF			
	$q=5/9$		$q=6/9$		$q=6/9$	Nash		$q=5/9$		$q=6/9$	
	$T=20$ $M=116$	$T=40$ $M=56$	$T=20$ $M=90$	$T=40$ $M=60$	$T=6$ $M=45$	$q=5/9$	$q=6/9$	$T=20$	$T=40$	$T=20$	$T=40$
First 6 (%)	36	36	50	53	64	98	99	27	26	44	44
First 10 (%)	15	14	31	42		100	100	9	11	28	27
First 20 (%)	4	2	13	32		100	100	1	1	15	14
First 40 (%)		0		13		100	100		0		7

TABLE 5

Percentages of temporary cascades by treatment

	Our data				AH data			QRE–BRF			
	$q=5/9$		$q=6/9$		$q=6/9$	Nash		$q=5/9$		$q=6/9$	
	$T=20$ $M=116$	$T=40$ $M=56$	$T=20$ $M=90$	$T=40$ $M=60$	$T=6$ $M=45$	$q=5/9$	$q=6/9$	$T=20$	$T=40$	$T=20$	$T=40$
First 6 (%)	58	55	42	38	27	0	0	64	62	50	48
First 10 (%)	84	84	69	55		0	0	88	87	71	71
First 20 (%)	96	98	87	68		0	0	99	99	85	86
First 40 (%)		100		87		0	0		100		93

allowing an opportunity to observe long cascades, the length distribution of temporary cascades, and the self-correcting property. As Table 4 clearly demonstrates, pure cascades essentially do not happen in the longer matches. The cascades that persisted in the AH experiments simply *appear* to be pure cascades, a likely artefact of the short horizon. Our numbers are comparable with those of AH when we consider only the first six decision-makers in our sequences. These numbers are given in the row marked "First 6" in Table 4. In contrast, we observe pure cascades in only 17 our of 206 sequences with $T=20$ decision-makers and only 8 of 116 sequences with $T=40$ decision-makers.

The final columns of Table 4 give the predicted frequency of pure cascades according to the Nash equilibrium based on actual signal draws and out-of-sample predictions from the QRE–BRF model (which we explain and discuss in a later section). The Nash equilibrium probability of a pure cascade with T decision-makers is $1-(q(1-q))^{T/2}$.

The data contradict the Nash predictions in three ways. First, there are far fewer pure cascades than theory predicts. Second, there were far fewer than were observed in past experiments with very short decision sequences. According to theory, the frequency of pure cascades should increase with T but in fact the data show the opposite. Third, the frequency of pure cascades in the data is steeply increasing in q, while the Nash equilibrium predicts almost no effect. In our data, pure cascades occurred nearly five times as frequently in the $q=6/9$ treatment than when $q=5/9$ (20/150 compared with 5/172).[26]

In contrast to pure cascades, temporary cascades are common in all treatments. Table 5 shows the frequency of temporary cascades in our data. The rows and columns mirror Table 4,

26. Further evidence indicates that this continues to increase with q. In a single additional session with $q=3/4$ and $T=20$, we observed pure cascades in $28/30$ sequences.

TABLE 6

Number and lengths of cascades by treatment

		$q = 5/9$		$q = 6/9$	
		$T = 20$ $M = 116$	$T = 40$ $M = 56$	$T = 20$ $M = 90$	$T = 40$ $M = 60$
Average number cascades	Our data	3·47	7·54	2·99	3·73
	QRE–BRF	3·88	7·30	2·88	4·21
	Nash	1·00	1·00	1·00	1·00
Average length cascades	Our data	2·43	2·00	3·27	7·83
	QRE–BRF	1·54	2·26	4·15	6·39
	Nash	17·32	37·25	17·43	37·44

but the entries now indicate the proportion of sequences in a given treatment that exhibit at least one temporary cascade that falls apart. Clearly, for large T, essentially all cascades we observe are temporary. With the short horizon of the AH experiment, temporary cascades occur only in about one-fourth of the sequences.

4.2. *Number and lengths of temporary cascades*

With larger T, we almost always observe *multiple* temporary cascades along a single sequence. Table 6 (top) displays the average number of cascades in each treatment. The number of temporary cascades rises with the sequence length, T, and falls with the signal precision, q, not only on average, but also in the sense of first-order stochastic dominance; see the top panel in Figure 1. This evidence supports hypotheses (C3) and (C4). The table and the figure also show the Nash prediction of exactly one cascade per sequence, independent of q and T, and out-of-sample predictions generated by the QRE–BRF model (discussed later).

Figure 2 graphs separately for each treatment, the empirical probability of collapse as a function of the duration of the cascade: that is, the probability of a collapse in period $t + s$, given the cascade started in period t. This probability is sharply decreasing in s. In other words, longer cascades are more stable (Kübler and Weizsäcker, 2005), which is predicted by QRE but is not true in the Nash equilibrium. This finding supports hypothesis (C5).

The average length of temporary cascades for each treatment is displayed in Table 6 (bottom), and the complete distributions of length are shown in Figure 1 (bottom). Average length of temporary cascades rises with the sequence length, T, when $q = 6/9$ but not when $q = 5/9$, with the difference insignificant at the 5% level in the latter case. Average length also rises with the signal precision, q, for both $T = 20$ and $T = 40$. Thus, we find strong support for (C7) but only weak support for (C6). A comparison of the entire distribution of lengths is given in the top panel of Figure 1. The table and the figure also show the Nash prediction of exactly 1 cascade per sequence, independent of q and T, and out-of-sample predictions generated by the QRE–BRF model (discussed later).

4.3. *Off-the-equilibrium-path behaviour*

Given that the vast majority (92%) of cascades are *temporary* and short in duration, and nearly all (90%) sequences in our data exhibit *multiple* cascades, an immediate conclusion is that there are many choices off the (Nash) equilibrium path. Table 2 in the Introduction characterizes a subset of these choices for the different treatments as a function of the deviating decision-maker's signal. The table shows the behaviour of what we call *cascade breakers*, since these are all terminal decisions of a temporary cascade.

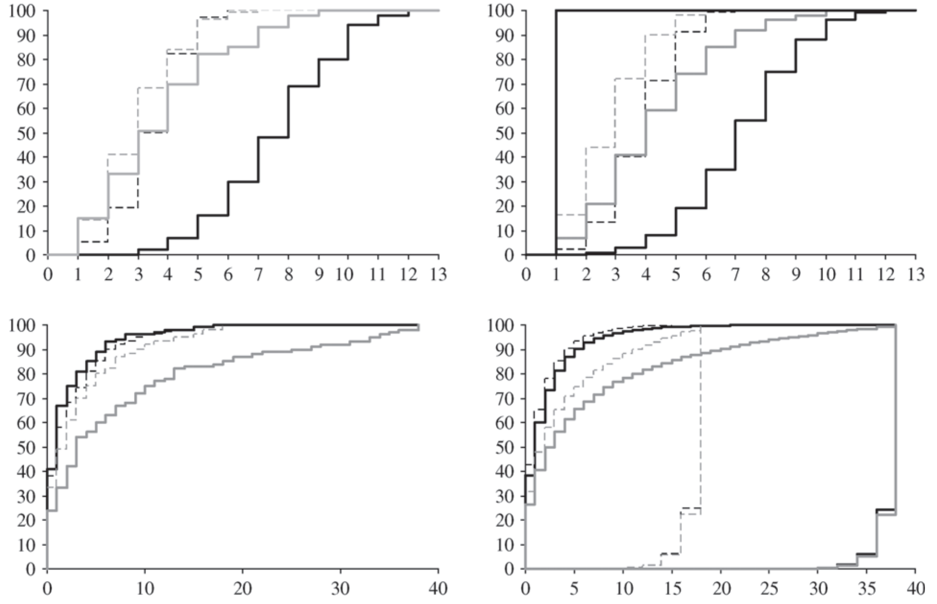

FIGURE 1

The left panels show the observed cumulative distributions of the number of cascades (top) and of cascade lengths (bottom), colour coded by treatment: dark (light) grey lines correspond to $q = 5/9$ ($q = 6/9$) and they are solid (broken) for $T = 40$ ($T = 20$). The right panels show predictions of the Nash and QRE–BRF models. In the top right panel, the solid line that jumps to 100% at 1 corresponds to Nash predictions and the other lines to the QRE–BRF predictions. In the bottom right panel, the lines that jump to 100% at $T-2$ correspond to Nash predictions and the others to QRE–BRF predictions

Over all treatments, cascades were broken a total of 1081 times. These contrary actions were *five times more likely* to be taken by subjects with contradictory signals than with confirmatory signals (898 compared with 183). In fact, if we compare the *rates* of breaking cascades for decision-makers with contradictory vs. confirmatory signals, their difference is even starker (37% compared with 6%). This supports hypothesis (SC5).

The behaviour of decision-makers immediately following a cascade breaker also plays a critical role in the dynamics. Because the first break is so informative, a second break moves beliefs close to 0·5, essentially eliminating the trend in beliefs that had developed during the cascade.

As expected, the probability of a second break by the next decision-maker is sharply increased. Approximately 75% of the decision-makers immediately following a cascade break follow their signals. A player who observes a signal consistent with the recent cascade, of course, should rationally follow the cascade, a prediction that is borne out by our data: 90% of these decision-makers follow the action corresponding to the recently broken cascade. Only 10% are *secondary deviators* who follow the recent break. Thus, they behave roughly the same as they would have if the cascade had never been broken. Those who received contradictory signals behaved much differently. Well over half (56%) of the decision-makers with contradictory signals are secondary deviators. Pooling over all treatments, they outnumber the secondary deviators with confirmatory signals by a factor of five to one (277 compared with 58). Table 7 gives a complete breakdown of the choices directly following a cascade break, by treatment.

The two key conclusions of this subsection are that play off the equilibrium path *occurs frequently* and is *highly informative*, setting the stage for self-correction. As a result, we will find that the long-run implications of the standard theory are mostly contradicted by the data.

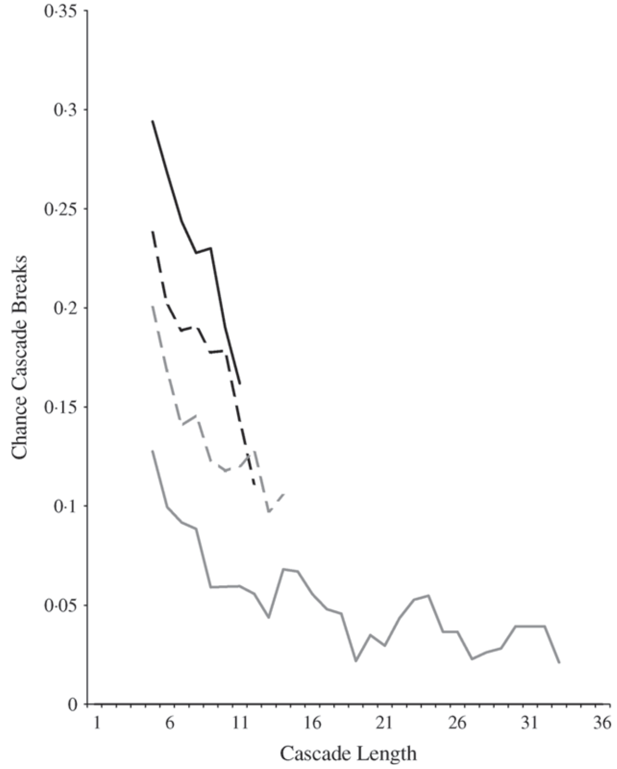

FIGURE 2

Chance of cascade breaking as a function of cascade length. The lines show five-period moving averages of the probability of a break in each of the treatments (colour coded as in Figure 1)

TABLE 7

Percentages of choices confirming/contradicting the recent cascade after a break

	$T = 20$			$T = 40$		
	Decision\Signal	Confirming	Contrary	Decision\Signal	Confirming	Contrary
$q = 5/9$	Confirming	42·8%	20·9%	Confirming	45·4%	20·3%
	Contrary	3·3%	33·0%	Contrary	8·4%	25·9%
	#Obs = 306			#Obs = 379		
	Decision\Signal	Confirming	Contrary	Decision\Signal	Confirming	Contrary
$q = 6/9$	Confirming	48·4%	14·7%	Confirming	58·2%	26·7%
	Contrary	6·3%	30·5%	Contrary	2·4%	12·7%
	#Obs = 190			#Obs = 165		

4.4. *Repeated and reversed cascades: self-correction*

Since this off-path behaviour is central to the dynamic properties of QRE (where such behaviour is actually *not* off-path) and to the resulting convergence of beliefs, our experimental design with much longer sequences allows us to better observe the kinds of complex dynamics predicted by the theory, in particular, the phenomenon of self-correction.

TABLE 8

Frequency of repeated and reversed cascades by treatment

		$q=5/9$		$q=6/9$	
		$T=20$ $M=116$	$T=40$ $M=56$	$T=20$ $M=90$	$T=40$ $M=60$
Average number repeat cascades	Our data	2·14	5·98	1·69	2·60
	QRE–BRF	2·39	5·37	1·62	2·91
	Nash	0·00	0·00	0·00	0·00
Average number reverse cascades	Our data	0·34	0·55	0·30	0·13
	QRE–BRF	0·50	0·93	0·26	0·30
	Nash	0·00	0·00	0·00	0·00

TABLE 9

Transitions between correct and incorrect cascades in our data

	$T=20$			$T=40$		
$q=5/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	92·7%	7·3%	Correct	93·6%	6·4%
	Incorrect	22·7%	77·3%	Incorrect	11·0%	89·0%
# Correct	Total = 252	Start = 65	Final = 78	Total = 237	Start = 29	Final = 34
# Incorrect	Total = 151	Start = 51	Final = 38	Total = 185	Start = 27	Final = 22
$q=6/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	91·4%	8·6%	Correct	98·7%	1·3%
	Incorrect	30·5%	69·5%	Incorrect	20·0%	80·0%
# Correct	Total = 197	Start = 66	Final = 71	Total = 186	Start = 48	Final = 52
# Incorrect	Total = 72	Start = 24	Final = 19	Total = 38	Start = 12	Final = 8

Table 8 shows the average number of repeated and reversed cascades per sequence, by treatment, and also gives theoretical expectations according to the Nash and out-of-sample QRE–BRF predictions (explained later). While such cascades are not possible in the Nash equilibrium, the observed number of reversed and repeated cascades are predicted rather well by QRE–BRF.

Table 9 shows how frequently correct and incorrect cascades repeat or reverse themselves.[27] The number of repeat cascades is increasing in T and decreasing in q, which is consistent with the QRE model.

Averaging over the four treatments shows that when a correct cascade breaks, it reverses to an incorrect one in approximately 6% of all cases (39/637). In contrast, an incorrect cascade that breaks leads to a self-corrected cascade in more than 21% of all cases (66/369). This confirms hypotheses (SC2) and (SC3).

Table 9 also lists the initial, final, and total number of correct and incorrect cascades by treatment. In all four treatments, the fraction of incorrect cascades is always lower among the final cascades compared with the initial cascades. Overall, initial cascades were incorrect nearly 35% of the time (114/322) and final cascades were incorrect only 27% of the time (87/322). This supports hypothesis (SC4).

27. The percentages listed ignore terminal cascades, since they can neither repeat nor reverse, by definition.

4.5. *Efficiency*

How frequently are actions correct? How does this change over time? And how does this change as a function of signal informativeness? These questions can be directly answered in our data by checking the proportion of correct decisions, since both the state and the action of each individual are observed in the data.

There are two important observations to note before delving into the analysis of the efficiency results. First, the probability of a correct decision, and the way that probability changes over time, will be much different for decision-makers who received correct vs. incorrect signals. Decision-makers with incorrect signals will do badly at the beginning, but will do increasingly well over time as the true state is eventually learnt. Decision-makers with correct signals will do very well at the beginning (perfectly in the Nash equilibrium). In the medium run, they will do less well on average than at the beginning because of the possibility of being in a false temporary cascade. Eventually, the public belief puts probability 1 on the correct state, so in the long run, they will do as well as at the beginning. Second, overall efficiency is highly sensitive to the specific realized sequence of individual signals and also the realized action choices (which are stochastic in the quantal response model). Since in 20 rounds there are over one million possible signal sequences (and many more signal–action sequences), our experimental data represent only a small fraction of the possible sequences. Therefore, there is a lot of sample variation.

Figure 3 shows the time dependence of decision accuracy, by treatment. The middle column displays the actual data, averaged across all experimental sequences, with the four rows each corresponding to a treatment: ($q = 6/9$, $T = 20$) top row, ($q = 6/9$, $T = 40$) second row, ($q = 5/9$, $T = 20$) third row, and ($q = 5/9$, $T = 40$) bottom row. In each graph, the thick solid black line shows the fraction of correct choices for all signals; the dashed (upper) and thinner (lower) lines display the fraction of correct choices for correct and incorrect signals, respectively.

It is useful to contrast the data with the efficiency predictions of Nash equilibrium, which are displayed in the left column of the same graph, again based on the actual signal draws in the experiment. In the Nash equilibrium, decision accuracy quickly becomes independent of signals, reflecting the formation of pure cascades where all learning stops, and all future decisions are the same.[28] The decision accuracy for (in)correct signals (rises) falls for a few rounds and then levels off. As a result, the unconditional decision accuracy increases for only a short amount of time as nearly all cascades are formed in the first five periods and never break. This contrasts sharply with the dynamics in the actual data, where unconditional decision accuracy continues to rise as the sequence of decision-makers passes through cycles of temporary cascades that break and re-form.

There is a strong signal dependence that persists throughout the experiment. The decision accuracy for incorrect signals is always less than for correct signals in the actual data *in every round t*, providing strong support for (E2). For incorrect signals, there is a clear and persistent upward trend in decision accuracy (due to information aggregation), and there is a small, early downward trend for decision-makers with correct signals, as hypothesized (the interim part of E1). For decision-makers with correct signals, this levels off and even reverses sign later, because later cascades are more likely to be correct due to the phenomenon of self-correction.

For a more formal statistical test of hypotheses E1–E3, Table 10 shows the results of a Probit regression with six independent explanatory variables: t, q, $q * t$, signal, signal$*t$, match. Signal is a dummy variable that takes on the value of 1 if the signal is correct. The variable $q * t$

28. Decision-makers in the $q = 5/9$, $T = 20$ treatment by chance drew many more correct signals in the early rounds than did decision-makers in the $q = 5/9$, $T = 40$ treatment. This is reflected in the graph of the Nash predictions, where decision accuracy remains stuck at 0·5 for the $T = 40$ treatment, but is above 0·6 for the $T = 20$ treatment.

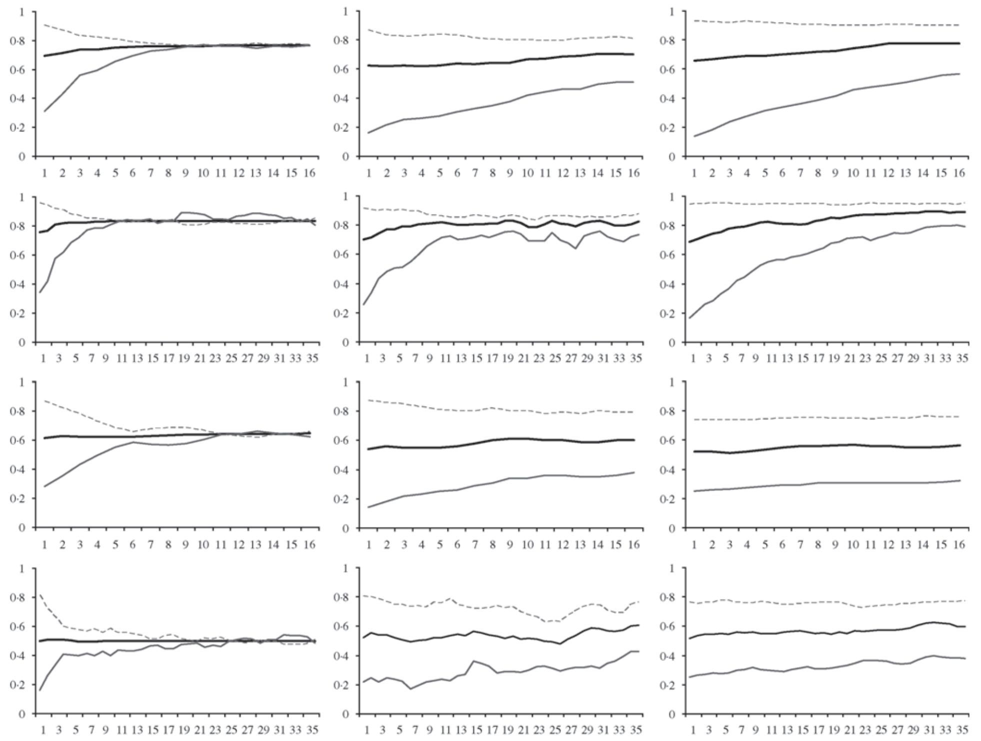

FIGURE 3

Decision accuracy along the sequence of decision-makers by treatment: ($q = 6/9$, $T = 20$) top row, ($q = 6/9$, $T = 40$) second row, ($q = 5/9$, $T = 20$) third row, and ($q = 5/9$, $T = 40$) bottom row. In each graph, the thick solid line shows the fraction of correct choices for all signals, the dashed line for correct signals, and the thin line for incorrect signals. The lines show moving averages: a point at time t represents average decision accuracy between $t-2$ and $t+2$ for $3 \leq t \leq T-2$. The left column gives Nash predictions, the middle column data, and the right column gives QRE–BRF simulations, all based on the actual signals used in the experiment

is an interaction of signal informativeness and time period,[29] which, according to hypothesis H4, should be positive. The variable (signal $* t$) is an interaction between time and signal correctness. From hypothesis H1, the effect of t on decision accuracy should be positive only for incorrect signals, with a possible small negative effect for correct signals. Match is a variable that is included to control for possible experience effects.[30] Notice that we do not include T in the regression because the theory does not predict any effect except through the variable t.

The second column of Table 10 shows the estimated coefficients with standard errors in parentheses. All coefficients have the expected sign and are statistically significant. These results deserve closer inspection for at least two reasons. First, the regression is not based on any kind of structural model of decision-making. Second, there are obvious dependencies in the data and unmodelled sources of error, including quantal response errors and variation in signal sequences. (The third and fourth columns of the table are discussed later.)

29. Here $q * t$ equals 0 if $q = 5/9$ and $q * t$ equals t if $q = 6/9$.

30. Match=1 corresponds to the first sequence in a session and ranges up to 20 or 30 depending on whether T equals 40 or 20, respectively.

TABLE 10

Probit estimation of the effects of q and t on efficiency

Dependent variable Correct choice	Data	Simulation 1	Simulation 2
Constant	−1·57(0·30)	−2·58(0·33)	−3·27(0·32)
q	1·26(0·51)	2·52(0·55)	4·51(0·53)
t	0·021(0·0025)	0·033(0·0026)	0·019(0·0025)
q^*t	0·017(0·0029)	0·016(0·0031)	0·012(0·0031)
Signal$^*t$	−0·037(0·0029)	−0·049(0·0031)	−0·027(0·0030)
Signal	1·76(0·055)	2·23(0·059)	1·71(0·057)
Match	0·0047(0·0021)	0·0077(0·0022)	−0·0015(0·0006)
# Obs	8760	8760	8760
$\log L$	−4620	−4021	−4178

4.6. *Summary of results*

Here we summarize our findings by relating them to the properties of the logit QRE discussed in Section 2.3.

- (C1) and (C2): *The occurrence of pure cascades decreases with T and increases with q.* The effect of T is obvious from comparing the different rows in Table 4. Both for $q = 5/9$ and $q = 6/9$, the percentages of pure cascades fall quickly with each successive row. Comparing columns 1 and 3 and columns 2 and 4 in Table 4 shows the effect of signal informativeness.
- (C3 and C4): *The number of cascades increases with T and decreases with q.* See Table 6 and Figure 1. Longer sequences have more cascades because they allow for more cycles of formation and collapse. These effects are barely noticeable in short sequences: AH's experiment averaged slightly more than one cascade per sequence.
- (C5): *The probability of collapse sharply decreases as a function of the duration of the cascade.* See Figure 2.
- (C6 and C7): *Cascades lengths increase with T for $q = 6/9$ and increase with q.* The effect of T can be decomposed as follows. First, and most obvious, if T is short then some cascades that would have lasted longer are interrupted at T. Second, by (C5) longer cascades are less likely to break. The two effects combined result in a fat tail of the length distribution and in a mass of cascades at $T-2$, see Table 6 and Figure 1. The effect of T is not observed in the $q = 5/9$ data, where the distributions of cascade lengths are very similar for the $T = 20$ and $T = 40$ treatments.
- (SC1): *Correct cascades last longer on average.* The observed average lengths of (correct, incorrect) cascades in the different treatments are $(2{\cdot}55, 2{\cdot}24)$ for $q = 5/9$ and $T = 20$, $(2{\cdot}08, 1{\cdot}91)$ for $q = 5/9$ and $T = 40$, $(3{\cdot}42, 2{\cdot}85)$ for $q = 6/9$ and $T = 20$, and $(8{\cdot}31, 5{\cdot}50)$ for $q = 6/9$ and $T = 40$.
- (SC2) and (SC3): *Reverse cascades are usually self-correcting, and repeat cascades are usually correct.* See Table 9. Across the four treatments, the probability that a reversed cascade is self-correcting is 63% (even though there are many more correct than incorrect cascades to reverse from). It is this feature of the dynamics that produces the full information aggregation result of Proposition 1.
- (SC4): *Later cascades are correct more frequently than earlier ones.* See Table 9, which lists the number of (in)correct cascades among initial and final cascades.
- (SC5): *Cascades are almost always broken by decision-makers with contradictory signals.* See Table 2.

- (E1): Ex ante *efficiency is increasing in t.* Efficiency is increasing in t, conditional on an incorrect signal. Efficiency is initially decreasing in t conditional on a correct signal, but this eventually reverses (see Figure 3).
- (E2): *Correct signals lead to more efficient decisions than do incorrect signals.* Again, see Figure 3.
- (E3): *More informative signals lead to more efficient decisions.* See Figure 2 and Table 10.

The final three hypotheses, (B1)–(B3), address the evolution of beliefs during a sequence and are discussed in Section 6. The next section describes the QRE estimation and our BRF model.

5. RESULTS II: ESTIMATION

We start by describing the estimation procedure for the basic logit QRE model. The only parameter is the slope of the logit response curve, which in the context of these games can be interpreted as a proxy for rationality, experience, and task performance skill. In subsequent subsections, we jointly estimate logit and other parameters, using standard maximum likelihood estimation. For comparability, we choose to normalize pay-offs in all experiments to equal 1 if a subject guesses the state correctly and 0 otherwise.[31]

Since subjects' choice behaviour depends on λ, public beliefs follow a stochastic process that depends on λ. The evolution of the public belief can be solved recursively (see equations (2.4) and (2.5)), so implicitly we can write $p_t(c_1,\ldots,c_{t-1} \mid \lambda)$. Given $\{\lambda, s_t, (c_1,\ldots,c_{t-1})\}$, the probability of observing player t choose A is

$$P(c_t = A \mid \lambda, s_t, c_1,\ldots,c_{t-1}) = \frac{1}{1+\exp(\lambda(1-2\pi_t^{s_t}(p_t(c_1,\ldots,c_{t-1} \mid \lambda))))}, \qquad (5.1)$$

and $P(c_t = B \mid \lambda, s_t, c_1,\ldots,c_{t-1}) = 1 - P(c_t = A \mid \lambda, s_t, c_1,\ldots,c_{t-1})$. Therefore, the likelihood of a particular sequence of choices, $c = (c_1,\ldots,c_T)$, given the sequence of signals is simply

$$l(c \mid \lambda) = \prod_{t=1}^{T} P(c_t \mid \lambda, s_t, c_1,\ldots,c_{t-1}).$$

Finally, assuming independence across sequences, the likelihood of observing a set of M sequences $\{c^1,\ldots,c^M\}$ is just

$$L(c^1,\ldots,c^M \mid \lambda) = \prod_{m=1}^{M} l(c^m \mid \lambda).$$

The estimation results for the logit QRE model are given in Table 11. The λ estimates for the four treatments are quite stable, and the pooled estimate is close to that estimated from the AH data.[32]

Since comparison with Nash equilibrium does not provide a particularly informative benchmark for the logit QRE, the following three subsections consider extensions and alternatives

31. Recall that in the experiment subjects received \$1 for a correct choice and \$0·10 for an incorrect choice. The difference of \$0·9 is normalized to 1 unit in the estimations. Without this normalization, the estimates of λ reported below would be multiplied by a factor of $1/0{\cdot}9$.

32. Notice that the estimated value of λ for the $(q = 5/9, T = 20)$ treatment is somewhat greater than the other three treatments. This may reflect a subject pool effect, since that treatment was the only one that used mostly Caltech students. (The reason that the design is not balanced with respect to subject pool is that Caltech's laboratory has a maximum capacity of 32 subjects.) In the estimations reported below, we found no other notable differences in parameter estimates, so subject pool seems to play a minor role in our data.

TABLE 11

Parameter estimates for the different models with standard errors in parentheses

	Our data					AH data
	$p=5/9$		$p=6/9$			$p=6/9$
	$T=20$	$T=40$	$T=20$	$T=40$	Pooled	$T=6$
# Obs	2320	2240	1800	2400	8760	270
QRE						
λ	11·36(0·42)	7·19(0·32)	4·38(0·18)	4·69(0·19)	6·12(0·14)	6·62(0·72)
$\log L$	−981·0	−1181·4	−682·0	−634·0	−3650·3	−79·0
QRE–BRF						
α	2·33(0·18)	2·97(0·36)	2·01(0·16)	1·67(0·16)	2·46(0·10)	1·51(0·19)
λ	7·07(0·45)	3·68(0·32)	3·47(0·16)	4·09(0·18)	4·23(0·11)	5·90(0·76)
$\log L$	−930·7	−1147·6	−653·0	−622·5	−3466·0	−74·5
QRNE						
λ_A	14·45(0·62)	9·82(0·49)	5·16(0·23)	4·74(0·18)	6·32(0·14)	7·93(0·92)
λ_B	4·07(0·37)	1·86(0·18)	1·86(0·18)	3·45(0·33)	4·48(0·28)	3·78(0·66)
$\log L$	−947·7	−1156·3	−660·8	−627·9	−3636·6	−74·7
QRNE–BRF						
α	3·24(0·34)	2·64(0·41)	1·82(0·24)	1·54(0·16)	2·59(0·12)	1·75(0·23)
λ_A	5·43(0·44)	4·06(0·51)	3·65(0·27)	4·19(0·20)	4·09(0·12)	5·35(0·83)
λ_B	12·56(1·87)	3·25(0·47)	2·93(0·52)	3·40(0·34)	4·92(0·33)	15·68(10·58)
$\log L$	−925·6	−1147·1	−652·5	−620·5	−3462·8	−73·3
CH						
τ	1·67(0·06)	1·24(0·04)	1·96(0·04)	2·82(0·03)	1·91(0·02)	2·20(0·22)
$\log L$	−964·0	−1180·4	−694·3	−656·6	−3636·1	−77·1
CH–BRF						
τ	1·98(0·08)	1·40(0·07)	2·03(0·08)	2·87(0·08)	2·06(0·04)	2·45(0·27)
α	1·49(0·10)	0·97(0·12)	1·54(0·06)	1·65(0·07)	1·34(0·04)	0·99(0·07)
$\log L$	−925·7	−1167·2	−683·1	−650·0	−3537·5	−74·3
QRE–CH						
τ	2·00(0·11)	1·67(0·14)	2·52(0·20)	3·63(0·23)	2·54(0·08)	2·44(0·25)
λ	26·45(3·31)	16·99(2·33)	7·07(0·86)	6·23(0·51)	13·12(0·75)	28·34(14·16)
$\log L$	−940·7	−1162·1	−672·3	−650·0	−3486·3	−74·3
QRE–CH–BRF						
α	1·91(0·16)	2·67(0·27)	1·90(0·16)	1·50(0·15)	1·81(0·08)	1·36(0·32)
τ	2·56(0·23)	3·23(0·73)	3·70(0·73)	3·80(0·28)	2·90(0·10)	3·54(2·28)
λ	12·77(1·80)	4·50(0·73)	3·97(0·47)	5·21(0·45)	7·69(0·50)	7·47(3·96)
$\log L$	−911·9	−1144·3	−652·0	−616·1	−3411·3	−73·9

to the basic model. This allows us to assess the extent to which the choice behaviour in our data is explained by quantal response type decision errors as opposed to other sources, such as non-Bayesian updating and non-rational expectations.[33] Using parametric specifications, we measure the extent of certain types of these biases in the data.

5.1. *Incorporating the base rate fallacy*

In their seminal article, Kahneman and Tversky (1973) present experimental evidence showing that individuals' behaviour is often at odds with Bayesian updating. As noted in the introduction, there is considerable evidence in the literature on cascade experiments that players are

33. Huck and Oechssler (2000) find evidence of non-Bayesian updating in a similar context.

non-Bayesian. We explore two of these here. First, a particularly prevalent judgement bias is the BRF, or as Camerer (1995, pp. 597–601) more accurately calls it, "base rate neglect". In the context of our social learning model, the BRF would imply that agents weight the public prior too little relative to their own signal. Because past experiments have been suggestive of these effects, we construct an analytical model of this and estimate it using the error structure of the logit equilibrium.[34] We formalize this idea as a non-Bayesian updating process in which the private signal is counted by the decision-maker as α signals, where $\alpha \in (0,\infty)$.[35] Rational agents correspond to $\alpha = 1$, while agents have progressively more severe BRF as α increases above 1.[36]

While agents overweight their private signals, we retain the assumption that they have rational expectations about others' behaviour. This implicitly assumes that α is common knowledge (as well as λ). The updating rules in (2.1) and (2.2) now become

$$\pi_t^a(p_t \mid \alpha) = \frac{q^\alpha\, p_t}{q^\alpha\, p_t + (1-q)^\alpha (1-p_t)} \tag{5.2}$$

and

$$\pi_t^b(p_t \mid \alpha) = \frac{(1-q)^\alpha\, p_t}{(1-q)^\alpha\, p_t + q^\alpha (1-p_t)}, \tag{5.3}$$

respectively. From these equations, it is easy to see that for $\alpha > 1$ the learning process is faster as agents' choices depend more on their own signals, in the sense that the expected change in posterior is greater. Indeed, when α tends to infinity and decision-making is perfectly rational ($\lambda = \infty$), the Bayesian updating process on public beliefs would be equivalent to having full information about previous signals.

The public belief, p_t, in equations (5.2) and (5.3) is derived recursively using (2.3)–(2.5). In particular, this means that subjects not only overweight signals, but also take into account that other subjects overweight signals too, and the public belief is updated accordingly. Thus, for $\alpha > 1$, the public belief is updated more quickly than in the pure Bayesian model.

The estimation results for the QRE–BRF model are reported in the second panel of Table 11. For all treatments, the BRF parameter, α, is significantly greater than 1.[37] To test for significance, we can simply compare the loglikelihood of the QRE–BRF model with that of the constrained model (with $\alpha = 1$) in the top panel. Obviously, the BRF parameter is highly significant.[38] Furthermore, the constrained model yields a significantly (at the 0·01 level) higher estimate of λ for all treatments.

There is at least one alternative interpretation to the finding that subjects respond too strongly to their signal. By doing so, they are giving better information to later decision-makers, which increases efficiency and raises the expected utility of the other players in the game. Evidence from experiments on public goods and some game theory experiments suggest some degree of altruism by subjects. Conceivably, what we are calling a base rate neglect (or overweighting of signals) may simply be a manifestation of altruistic behaviour. However, there is some counter evidence that suggests this is probably not the case. First, if altruism is the motivating force, one

34. Some error structure is required for the estimation because the α-BRF model is deterministic.

35. This could also be loosely interpreted as a parametric model of "overconfidence" bias in the sense of Griffin and Tversky (1992). Nöth and Weber (2003) and Kariv (2005) use this terminology.

36. Values of $\alpha < 1$ correspond to underweighting the signal, or "conservatism" bias, as discussed in Edwards (1968) and Camerer (1995, pp. 601–602). Although this latter kind of bias has less support in the experimental literature, it is sufficiently plausible that we choose not to assume it away.

37. Similar results are reported by Çelen and Kariv (2004).

38. For the pooled data, the difference in loglikelihoods is nearly 200. A simple t-test also rejects the hypothesis that $\alpha = 1$, with a t-statistic of 14·6. Tests conducted for the AH data also reject the constrained model, with a slightly lower estimate of α.

would expect higher estimates of α for $T = 40$ than for $T = 20$. This is not the case. Second, one would expect less overweighting of signals in later periods than in earlier periods. We tested for this and found no significant effect. Therefore, our interpretation is not that subjects are behaving altruistically, but rather the source of the distortion is a probability judgement fallacy.

5.2. *Incorporating non-rational expectations*

Rather than simply overweighting private information relative to the base rate (public belief), it is possible that players update incorrectly because they do not have rational expectations about the driving parameters of the model. The QRE model implicitly assumes that λ is constant across the population and common knowledge. In particular, if players believed other players' λ were lower than it truly was, then beliefs, and hence choice dynamics, would be qualitatively different from the previous models. For example, if choices are believed to be generated by a noisier process, players draw weaker inferences about predecessors' signals from observing their choices, while the opposite would be true if choices were believed to be closer to perfect best response. Accordingly, we consider a model that allows for separate belief and action precision parameters, as proposed by Weizsäcker (2003). These different parameters are labelled λ_a (action lambda) and λ_b (belief lambda). That is, players choice probabilities follow the logit choice function with parameter λ_a but they believe that other players' choice probabilities follow a logit choice function with parameter λ_b.[39] We call this the *non-rational expectations model* or QRNE model.

The estimation results for the QRNE model are also given in Table 11. While this two-parameter model performs significantly better than the QRE model, the increase in likelihood is smaller in magnitude than the increase in QRE–BRF relative to the simple QRE.

An advantage of using the QRE model is that we can explore the relative importance of different biases, by nesting them in the same model. In this case, we can see whether the BRF bias is more or less important in our data compared with updating failures due to irrational expectations about other players' error probabilities. When BRF and QRNE are combined so that the model includes both sources of bias, the action and belief λ are virtually identical when estimated from the pooled data, and the increase in likelihood from the QRE–BRF model is barely significant.

5.3. *Incorporating cognitive heterogeneity*

It is instructive to consider other models with non-quantal response sources of noise which could also potentially explain our data. This helps to check the validity of our basic story for choice behaviour, in light of the observation that the Nash equilibrium does not provide a way to challenge any of the predictions of QRE. One natural question to ask is where the source of scatter (error) in our data is really coming from. In QRE, it is assumed to come entirely from pay-off-monotone choice errors, and this behaviour is assumed to be homogeneous across the population. An alternative possibility is that this apparent noise in the data is due to some kind of underlying heterogeneity. We explore one possible model of heterogeneity in this section.

Although there are many options, a natural first step is to suppose that some players behave completely randomly, while other players optimize against such behaviour. Camerer, Ho and Chong (2004) extend this idea to allow for multiple levels of sophistication.[40] Specifically, level 0 players are random, and all other players use optimal strategies given their beliefs. Level 1 players believe all the other players are level 0, level 2 players believe all others are a mixture of level 0 and level 1, and so forth. The proportion of level *k* players in the population is given by a Poisson

39. See Kübler and Weizsäcker (2004) for a more extensive discussion of this model.

40. Stahl and Wilson (1995) explored a related but different model with levels of sophistication to study behaviour in experimental games. See Camerer *et al.* (2004) for a discussion of the differences between the two models.

distribution with parameter τ. That is, the probability of a level k player in the population, given the Poisson parameter τ is equal to $\frac{\tau^k e^\tau}{k!}$. Thus, for example, if $\tau = 1{\cdot}5$ then the distribution of types $0, 1, 2, 3, \ldots$ is equal to $(0{\cdot}22, 0{\cdot}33, 0{\cdot}25, 0{\cdot}125, \ldots)$. Players are assumed to have truncated rational expectations, that is, level k players believe all other players are a mixture of levels less than k, with their relative probabilities given by the true Poisson distribution. Thus, again using the example of $\tau = 1{\cdot}5$, 22% of the players are simply randomizing, 33% are optimizing assuming they face only rational players, 25% are optimizing assuming they face a mixture of level 0 and level 1 in proportions equal to $\frac{2}{5}$ and $\frac{3}{5}$, and so forth. Therefore, assuming the model is correct, very high-level types have very accurate beliefs about the distribution of types. This implies they also have accurate beliefs about the distribution of strategies in the population, and therefore they are almost optimizing. This is called the *CH model*.

The presence of randomizing level 0 players will lead higher level players to implicitly discount the information contained in the choices of their predecessors. In this way, the CH model can pick up some of the same features of the data as QRE. To see this, it is instructive to look at exactly what the behaviour of the lowest three types are. Level 0's of course are just random. Level 1's simply follow their own signal, since they assume there is no useful information in the observations of previous decision-makers (they are believed to be totally random). Level 2's optimize against a mixture of such players, so they simply act as if each previous decision is a noisy (but informative) signal about the signals of earlier decision-makers. Again using the example of $\tau = 1{\cdot}5$, if the second mover is a level 2 player and observed the first player choose A choices, they believe that the first mover received an A signal with probability $\frac{4}{5}$ and a B signal with probability $\frac{1}{5}$. Thus, such a player's posterior on state A will be less than q. That is, level 2's have dampened updating, but also note that level 2's will reach a point quickly where they no longer follow their own signal. In the example above, they will act exactly like a player following the Nash equilibrium and will herd after one of the decisions has been chosen two more times than the other decision. (This is independent of q.) Furthermore, like QRE, CH is "complete" in the sense that it is consistent with any sequence of choices and signals. Hence we can obtain maximum likelihood estimates of the parameter τ via the same methodology, without using QRE, see Table 11.

We also estimate CH together with QRE to allow for further comparison with QRE. To do so, we suppose that each agent is assigned a level k in the hierarchy, as in CH, but quantal responds to her beliefs, as in QRE. Thus, QRE–CH is a model parameterized by (τ, λ), which are assumed to be common knowledge. All three models (CH, QRE, and QRE–CH) are then re-estimated with the inclusion of the BRF parameter α, to allow for the possibility of over- or underweighting of private information in each case, see Table 11. Note that the estimates for the combined QRE–CH–BRF model are stable across data-sets and generally result in the highest likelihood. All three are significant factors, based on likelihood ratio tests, and leaving out any one of these factors changes the magnitudes of the other estimates.

A surprising finding is that the estimate for τ is larger in magnitude than has been typically found in other settings. Camerer *et al.* (2004) report estimates in the range of 1·5–2·5, while our estimate in the combined model is 2·9 (with a standard error of 0·10). This appears to be due to an interaction between τ, λ, and α. The estimate of τ in the pure CH model is 1·9, and its estimate in the QRE–CH model (without BRF) is 2·5. Combining QRE and CH also leads to substantially larger estimates of λ. The reason for this is that both are rationality parameters that substitute for each other. The 0 types in the CH model absorb a lot of the randomness in the QRE model. In other words, the random behaviour that can only be explained by 0 types in the CH model is also explained by quantal response randomness. Hence, we find relatively low values of either parameter if the models are estimated separately, but both increase significantly when the models are combined.

5.4. *Implications of estimates for the data*

The QRE–BRF model is simple and intuitively appealing and we use it to create simulated data for comparisons with the actual data.[41] For each of the four treatments, we used the data from the *other* three treatments to obtain out-of-sample estimates for λ and α. We then applied the out-of-sample estimates to the signals realized in the experiment to obtain simulated choices for the treatment. Based on this simulated data-set, we computed descriptive statistics about the numbers, lengths, and types of cascades: pure and temporary, repeated and reversed, self-correcting, etc. These are reported in the right two columns of Tables 4 and 5 (pure and temporary cascades, respectively) and the second and fifth rows of Tables 6 and 8 (numbers/lengths of cascades and reversals, respectively). Because the simulations were constructed using out-of-sample estimates of λ and α, they represent out-of-sample predictions of the properties of cascades in our data, which makes a comparison with the actual data meaningful. Indeed, the match with the actual data is quite remarkable.

We are also able to construct out-of-sample simulated efficiency dynamics in the same way for each of the four treatments, again using the actual sample draws. These are displayed in the four charts in the right-hand column of Figure 3. Again, it reproduced the patterns observed in the data.

To check the robustness of our findings and to check it against the theoretical model, we generated two simulated data-sets based on the QRE–BRF model, using the pooled estimates $\lambda = 4{\cdot}23$ and $\alpha = 2{\cdot}46$. The first of these simulations uses the same signal sequences as in the laboratory experiment, but decisions are generated by the QRE–BRF model. The second simulation uses a completely new draw of signal sequences. The Probit estimations based on the simulated data-sets are reported in columns 3 and 4 of Table 10. While there are some small differences in magnitude, all coefficients of theoretical interest are significant with the correct sign.[42] Note that the loglikelihoods for the simulated data are higher than for the real data. This is likely caused by the fact that the simulations assume homogeneous agents, while we would expect some heterogeneity to be present in the laboratory data.

6. RESULTS III: ESTIMATED BELIEF TRAJECTORIES

We use belief estimates generated from the QRE–BRF model to examine both the informational efficiency and to address hypotheses about the evolution of beliefs (B1–B3). How well is the information from private signals aggregated? How high is the public belief on the correct alternative after a sequence of decisions? How does this vary with our treatment variables, q and T?

6.1. *Informational efficiency*

As shown in Proposition 1, in a QRE the public belief about the correct alternative increases on average with t and converges to 1 as T approaches infinity. The convergence is slower for the $q = 5/9$ treatments than for the $q = 6/9$ treatments. Of course, in any finite sequence, information cannot possibly reveal the correct alternative because of a combination of noise in the signal

41. The QRE–CH–BRF model would have been an alternative model for simulation, but the additional randomness of 0-level types would have necessitated many more simulated sequences. Because the fit improvement over QRE–BRF is negligible, we decided to use the simpler QRE–BRF model for our simulations.

42. The only notable difference is the experience variable, which is not significant in the simulation using a new batch of signal sequences, suggesting that its significance was spurious, due to more favourable order of signals in later matches. (Indeed, there is no reason that experience should have had a significant effect in the first simulation.) In any case, the magnitude of the experience effects, to the extent they may possibly not be spurious, is negligible.

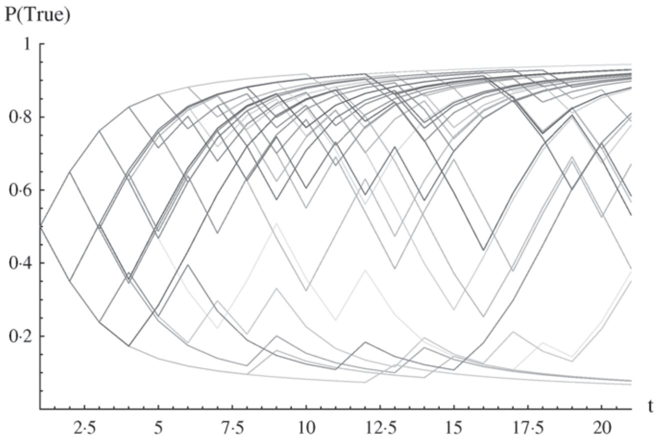

FIGURE 4

Estimated belief trajectories using the QRE–BRF model for all sequences in one of the ($q = 6/9$, $T = 20$) sessions

generation process and noise in the decision-making process. Moreover, this noise in signal generation is compounded by strategic considerations that affect the social learning process.

Although we do not observe beliefs directly, we can use the theoretical QRE–BRF model together with the observed choice data to obtain estimated public belief paths.[43] This is done for every sequence in the experiment. Using the pooled estimates, $\lambda = 4{\cdot}23$ and $\alpha = 2{\cdot}46$, each sequence of action choices implies a unique public belief. This is illustrated in Figure 4, which shows the belief paths for all sequences in one of the $q = 6/9$ and $T = 20$ sessions. The belief trajectories for other sessions exhibit similar features. Here, the horizontal axis represents the sequence of decisions and the vertical axis the belief about the correct alternative. Each upward tick in the belief paths corresponds to a correct choice and each downward tick to an incorrect choice. Theoretically, for long enough sequences, the belief paths for almost all sequences should converge to 1.

The simplest way to test Hypotheses (B1)–(B3) is to average the public belief about the correct alternative across all sequences for a given treatment. This produces the four curves in the upper left panel of Figure 5. The other three panels depict simulated average beliefs using the QRE–BRF model (upper right panel), the Nash model (lower left panel), and a model where agents simply follow their private signals (lower right panel). Notice that the beliefs implied by the QRE–BRF model resemble those estimated from the actual choices, while the other two models produce very different belief paths. Furthermore, the curves in the upper left panel are obviously consistent with the theoretical hypotheses (B1)–(B3).[44]

The comparison between the different q treatments is a weak test since the paths are constructed using the theoretical model. That is, even if the sequences of signals and decisions were exactly the same for all sequences in $q = 6/9$ and $q = 5/9$ session, the $q = 6/9$ curves necessarily would lie strictly above the $q = 5/9$ curves. That said, the ordering also reflects a salient difference between our $q = 5/9$ and $q = 6/9$ data, namely that cascades fall apart more quickly and are more often incorrect in the $q = 5/9$ data than in the $q = 6/9$ data (see Tables 5–8 of

43. Dominitz and Hung (2004) report a social learning experiment using a belief elicitation procedure.

44. The Nash predictions in the lower leftmost panel show that the difference between the two $q = 6/9$ treatments is caused by the particular signals drawn in these treatments.

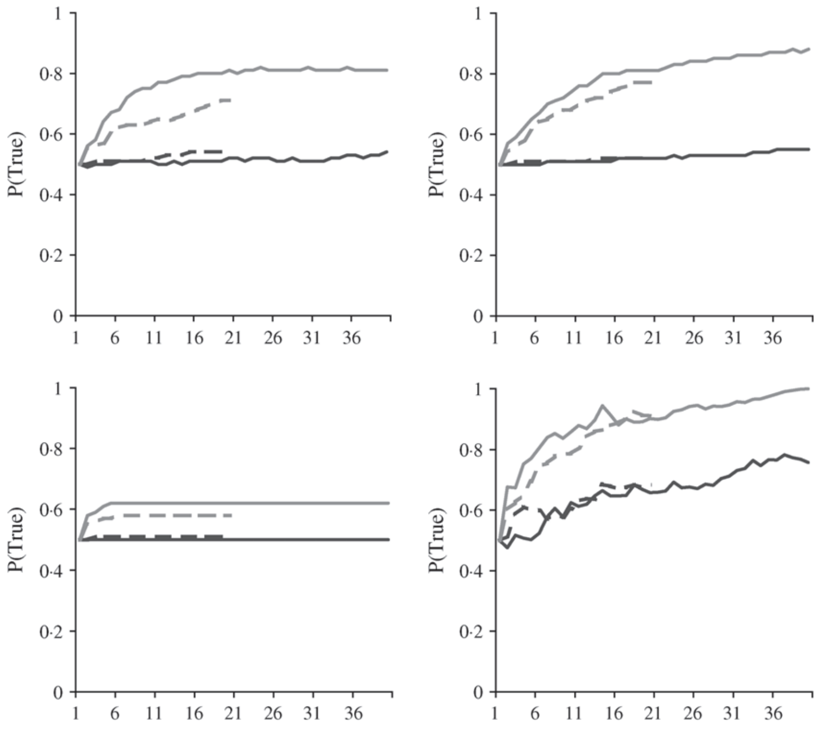

FIGURE 5

Estimated public beliefs about the true state by treatment (colour coded as in Figure 1) averaged over all sessions. All four panels are based on the sequence of signals employed in the experiment. Furthermore, (i) in the upper left panel, beliefs are based on observed decisions using the estimated parameters of the QRE–BRF model, (ii) in the upper right panel, beliefs are based on the average of 100 QRE–BRF simulations of decisions, (iii) in the lower left panel, beliefs are based on Nash decisions, and (iv) in the lower right panel, beliefs are based on fully revealing decisions, that is, when choices are based on private information only

the previous section). However, that the curves are increasing in t is *not* an artefact of the construction, but simply reflects the fact that there are more good cascades and fewer bad cascades towards the end of a sequence than towards the beginning. In summary, we find strong support for hypotheses (B1), (B2), and (B3).

Finally, it is natural to ask whether efficiency is higher under the QRE–BRF model than it is under the standard Nash model. Information is aggregated better under QRE–BRF (see Proposition 1), but decision-making is worse in this case as subjects are prone to errors. Figure 5 shows that efficiency levels are increasing with time under the QRE–BRF model throughout the duration of the experiment. In fact, in the long run as T grows large, beliefs in the QRE–BRF model converge to the true state so that private beliefs and public coincide, independent of signals. Using the pooled data to estimate the parameters of the QRE–BRF model, we can compute the asymptotic decision accuracy: 0·99, that is, almost full allocative efficiency is achieved in this limit.

7. CONCLUSION

This paper reports the results of an information cascade experiment with two novel features: longer sequences of decisions and systematic variation of signal informativeness. According to standard game theory, neither of these treatments should be interesting and neither should produce significantly different results. We find, however, that both of these treatment effects are strong and significant, with important implications for social learning, information aggregation, and efficiency.

The longer sequences have several interesting features. First, there is almost a complete absence of pure cascades, a proliferation of temporary cascades, including many repeated, reversed, and self-correcting cascades. Standard theory predicts that longer sequences will have more permanent cascades, and that temporary, repeated, reversed, and self-corrected cascades never occur. Relatively uninformative signals lead to less stable dynamics, in the sense that cascades are much shorter, more frequent, and reverse more often. These subtle but important features of the dynamics are impossible to detect in the short sequences employed in previous experiments.

To explain the observed features of the dynamics and the dependence on signal informativeness, we consider the logit QRE. In addition, we apply QRE as a structural model to estimate base rate neglect and to test for heterogeneity in levels of rationality. We find both to be significant factors in observed behaviour. In particular, subjects tend to overweight their signals, or, alternatively, underweight the public prior generated by past publicly observed choices.

Our experimental results confirm a wide range of hypotheses about the number and frequency of different kinds of cascades, efficiency, and belief dynamics. Most of these hypotheses follow logically from the informativeness of signals and the basic property of QRE: deviations from rationality occur, their likelihood is inversely related to their cost, and decision-makers correctly believe that other decision-makers behave this way. In the context of information cascades, this implies that cascade breakers more often than not hold contrary signals, and, hence, that deviations from cascades can be highly informative. Learning continues in a QRE even after a cascade forms or breaks, and temporary, repeated, reversed, and self-correcting cascades arise as equilibrium phenomena. While standard cascade theory predicts that learning ceases after a few initial decisions, our data show that information is continuously being aggregated, providing evidence for the QRE prediction that for long enough sequences public beliefs would be approximately correct.

APPENDIX: PROOF OF PROPOSITION 1

Proofs of (i) *and* (ii). The proof of (i) is by induction. Recall that $p_1 = \frac{1}{2}$, so we only need to show that $0 < p_t < 1$ implies $0 < p_t^- < p_t < p_t^+ < 1$. Equation (2.4) can be expanded as

$$p_t^+ = \frac{qp_t(1-F_\lambda(1-2\pi_t^a)) + (1-q)p_t(1-F_\lambda(1-2\pi_t^b))}{(qp_t+(1-q)(1-p_t))(1-F_\lambda(1-2\pi_t^a)) + ((1-q)p_t + q(1-p_t))(1-F_\lambda(1-2\pi_t^b))}, \tag{A.1}$$

with $1 > \pi_t^a > \pi_t^b > 0$ defined in (2.1) and (2.2), and $F_\lambda(x) = 1/(1+\exp(-\lambda x))$ the logistic distribution with parameter λ and support $(-\infty, \infty)$. Since $\frac{1}{2} < q < 1$ and $0 < p_t < 1$ by assumption, the denominator exceeds the numerator: $p_t^+ < 1$. A direct computation shows

$$p_t^+ - p_t = \frac{p_t(1-p_t)(2q-1)(F_\lambda(1-2\pi_t^b) - F_\lambda(1-2\pi_t^a))}{(qp_t+(1-q)(1-p_t))(1-F_\lambda(1-2\pi_t^a)) + ((1-q)p_t + q(1-p_t))(1-F_\lambda(1-2\pi_t^b))}, \tag{A.2}$$

which is strictly positive because $\pi_t^a > \pi_t^b$. The proof that $0 < p_t^- < p_t$ is similar. ‖

Proofs of (iii) *and* (iv). Let $\ell_t = (1-p_t)/p_t$ denote the likelihood ratio that B is correct. For all $t \in \mathcal{T}$ we have

$$E(\ell_{t+1} \mid \omega = A, \ell_t) = \ell_t, \tag{A.3}$$

that is, the likelihood ratio constitutes a martingale, a basic property of Bayesian updating. Note that p_t is a strictly convex transformation of the likelihood ratio ($p_t = (\ell_t + 1)^{-1}$), so

$$E(p_{t+1} \mid \omega = A, p_t) = E((\ell_{t+1}+1)^{-1} \mid \omega = A, \ell_t) > (E(\ell_{t+1}+1 \mid \omega = A, \ell_t))^{-1} = p_t, \tag{A.4}$$

by Jensen's inequality and the fact that $\ell_t^+ \neq \ell_t^-$, see (ii). We sketch the proof of (iv). See Goeree, Palfrey and Rogers (2006) for proof details, and Smith and Sørensen (2000) for a similar argument if there are continuous signals with unbounded beliefs. First, limit points of the stochastic belief process $\{p_t\}_{t=1,2,\ldots}$ have to be invariant under the belief updating process. But (ii) implies that $p_{t+1} \neq p_t$ when $p_t \neq \{0,1\}$, so the only invariant points are 0 and 1. Next, the martingale convergence theorem implies that ℓ_t converges almost surely to a limit random variable ℓ_∞ with finite expectation. Hence, $\ell_\infty < \infty$ with probability 1, which implies that $p_\infty > 0$ with probability 1 and p_t thus converges to 1 almost surely. ‖

Acknowledgements. Financial support from the National Science Foundation (SES-0551014 and SES-0079301) and the Alfred P. Sloan Foundation is gratefully acknowledged. The theory and experimental design was partially completed, and pilot experiments were conducted in collaboration with Richard McKelvey, who died in April 2002. He is not responsible for any errors in the paper. We acknowledge helpful comments from Boğacen Çelen, Terry Sovinsky, three anonymous referees, the managing editor, seminar participants at GREQAM, Harvard University, Johns Hopkins University, NYU, Penn State University, Princeton University, UCLA, Universitat Autonoma de Barcelona, University of Edinburgh, Washington University, the 2003 annual meeting of ESA in Pittsburgh, the 2003 Malaga Workshop on Social Choice and Welfare Economics, the 2003 SAET meetings in Rhodos, the 2003 ESSET meetings in Gerzensee, the 2004 PIER conference on Political Economy, and the 2004 Summer Festival on Game Theory at Stony Brook.

REFERENCES

ANDERSON, L. (2001), "Payoff Effects in Information Cascade Experiments", *Economic Inquiry*, **39** (4), 609–615.

ANDERSON, L. and HOLT, C. (1997), "Information Cascades in the Laboratory", *American Economic Review*, **87** (5), 847–862.

BANERJEE, A. (1992), "A Simple Model of Herd Behavior", *Quarterly Journal of Economics*, **107**, 797–817.

BIKHCHANDANI, S., HIRSHLEIFER, D. and WELCH, I. (1992), "A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades", *Journal of Political Economy*, **100** (October), 992–1026.

CAMERER, C. (1995), "Individual Decision Making", in J. Kagel and A. Roth (eds.) *The Handbook of Experimental Economics* (Princeton: Princeton University Press) 587–703.

CAMERER, C., HO, T. and CHONG, J. (2004), "A Cognitive Hierarchy Model of Behavior in Games", *Quarterly Journal of Economics*, **119** (3), 861–898.

ÇELEN, B. and KARIV, S. (2004), "Distinguishing Informational Cascades from Herd Behavior in the Laboratory", *American Economic Review*, **94** (3), 484–497.

ÇELEN, B., KARIV, S. and SCHOTTER, A. (2005), "An Experimental Test of Advice and Social Learning" (Working Paper, New York University).

CHOI, S., GALE, D. and KARIV, S. (2005), "Learning in Networks: An Experimental Study" (Working Paper, New York University).

CIPRIANI, M. and GUARINO, A. (2005), "Herd Behavior in a Laboratory Financial Market", *American Economic Review*, **95** (5), 1427–1443.

DOMINITZ, J. and HUNG, A. (2004), "Homogeneous Actions and Heterogeneous Beliefs: Experimental Evidence on the Formation of Information Cascades" (Working Paper, Carnegie Mellon University).

DREHMANN, M., OECHSSLER, J. and ROIDER, S. (2005), "Herding and Contrarian Behavior in Financial Markets: An Internet Experiment", *American Economic Review*, **95** (5), 1403–1426.

EDWARDS, W. (1968), "Conservatism in Human Information Processing", in B. Kleinmuntz (ed.) *Formal Representation of Human Judgement* (New York: Wiley) 17–52.

GOEREE, J., HOLT, C. and PALFREY, T. (2005), "Regular Quantal Response Equilibrium", *Experimental Economics*, **8** (4), 347–367.

GOEREE, J., PALFREY, T. and ROGERS, B. (2006), "Social Learning with Private and Common Values", *Economic Theory*, **28** (2), 245–264.

GRIFFIN, D. and TVERSKY, A. (1992), "The Weighting of Evidence and the Determinants of Confidence", *Cognitive Psychology*, **24** (3), 411–435.

HUCK, S. and OECHSSLER, J. (2000), "Informational Cascades in the Laboratory: Do They Occur for the Right Reasons?", *Journal of Economic Psychology*, **21** (6), 661–671.

HUNG, A. and PLOTT, C. (2001), "Information Cascades: Replication and an Extension to Majority Rule and Conformity-Rewarding Institutions", *American Economic Review*, **91** (5), 1508–1520.

KAHNEMAN, D. and TVERSKY, A. (1973), "On the Psychology of Prediction", *Psychological Review*, **80** (4), 237–251.

KARIV, S. (2005), "Overconfidence and Information Cascades" (Working Paper, University of California, Berkeley, CA).

KÜBLER, D. and WEIZSÄCKER, G. (2004), "Limited Depth of Reasoning and Failure of Cascade Formation in the Laboratory", *Review of Economic Studies*, **71** (2), 425–441.

KÜBLER, D. and WEIZSÄCKER, G. (2005), "Are Longer Cascades More Stable?", *Journal of the European Economic Association (Papers and Proceedings)*, **3** (2–3), 330–339.

MCKELVEY, R. and PALFREY, T. (1995), "Quantal Response Equilibria in Normal Form Games", *Games and Economic Behavior*, **10**, 6–38.

McKELVEY, R. and PALFREY, T. (1998), "Quantal Response Equilibria in Extensive Form Games", *Experimental Economics*, **1** (1), 9–41.

NÖTH, M. and WEBER, M. (2003), "Information Aggregation with Random Ordering: Cascades and Overconfidence", *Economic Journal*, **113**, 166–189.

OBERHAMMER, C. and STIEHLER, A. (2003), "Does Cascade Behavior Reflect Bayesian Updating: An Experimental Study" (Working Paper, Humboldt University).

SMITH, L. and SØRENSEN, P. (2000), "Pathological Outcomes of Observational Learning", *Econometrica*, **68**, 371–398.

STAHL, D. and WILSON, P. (1995), "On Players' Models of Other Players: Theory and Experimental Evidence", *Games and Economic Behavior*, **10**, 218–254.

WEIZSÄCKER, G. (2003), "Ignoring the Rationality of Others: Evidence from Experimental Normal Form Games", *Games and Economic Behavior*, **44**, 145–171.